

Appendix F: Effective Sample Sizes From Bootstrapping

William Stockhausen

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1 Introduction

One of the candidate models (22.02) presented to the Crab Plan Team (CPT) at its May, 2022 for the 2022 Tanner crab assessment meeting used effective sample sizes derived from a two-stage bootstrapping approach (Stewart and Hamel, 2014) as input sample sizes for NMFS EBS survey size compositions. The CPT requested that a version of 22.02 that incorporated aspects of other candidate models (see the main text for details) be included in the suite of candidate models presented at its September, 2022 meeting. The resulting model is denoted 22.08 in the main text, and this appendix provides a comparison between the “default” values used as sample sizes and those developed for 22.08 using bootstrapping.

2 Methods

A size composition is a set of numbers $\{n_z\}$ that describe the number of individuals in a sample that fall into a set of size bins $\{z\}$. If the sample size is N , then $\sum_z n_z = N$ and the size composition can be described in terms of the probabilities $p_z = \frac{n_z}{N}$ and the sample size (number of individuals measured) N . If the size composition is assumed to follow a multinomial distribution with probabilities p_z , then the variance of the number of individuals falling into size bin z , $V[n_z]$, is given by

$$V[n_z] = N \cdot p_z (1 - p_z) \tag{1}$$

But recognizing that $V[n_z] = V[N \cdot p_z] = N^2 \cdot V[p_z]$, this can be substituted into the lefthand side of (1), which can then be summed over z on both sides and solved for N to yield

$$\tilde{N} = \frac{\sum_z \cdot p_z (1 - p_z)}{\sum_z V[p_z]} \quad (2)$$

This equation holds for size compositions that follow a multinomial distribution, i.e., where the probability of an individual in the population being sampled is independent of any other individual being sampled, but this requirement is rarely met in fishery data (see references in Stewart and Hamel, 2014). Instead, it provides a means of estimating what sample size (the “effective” sample size) would have generated size compositions with similar statistical characteristics if the population had been sampled in an independent manner consistent with the multinomial distribution.

The approach used here, as in Stewart and Hamel (2014), was to use the observed size composition proportions from the annual NMFS EBS shelf survey for a given year as p_z and to resample the observed data in a manner consistent with the survey design to yield a “bootstrapped” size composition with proportions b_z . For each survey, hauls were first randomly-selected with replacement within the standard survey strata used for Tanner crab. For each selected haul, individuals of the sex and maturity category of interest were then randomly-selected with replacement from the haul data. The number of hauls selected within a stratum, and the number of individuals selected within a haul, were the same as in the associated survey. The survey-level size composition, using 5-mm CW size bins from 25-180 mm CW, was then calculated using the standard design-based approach. This two-stage resampling provided a single bootstrap replicate of the size composition. For each bootstrap replicate, $V[p_z]$ was estimated using $(p_z - b_z)^2$ for all size bins, then the estimated effective N, N_{eff} was calculated using

$$\tilde{N}_{eff} = \frac{\sum_z \cdot p_z (1 - p_z)}{\sum_z (p_z - b_z)^2} \quad (3)$$

(equivalent to eq. 6 in Stewart and Hamel, 2014). This was repeated 400 times for each survey to generate the arithmetic and harmonic means for N_{eff} for each survey.

3 Results

The range of bootstrapped size compositions are shown by sex and time period in Figures ?? to ?. Time series of the number of crabs measured, the input sample sizes used in models other than 22.08, the number of stations at which crab were caught, and the arithmetic and harmonic means of the estimated effective N’s are shown in Tables 1 and 2 and plotted as time series in Figures ?? to ?. In general, the numbers of measured crab are far larger than the estimated effective N’s, while these are generally somewhat larger than the default sample sizes used in models other than 22.08.

4 References

- McAllister, M.K., and Ianelli, J.N. 1997. Bayesian stock assessment using catch-age data and the sampling - importance resampling algorithm. *Can. J. Fish. Aquat. Sci.* 54(2): 284–300. doi:10.1139/f96-285.
- Stewart, I.J., and O.S. Hamel. 2014. Bootstrapping of sample sizes for length- or age-composition data used in stock assessments. *Can. J. Fish. Aquat. Sci.* 71: 581–588. <dx.doi.org/10.1139/cjfas-2013-0289>.

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5 Tables

Table 1: Size composition sampling information for Tanner crab in the NMFS EBS shelf survey, 1975-1999. default: default sample size for assessment model; measured: number of crab measured; non-0 hauls: number of hauls with measured crab; avg(N): arithmetic mean effective sample size; har(N): harmonic mean effective sample size. Values have been rounded to integers.

year	female										male				
	immature					mature					all				
	default	measured	non-0 hauls	avg(N)	har(N)	default	measured	non-0 hauls	avg(N)	har(N)	default	measured	non-0 hauls	avg(N)	har(N)
1975	19	1,047	73	72	33	47	2,567	95	275	150	134	7,287	127	253	161
1976	29	1,097	88	62	31	43	1,615	96	174	113	127	4,734	132	262	167
1977	22	776	69	48	24	55	1,921	83	438	187	122	4,234	117	154	95
1978	43	1,949	88	89	38	43	1,945	103	220	120	115	5,227	158	361	238
1979	30	429	43	73	34	42	597	51	75	30	128	1,829	110	133	77
1980	27	1,491	103	118	66	37	2,041	108	193	65	136	7,530	175	517	314
1981	11	579	71	89	55	50	2,525	122	115	55	138	6,988	182	732	480
1982	19	823	87	109	70	64	2,841	129	157	63	117	5,204	202	610	437
1983	46	2,113	102	113	53	52	2,355	115	264	132	102	4,648	187	225	137
1984	50	1,879	135	147	80	48	1,815	107	173	77	102	3,854	184	328	196
1985	47	847	141	125	82	46	829	91	90	46	106	1,900	188	288	225
1986	61	1,588	162	110	67	20	522	107	145	95	120	3,137	228	216	143
1987	73	4,230	189	165	90	15	837	129	180	107	112	6,463	229	334	199
1988	52	3,735	206	230	144	32	2,283	169	272	148	116	8,312	253	340	216
1989	45	3,271	204	118	76	29	2,123	170	279	148	126	9,245	243	241	155
1990	40	3,114	198	159	88	38	3,013	178	403	253	122	9,598	253	503	357
1991	28	2,259	163	115	64	48	3,851	174	362	174	124	9,946	241	443	304
1992	26	1,494	107	146	96	53	3,025	167	388	222	121	6,929	231	483	353
1993	21	869	99	112	69	45	1,882	155	384	233	134	5,593	230	665	494
1994	30	921	97	112	59	47	1,441	120	327	198	124	3,832	213	495	398
1995	35	834	115	90	61	50	1,197	116	235	139	116	2,789	191	320	232
1996	38	883	115	101	66	46	1,072	125	197	124	116	2,705	190	264	184
1997	63	1,329	116	201	102	32	672	111	246	169	105	2,207	195	251	160
1998	65	1,710	146	195	115	19	504	96	195	124	116	3,052	195	344	244
1999	72	2,628	138	185	108	21	765	105	223	134	107	3,933	186	189	109

Table 2: Size composition sampling information for Tanner crab in the NMFS EBS shelf survey, 2000-2022. default: default sample size for assessment model; measured: number of crab measured; non-0 hauls: number of hauls with measured crab; avg(N): arithmetic mean effective sample size; har(N): harmonic mean effective sample size. Values have been rounded to integers.

year	female										male				
	immature					mature					all				
	default	measured	non-0 hauls	avg(N)	har(N)	default	measured	non-0 hauls	avg(N)	har(N)	default	measured	non-0 hauls	avg(N)	har(N)
2000	65	2,249	142	198	115	17	587	89	195	122	118	4,117	206	347	229
2001	72	3,678	164	159	88	20	1,008	109	226	136	108	5,482	227	231	151
2002	72	3,585	155	138	68	17	850	105	129	73	110	5,459	213	233	141
2003	49	2,834	153	110	63	29	1,675	128	244	97	122	7,003	214	309	192
2004	63	3,922	175	222	118	17	1,083	124	143	70	120	7,468	257	370	262
2005	54	3,352	201	135	83	25	1,562	129	113	70	121	7,529	267	264	194
2006	46	4,364	211	172	104	28	2,659	180	261	154	126	12,035	271	471	279
2007	33	2,430	186	148	95	37	2,707	185	221	123	130	9,586	275	328	196
2008	30	1,747	153	112	74	41	2,363	167	269	160	129	7,389	253	722	536
2009	48	2,408	171	207	116	33	1,680	140	248	135	119	5,977	241	561	395
2010	58	3,180	186	165	103	22	1,186	126	190	113	121	6,624	240	400	280
2011	66	5,044	193	185	118	15	1,176	137	286	177	119	9,151	223	348	235
2012	53	3,611	195	203	114	24	1,662	144	167	96	123	8,386	230	388	254
2013	39	2,917	163	192	105	32	2,419	157	258	154	129	9,611	214	474	289
2014	29	2,211	165	153	88	27	2,066	148	295	165	143	10,861	235	722	550
2015	27	1,455	118	179	115	34	1,808	115	212	111	139	7,413	251	827	648
2016	27	1,373	110	143	81	32	1,618	100	240	114	141	7,073	266	636	453
2017	42	2,033	131	185	62	28	1,338	118	221	142	130	6,206	251	451	257
2018	66	4,666	196	233	115	17	1,228	120	341	202	117	8,251	250	449	232
2019	70	3,810	181	227	136	22	1,190	106	175	81	108	5,913	237	387	258
2021	51	3,015	189	128	54	34	1,991	148	305	157	115	6,721	235	239	106
2022	58	2,684	159	109	47	25	1,172	142	201	123	117	5,393	239	194	108

6 Figures

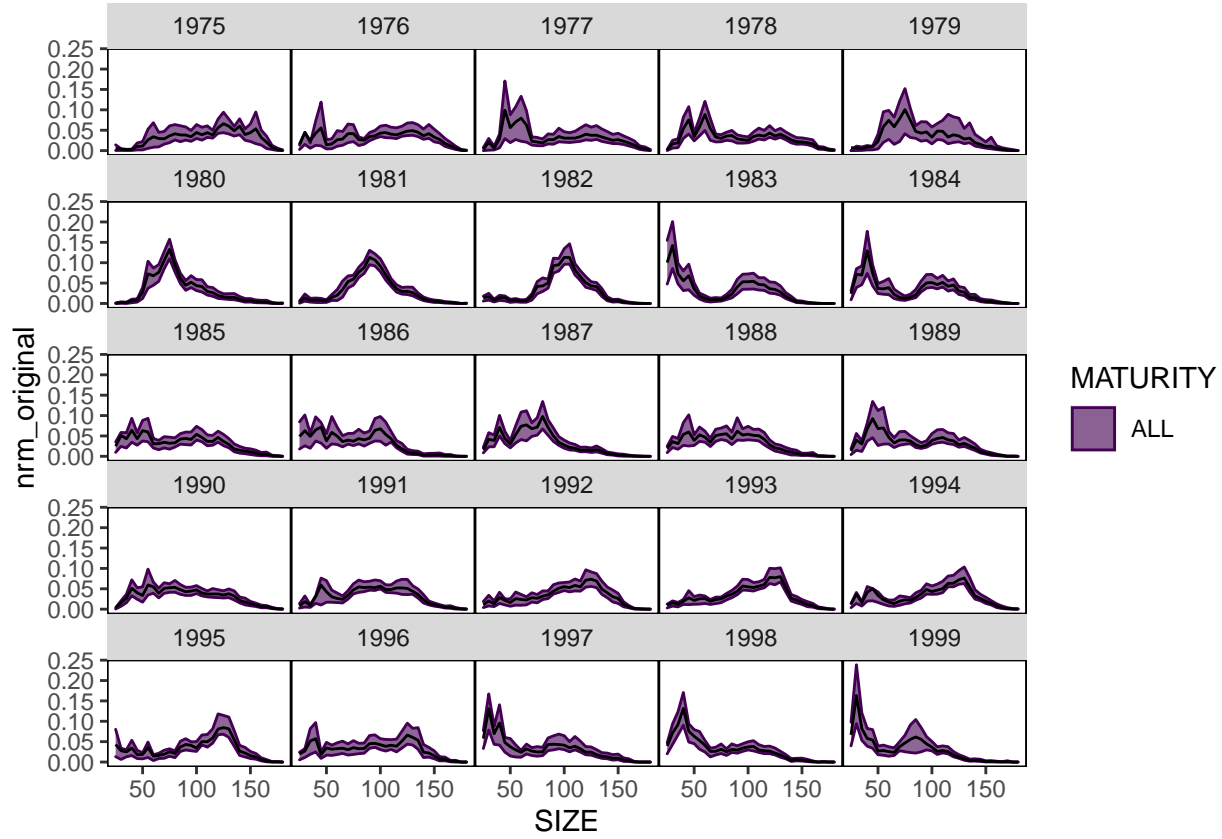


Figure 1: Confidence intervals from bootstrap analysis of NMFS EBS shelf survey size compositions for male Tanner crab, 1975-1999.

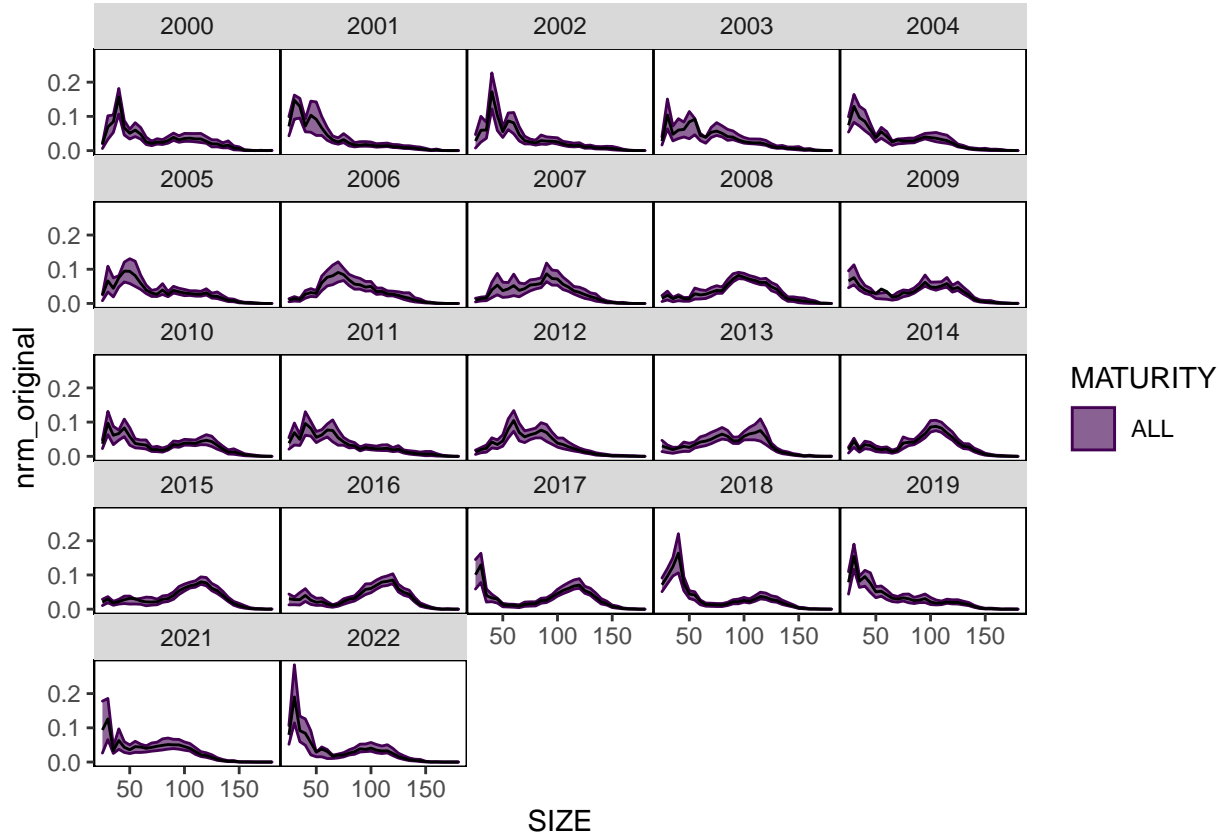


Figure 2: Confidence intervals from bootstrap analysis of NMFS EBS shelf survey size compositions for male Tanner crab, 2000+.

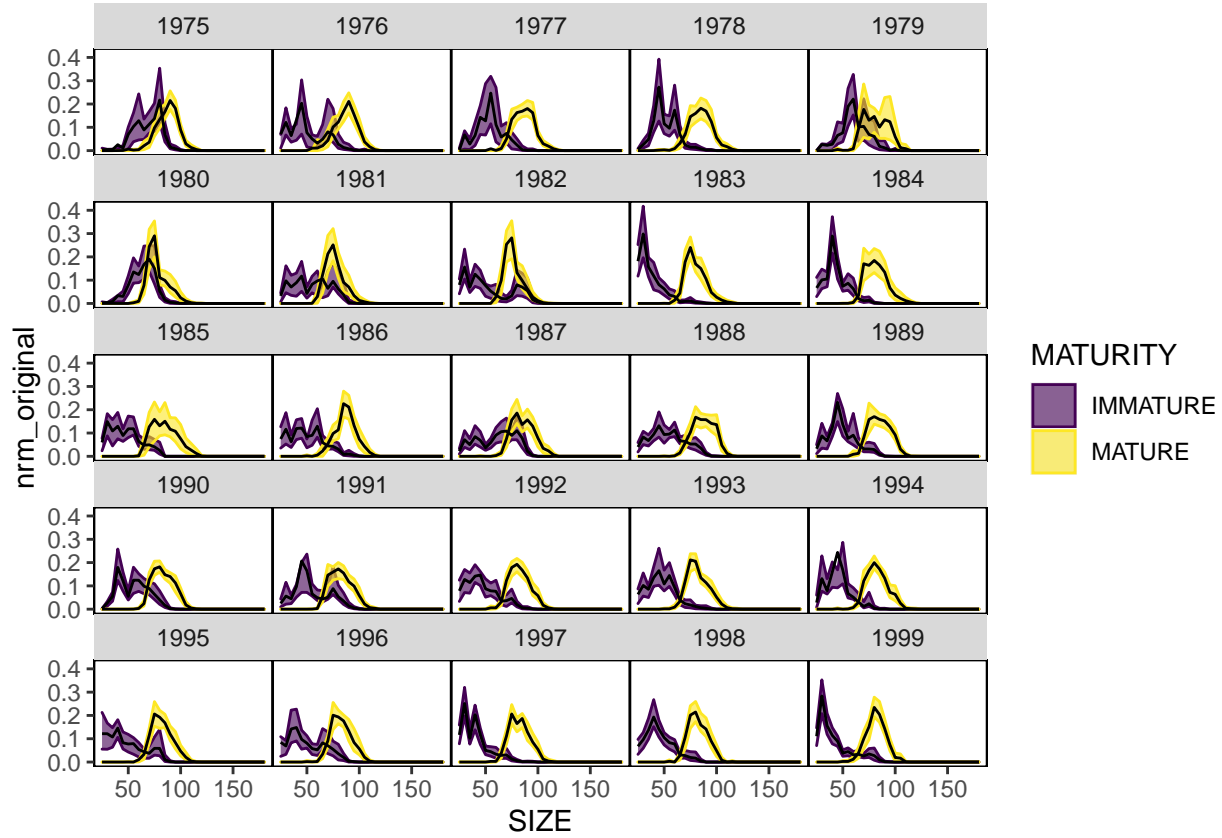


Figure 3: Confidence intervals from bootstrap analysis of NMFS EBS shelf survey size compositions for female Tanner crab, 1975-1999.

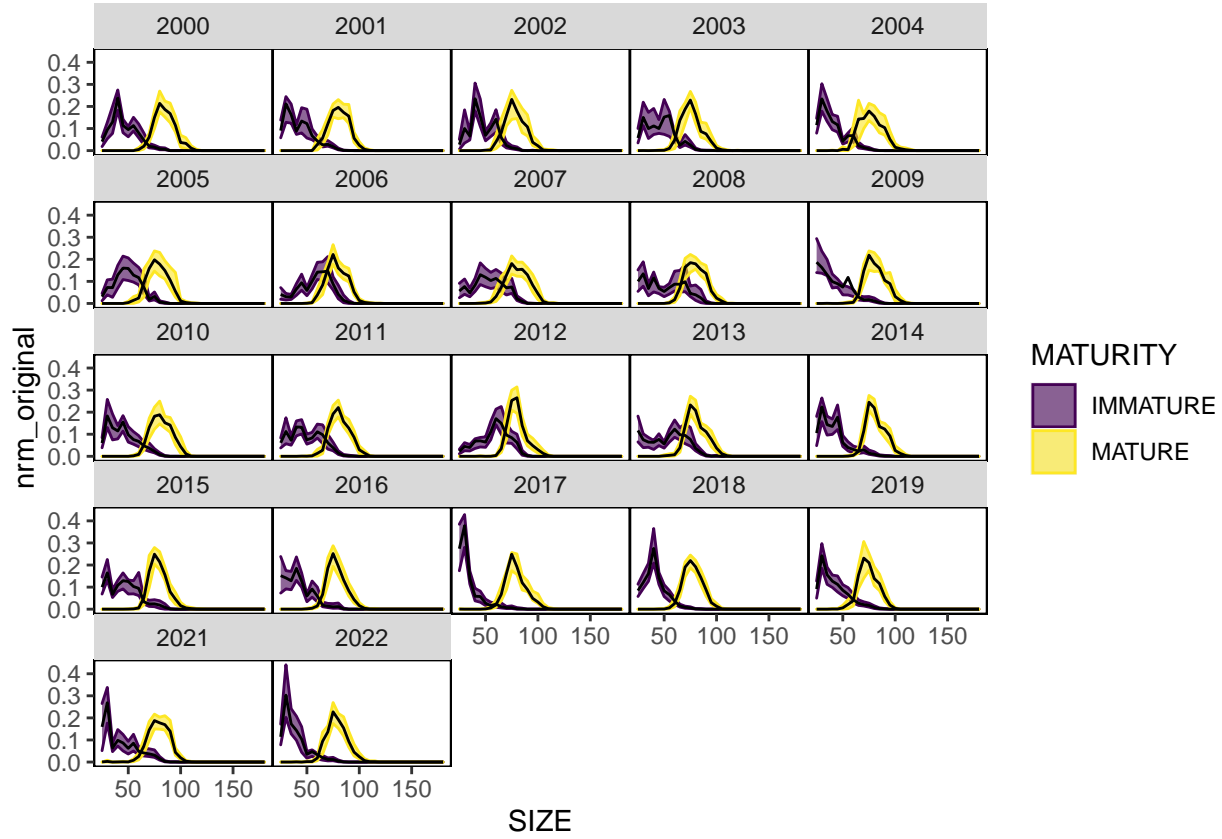


Figure 4: Confidence intervals from bootstrap analysis of NMFS EBS shelf survey size compositions for female Tanner crab, 2000+.

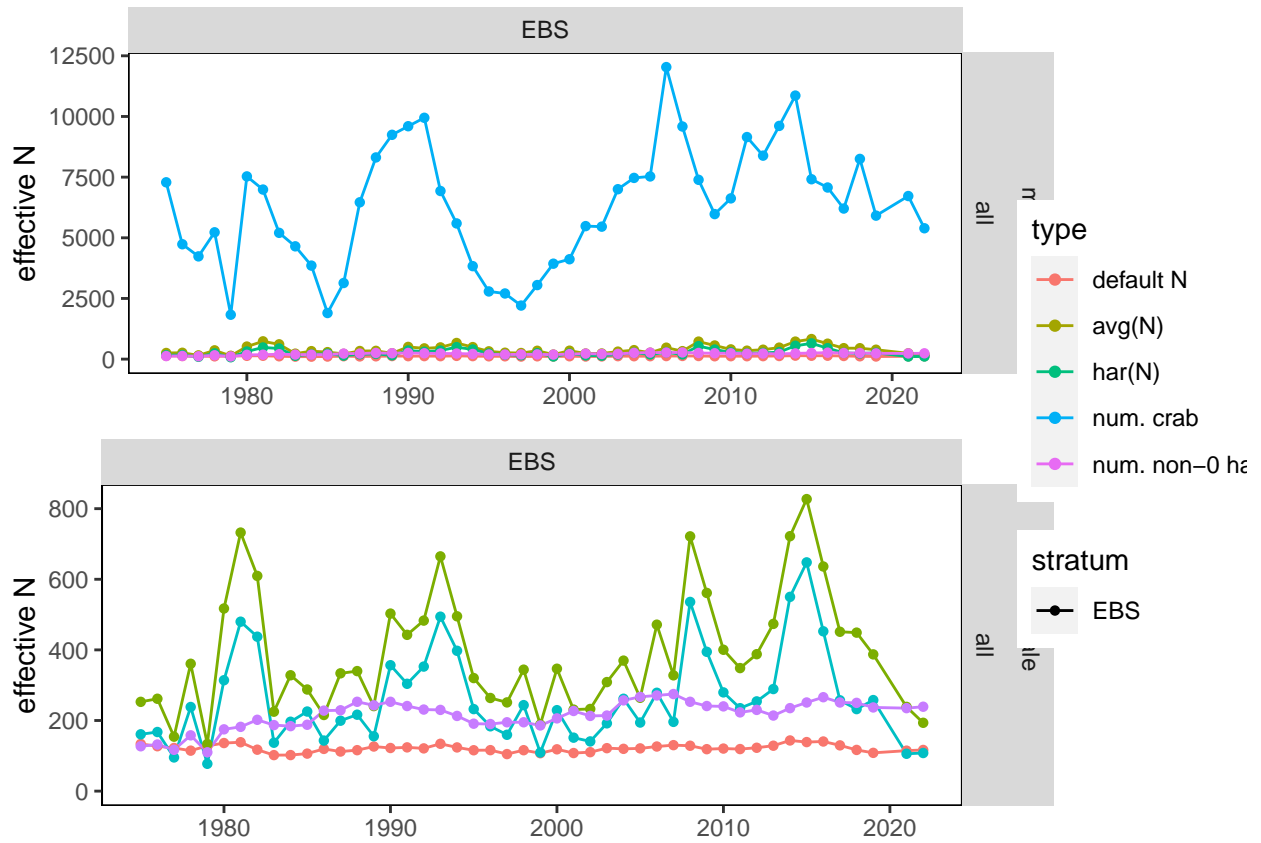


Figure 5: Comparison of the annual number of crab sampled (num. crab), default sample sizes (default N), effective sample sizes from bootstrapping (mean effective N: avg(N), harmonic mean effective N: har(N)), and the number of hauls with non-zero catch for male crab in the NMFS EBS survey.

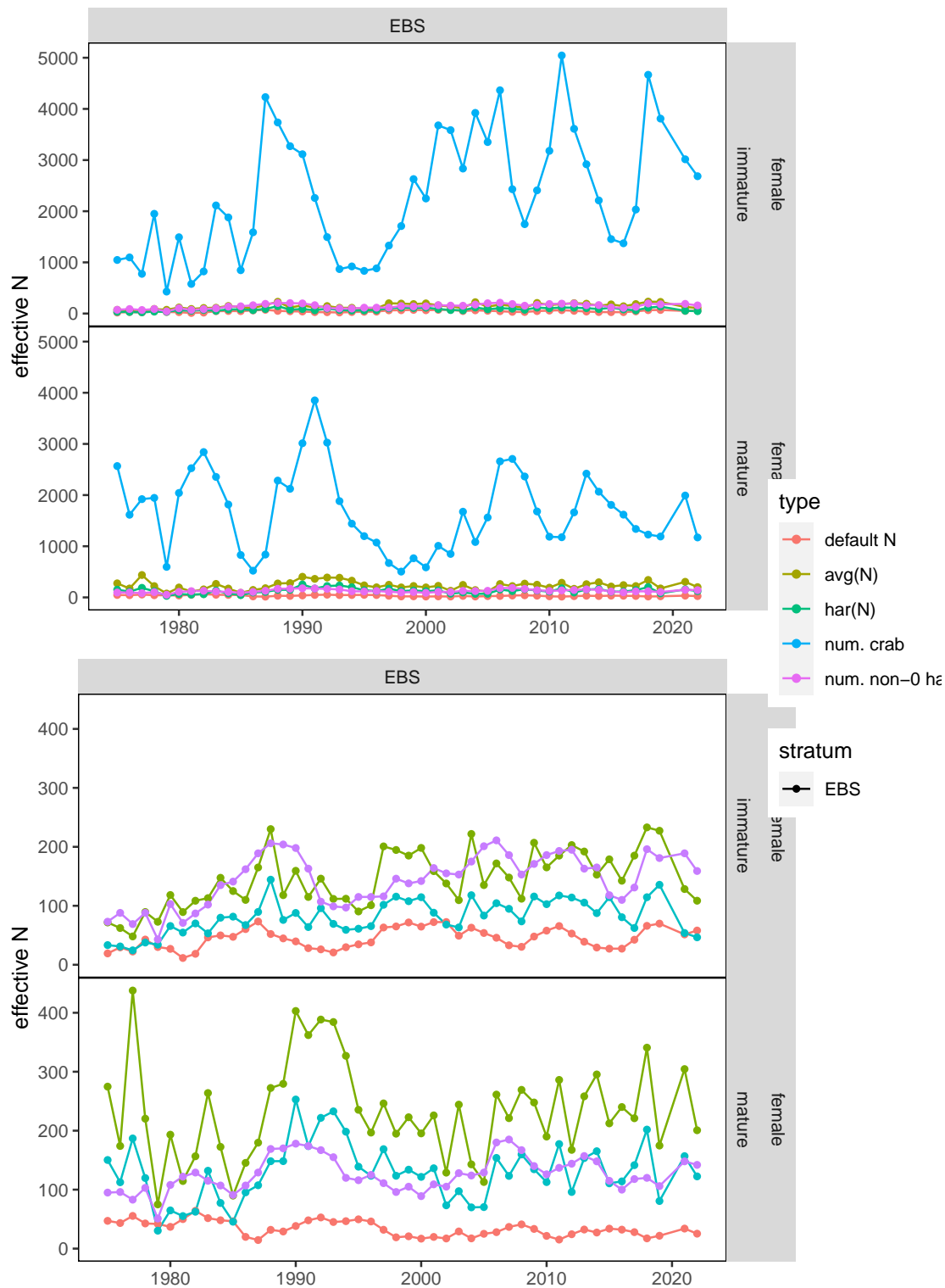


Figure 6: Comparison of the annual number of crab sampled (num. crab), default sample sizes (default N), effective sample sizes from bootstrapping (mean effective N: avg(N), harmonic mean effective N: har(N)), and the number of hauls with non-zero catch for female crab in the NMFS EBS survey.