Stock assessment for eastern Bering Sea Walleye Pollock: some preliminary alternative evaluations based on external review

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Abstract

Introduction

Three independent experts reviewed the stock assessment for eastern Bering Sea Walleye Pollock in May 2016. The terms of reference and presentations and their subsequent reports can be found at: www.tinyurl.com/pollockCIE2016. Several improvements to the assessment were recommended and those seen as highest priority that could be reasonably addressed this year include:

1. Modify the body weight-at-age estimation method to be based on increment rather than expected values
2. Fit the model to biomass indices rather than total population numbers
3. For the acoustic trawl data, use the time series that covers the water column down to a half meter from bottom rather than down to 3 meters from bottom.
4. Evaluate data weightings from first principles for input sample sizes
5. Evaluate whether weightings are appropriate given model fit
6. Consider components of variability of the F_{msy} estimation and the effect of the prior

The following sections are intended as a start to begin addressing these concerns.

1. Body weight-at-age estimation

Modern stock assessment methods that lead to scientific advice on sustainable fishing practices typically revolves around ensuring that fishing mortality rates are at or below values used as reference points. In most management settings, conservation measures are set based on catch biomass limits with some assumption about expected body mass-at-age (hereafter referred to as weight-at-age) to convert from modeled catch numbers (as specified based on the fishing mortality rates). Uncertainty estimates are typically concerned with the absolute values of the population numbers-at-age estimates and the stock productivity estimates leading to acceptable fishing mortality reference points. While uncertainty from these sources is obviously important for evaluating risks in management settings, the additional uncertainty due to unknown weight-at-age is typically ignored (Jaworski 2011)

For many fisheries settings empirical estimates of mean body mass-at-age are quite precise due to sampling design and effort. For example, the uncertainty of estimated mean body mass for the eastern Bering Sea (EBS) walleye pollock (*Gadus chalcogrammus*) for the main fished ages typically has coefficients of variation below 5%.

The model for predicting mean body weight-at-age in the fishery has two purposes, prediction of the current and future year values and their relative uncertainty. As shown in section 5 below, the uncertainty in average weight estimation is an important component.

Data

Fishery sampling for EBS pollock is extensive with large numbers of age, weight, and length measures sampled from the catch each year (Tables 1 and 2). NMFS observer sampling data on catch-at-length and age composition was estimated using the methods described by Kimura (1989) and modified by Dorn

(1992). Length-stratified age data are used to construct age-length keys for each stratum and sex. These keys are then applied to randomly sampled catch length frequency data. The stratum-specific age composition estimates are then weighted by the catch within each stratum to arrive at an overall age composition for each year. Data were collected through shore-side sampling and at-sea observers. The three strata for the EBS were: *i*) January–June (all areas, but mainly east of 170°W); *ii*) INPFC area 51 (east of 170°W) from July–December; and *iii*) INPFC area 52 (west of 170°W) from July–December. This method was used to derive the age compositions from 1991-2015 (the period for which all the necessary information is readily available).

The catch-at-age estimation method uses a two-stage bootstrap re-sampling of the data. Observed tows were first selected with replacement, followed by re-sampling actual lengths and age specimens given those sets of tows. This method allows an objective way to specify the effective sample size for fitting fishery age composition data within the assessment model. In addition, estimates of stratum-specific fishery mean weights-at-age (and variances) are provided which are useful for evaluating general patterns in growth and growth variability. For example, Ianelli *et al.* (2007) showed that seasonal aspects of pollock condition factor that could affect estimates of mean weight-at-age vary substantially within years. In 2016, the routine for estimating weights-at-age was updated to be adaptable to other stocks and converted into an R package. The values were re-computed for the period 1991-2014 (and include 2015) and estimated mean body weights-at-age were nearly identical to those previously used (Fig. 1). A detailed summary of the relative mean weight-at-age estimates is shown in a series of figures presented in Appendix 1.

As part of the response to the CIE review completed in May 2016, these calculation updates also included some new estimation methods including Francis (2011) method for estimating input effective sample sizes. This was done by the simple method as: $N_y = v_y / V_j\left(\bar{A}_{jy}\right)$, where $\bar{A}_{jy}$ is the mean age of the *j*th bootstrap in year *y*, V_j is the variance over the bootstrap samples, v_y, the variance of the observed composition in year *y* is calculated as $v_y = \sum_a a^2 p_{ay} - \left(\sum_a a p_{ay}\right)^2$ and p_{ay} is the proportion-at-age *a* in year *y*. Results applying this method suggest that the effective input sample sizes range from about 2 thousand to over 12 thousand fish (Table 3).

Models

The growth model followed the parameterization of Schnute and Fournier (1980), with the addition of cohort effects and annual year effects:

$$
\begin{aligned}
\hat{w}_{ij} &= \mu_j e^{\delta_i} && j = 1,\ \ i \geq 1 \\
\hat{w}_{ij} &= \hat{w}_{i-1,j-1} + \Delta_j e^{\zeta_i} && j > 1,\ \ i > 1 \\
\Delta_j &= \mu_{j\ \ 1} - \mu_j && j < J \\
\mu_j &= \alpha\left[L_1 + \left(L_2 - L_1\right)\left(\frac{1 - K^{j-1}}{1 - K^{J\ \ 1}}\right)\right]^3
\end{aligned}
\qquad 1
$$

with symbols defined in Table 4. The years and ages for model application can be specified independently of the data extent. As with Jaworski (2011) a series of prediction methods were evaluated against a measure of predictive performance. These alternative estimators for mean weight-at-age were developed based on evaluating a variety of potentially useful independent variables. Potential explanatory variables were evaluated provided that they would be available at the time of the assessment in each year (e.g., since the bottom trawl survey is used to collect temperature information, this may be useful to predict mean weights in the fishery). The objective function used to evaluate estimator performance was simply examining how well "out-of-sample" data were predicted. For example, for a particular estimator, the first iteration data from 1991-2000 were used to estimate the mean weights in 2001 and 2002. These estimated were then compared to the actual mean weights observed for 2001 and 2002. The second iteration repeated this process but used data from 1991-2001 to estimate 2002 and 2003 data for comparison with actual observations. This sequence was continued through to using data from 1991-2014 to estimate 2015 means (and compared with actual 2015 mean values). Since some age-groups are relatively more important than others to the fishery (in terms of prediction errors), comparisons of estimates with "observed" were weighted by the relative importance of different age-groups. The relative importance of different age-groups was computed by using the mean numbers-at-age estimated in the population from Ianelli *et al.* (2015) and accounting for the fishery selectivity and mean weight over that period. This weighting scheme is intended to favor estimators for age-groups that are most important to the fishery and is computed as:

$$\gamma_a = \frac{\bar{N}_a s_a \bar{w}_a}{\sum \bar{N}_a s_a \bar{w}_a}.$$

Then the estimator that performed best minimizes:

$\sum_{y=2006}^{2015} \sum_{t=y}^{y+1} \sum_{a=3}^{15} \gamma_a \left(w'_{t,a} - \hat{w}^k_{t,a} \right)^2$ where y is the "assessment" year, $\hat{w}^k_{t,a}$ is the k^th^ estimator for mean weight-at-age a, in year y, and $w'_{t,a}$ are the actual observations in year t. The vector for the g_a weighting was based on estimates from 2000-2015:

3	4	5	6	7	8	9	10	11	12	13	14	15
0.031	0.132	0.227	0.222	0.155	0.089	0.055	0.033	0.022	0.014	0.009	0.005	0.006

Parameter estimation

The estimation configurations tested included simple means to more complex year- and cohort- specific random effects approaches (Table 5) and was coded in both TMB (Kristensen *et al.,* 2016) and ADMB (Fournier *et al.*, 2012). The code used is available at http://goo.gl/h8So5Z .

Results

The projection model for the mean weights-at-age in retrospective fitting shows the high level of variability and relatively poor skill in model predictions (Fig. 2). Nonetheless, the performance was substantially improved with the inclusion of current year survey data and modeling the cohort and year effects (Fig. 3). A preliminary evaluation of observed factors that might affect growth changes was also conducted. Temperature anomalies appeared to have a poor relationship with growth increment year effects (Fig. 4).

Table 1. Numbers of pollock fishery samples measured for lengths and for length-weight by sex and strata, 1977-2015, as sampled by the NMFS observer program.

Length Frequency samples

	A Season		B Season SE		B Season NW		
Year	Males	Females	Males	Females	Males	Females	Total
1977	26,411	25,923	4,301	4,511	29,075	31,219	121,440
1978	25,110	31,653	9,829	9,524	46,349	46,072	168,537
1979	59,782	62,512	3,461	3,113	62,298	61,402	252,568
1980	42,726	42,577	3,380	3,464	47,030	49,037	188,214
1981	64,718	57,936	2,401	2,147	53,161	53,570	233,933
1982	74,172	70,073	16,265	14,885	181,606	163,272	520,273
1983	94,118	90,778	16,604	16,826	193,031	174,589	585,946
1984	158,329	161,876	106,654	105,234	243,877	217,362	993,332
1985	119,384	109,230	96,684	97,841	284,850	256,091	964,080
1986	186,505	189,497	135,444	123,413	164,546	131,322	930,727
1987	373,163	399,072	14,170	21,162	24,038	22,117	853,722
1991	160,491	148,236	166,117	150,261	141,085	139,852	906,042
1992	158,405	153,866	163,045	164,227	101,036	102,667	843,244
1993	143,296	133,711	148,299	140,402	27,262	28,522	621,490
1994	139,332	147,204	159,341	153,526	28,015	27,953	655,370
1995	131,287	128,389	179,312	154,520	16,170	16,356	626,032
1996	149,111	140,981	200,482	156,804	18,165	18,348	683,890
1997	124,953	104,115	116,448	107,630	60,192	53,191	566,527
1998	136,605	110,620	208,659	178,012	32,819	40,307	707,019
1999	36,258	32,630	38,840	35,695	16,282	18,339	178,044
2000	64,575	58,162	63,832	41,120	40,868	39,134	307,689
2001	79,333	75,633	54,119	51,268	44,295	45,836	350,483
2002	71,776	69,743	65,432	64,373	37,701	39,322	348,347
2003	74,995	77,612	49,469	53,053	51,799	53,463	360,390
2004	75,426	76,018	63,204	62,005	47,289	44,246	368,188
2005	76,627	69,543	43,205	33,886	68,878	63,088	355,225
2006	72,353	63,108	28,799	22,363	75,180	65,209	327,010
2007	62,827	60,522	32,945	25,518	75,128	69,116	326,054
2008	46,125	51,027	20,493	23,503	61,149	64,598	266,894
2009	46,051	44,080	19,877	18,579	50,451	53,344	232,379
2010	39,495	41,054	19,194	20,591	40,449	41,323	202,106
2011	58,822	62,617	60,254	65,057	51,137	48,084	345,971
2012	53,641	57,966	45,044	46,940	50,167	53,224	306,982
2013	52,303	62,336	37,434	44,709	49,484	49,903	296,168
2014	55,954	58,097	46,568	51,950	46,643	46,202	305,414
2015	55,646	56,507	45,074	41,218	46,237	43,084	287,766

Table 1. (continued) Numbers of pollock fishery samples measured for lengths and for length-weight by sex and strata, 1977-2015, as sampled by the NMFS observer program.

Length – weight samples

	A Season		B Season SE		B Season NW		
	Males	Females	Males	Females	Males	Females	Total
1977	1,222	1,338	137	166	1,461	1,664	5,988
1978	1,991	2,686	409	516	2,200	2,623	10,425
1979	2,709	3,151	152	209	1,469	1,566	9,256
1980	1,849	2,156	99	144	612	681	5,541
1981	1,821	2,045	51	52	1,623	1,810	7,402
1982	2,030	2,208	181	176	2,852	3,043	10,490
1983	1,199	1,200	144	122	3,268	3,447	9,380
1984	980	1,046	117	136	1,273	1,378	4,930
1985	520	499	46	55	426	488	2,034
1986	689	794	518	501	286	286	3,074
1987	1,351	1,466	25	33	72	63	3,010
1991	2,712	2,781	2,339	2,496	1,065	1,169	12,562
1992	1,517	1,582	1,911	1,970	588	566	8,134
1993	1,201	1,270	1,448	1,406	435	450	6,210
1994	1,552	1,630	1,569	1,577	162	171	6,661
1995	1,215	1,259	1,320	1,343	223	232	5,592
1996	2,094	2,135	1,409	1,384	1	1	7,024
1997	628	627	616	665	511	523	3,570
1998	1,852	1,946	959	923	327	350	6,357
1999	5,318	4,798	7,797	7,054	3,532	3,768	32,267
2000	12,421	11,318	12,374	7,809	7,977	7,738	59,637
2001	14,882	14,369	10,778	10,378	8,777	9,079	68,263
2002	14,004	13,541	12,883	12,942	7,202	7,648	68,220
2003	14,780	15,495	9,401	10,092	9,994	10,261	70,023
2004	7,690	7,890	6,819	6,847	4,603	4,321	38,170
2005	7,390	7,033	5,109	4,115	6,927	6,424	36,998
2006	7,324	6,989	5,085	4,068	6,842	6,356	36,664
2007	6,681	6,635	4,278	3,203	7,745	7,094	35,636
2008	4,256	4,787	2,056	2,563	5,950	6,316	25,928
2009	4,470	4,199	2,273	2,034	5,004	5,187	23,167
2010	4,536	5,272	2,261	2,749	4,125	4,618	23,561
2011	6,772	6,388	6,906	6,455	5,809	4,634	36,964
2012	5,500	5,981	4,508	4,774	4,928	5,348	31,039
2013	6,525	5,690	4,313	3,613	4,920	4,849	29,910
2014	5,675	5,871	4,753	5,180	4,785	4,652	30,916
2015	5,310	5,323	4,645	4,188	4,337	4,011	27,766

Table 2. Numbers of pollock fishery samples used for age determination estimates by sex and strata, 1977-2015, as sampled by the NMFS observer program.

	Number of samples aged						
	A Season		B Season SE		B Season NW		
	Males	Females	Males	Females	Males	Females	Total
1977	1,229	1,344	137	166	1,415	1,613	5,904
1978	1,992	2,686	407	514	2,188	2,611	10,398
1979	2,647	3,088	152	209	1,464	1,561	9,121
1980	1,854	2,158	93	138	606	675	5,524
1981	1,819	2,042	51	52	1,620	1,807	7,391
1982	2,030	2,210	181	176	2,865	3,062	10,524
1983	1,200	1,200	144	122	3,249	3,420	9,335
1984	980	1,046	117	136	1,272	1,379	4,930
1985	520	499	46	55	426	488	2,034
1986	689	794	518	501	286	286	3,074
1987	1,351	1,466	25	33	72	63	3,010
1991	420	423	272	265	320	341	2,041
1992	392	392	371	386	178	177	1,896
1993	444	473	503	493	124	122	2,159
1994	201	202	570	573	131	141	1,818
1995	298	316	436	417	123	131	1,721
1996	468	449	442	433	1	1	1,794
1997	433	436	284	311	326	326	2,116
1998	592	659	307	307	216	232	2,313
1999	540	500	730	727	306	298	3,100
2000	666	626	843	584	253	293	3,265
2001	598	560	724	688	178	205	2,951
2002	651	670	834	886	201	247	3,489
2003	583	644	652	680	260	274	3,092
2004	560	547	599	697	244	221	2,867
2005	611	597	613	489	419	421	3,149
2006	608	599	590	457	397	398	3,048
2007	639	627	586	482	583	570	3,485
2008	492	491	313	356	541	647	2,838
2009	488	416	285	325	400	434	2,346
2010	624	545	504	419	465	414	2,971
2011	581	808	579	659	404	396	3,427
2012	517	571	480	533	485	579	3,165
2013	703	666	517	402	568	526	3,381
2014	609	629	475	553	413	407	3,086
2015	653	642	511	491	502	509	3,308

Table 3. Sample size estimates derived from bootstrap variability and catch-at-age proportions.

Year	Mean age	CV Mean age	Effective N
1991	7.56	0.78%	2,639
1992	6.07	0.73%	5,667
1993	4.86	0.42%	12,546
1994	5.13	0.62%	2,474
1995	5.74	0.50%	3,010
1996	6.48	0.58%	2,085
1997	5.98	0.47%	4,891
1998	6.19	0.48%	3,701
1999	5.74	0.49%	5,310
2000	5.95	0.46%	5,521
2001	6.23	0.55%	3,201
2002	6.12	0.57%	3,475
2003	5.55	0.55%	5,024
2004	5.41	0.61%	3,695
2005	5.52	0.43%	4,257
2006	5.80	0.50%	3,539
2007	6.14	0.46%	4,235
2008	6.57	0.54%	3,733
2009	5.99	0.64%	4,607
2010	5.25	0.51%	6,928
2011	5.41	0.41%	6,883
2012	5.03	0.43%	6,629
2013	5.38	0.40%	5,705
2014	5.82	0.40%	4,284
2015	5.24	0.40%	8,411

Table 4. Equations and model parameters

Symbol	Descripion
$\hat{w}_{ij} = \mu_j e^{\delta_i} \quad j = 1, \quad i \geq 1$ $\hat{w}_{ij} = \hat{w}_{i-1,j-1} + \Delta_j e^{\zeta_i} \quad j > 1, \quad i > 1$ $\Delta_j = \mu_{j\ 1} - \mu_j \quad j < J$ $\mu_j = \alpha \left[L_1 + \left(L_2 - L_1 \right) \left(\frac{1 - K^{j-1}}{1 - K^{J\ 1}} \right) \right]^3$	Growth model
$\hat{w}_{ij}$	Expected mean weight-at-age j in year i
i , j	Index for year and age
μ_j	Mean length age j
Δ_j	Mean growth increment
α	Constant to scale lengths
$\delta_i\ \zeta_i$	Cohort and year effects
K, L_1, and L_2	Parameters of the von Bertalanffy growth

Table 5. Alternative methods evaluated for computing mean weight-at-age for EBS pollock.

Method	Description
Means	Mean fishery weights-at-age of most recent *n* years of data (*n* =1, 3, 5, and 10)
Year and Cohort	Year and cohort effect model
Year and Cohort with scaled survey data	Include scaled survey weights-at-age ($\hat{w}_{i,j}^{k-2} = \lambda_j w_{i,j}^{survey}$)
Year effect only (with scaled survey data)	Year effect model (a random effect parameter for each annual growth increment)

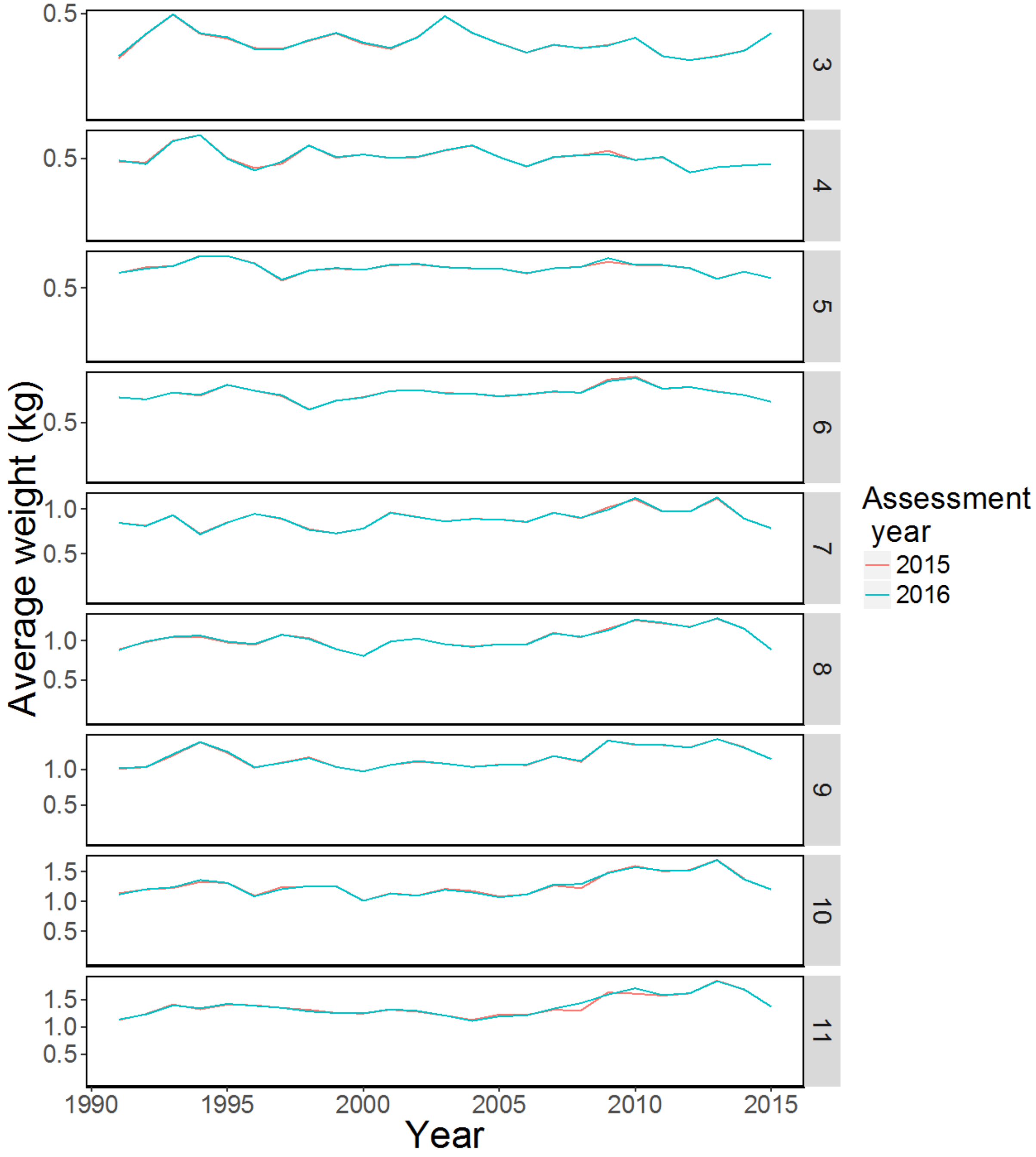

Figure 1. Comparisons of average fishery weight-at-age (kg) for EBS pollock, 1991-2015 from the 2015 assessment and the current revised estimates.

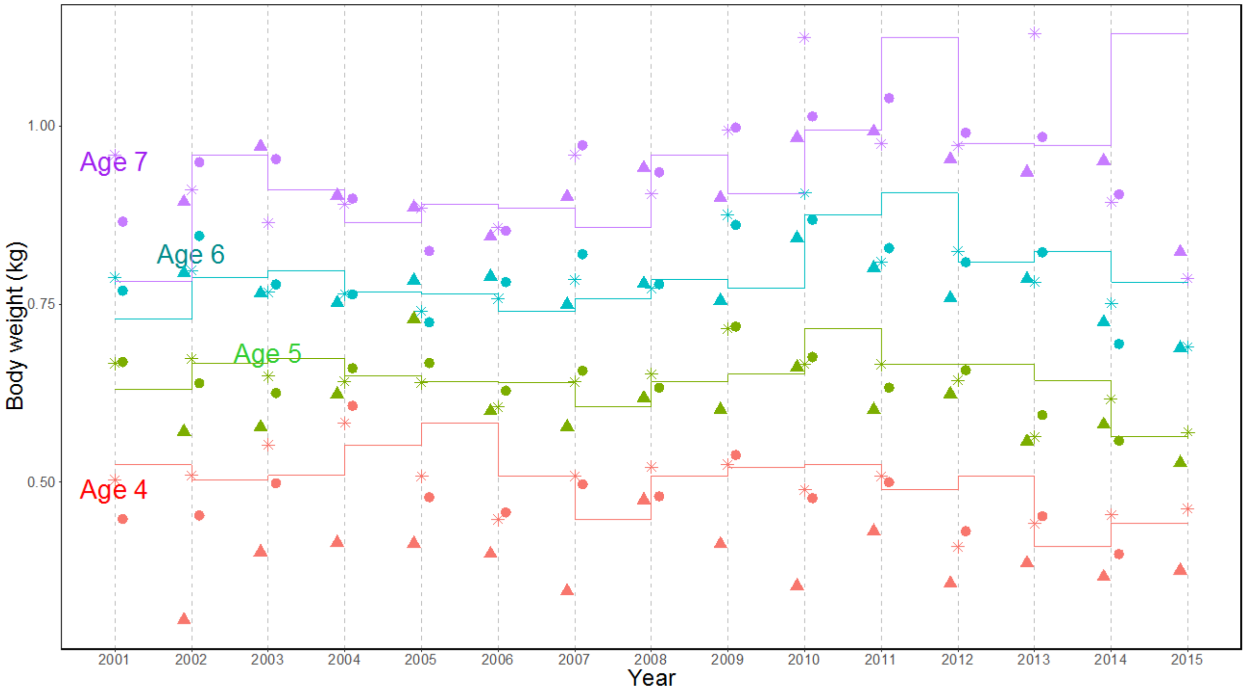

Figure 2. Projection results compared to data for fishery weights-at-ages 4-7. The lines represent estimates set equal to the most recent value for the current assessment year and next year whereas the solid bullets and triangles represent the modeled estimates for the current assessment year and next year, respectively. The stars represent the final realized estimates based on the observer data.

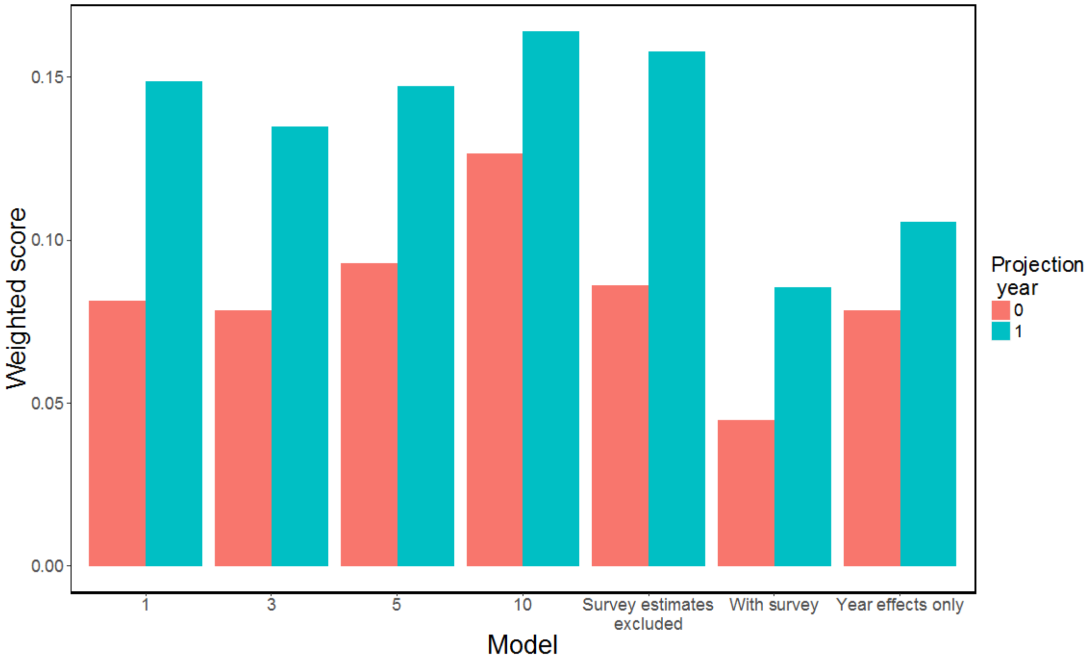

Figure 3. Scores of performance for different methods for projecting average body weight where projection year of 0 means current (assessment) year and 1 means the coming year used for ABC estimation. Models labeled 1, 3, 5, and 10 represent the means over that many most recent years. The right-most “Models” are random effects approaches with and without survey data included.

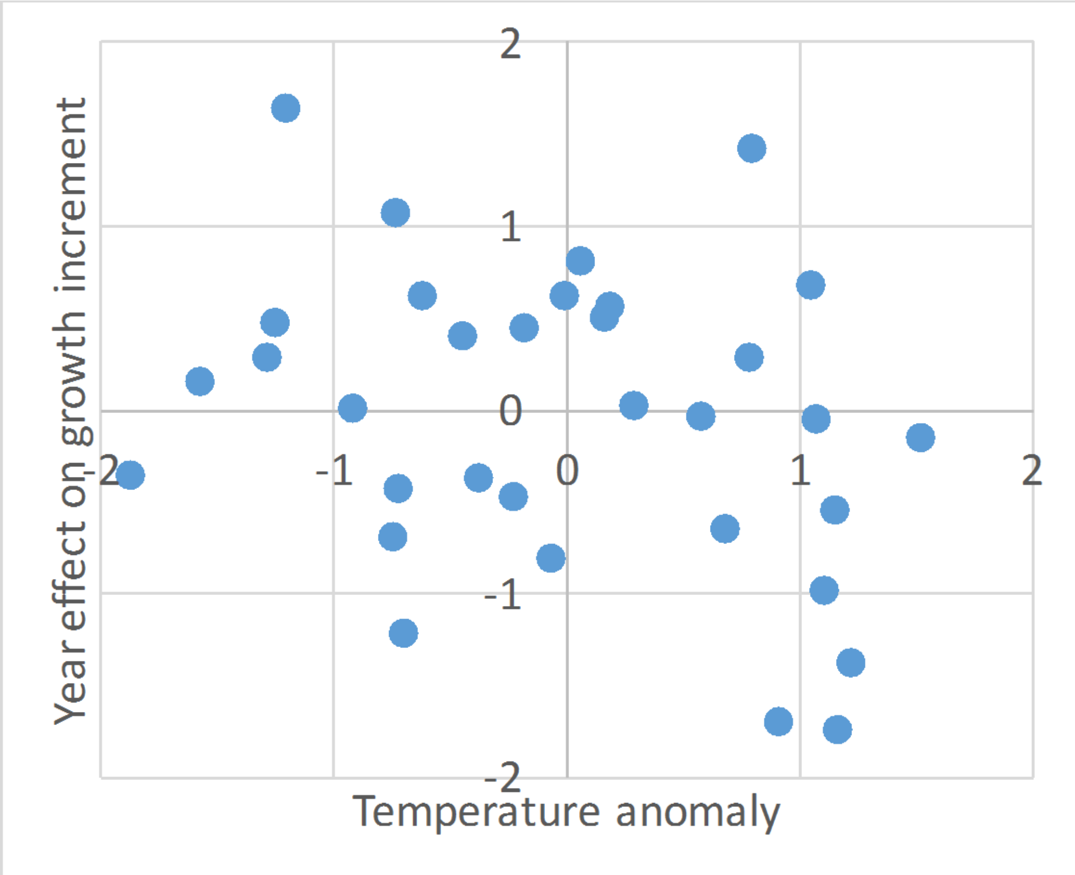

Figure 4. Estimated year effect on growth increment compared to Bering Sea temperature anomaly for EBS pollock.

2. and 3. Tuning model to survey biomass indices instead of numbers and alternative AT data

In the original development of the model, the option to tune to survey estimates of population numbers or biomass was available. The tradition of tuning this model using numbers should be re-evaluated. As such the following models were defined for incremental evaluations:

Model	Description
Model 0.0	The 2015 model used for management advice
Model 0	As Model 0.0 but using the design-based trawl survey estimates (and likelihoods) instead of the Kotwicki index
Model 1	As Model 0 but tuned to acoustic survey biomass instead of AT numbers
Model 2	As Model 0 but tuned to bottom trawl survey biomass instead of bottom trawl survey numbers
Model 3	As Model 0 but tuned to both acoustic and bottom trawl survey biomass estimates instead of numbers
Model 4	As Model 3 but using the acoustic trawl survey data covering the water column extended to 0.5 m from bottom instead of the traditional 3 m.

Note that model 0 was modified slightly from 0.0 to make for a clearer comparison of impacts of changes in subsequent models. The estimates from Lautenberger *et al.* (in press) extend the acoustic-trawl data from the near surface down to 0.5 m from the bottom. Generally, the trends are similar to the values which extended only down to 3.0 m (Table 6). These results suggest that, on average, about 26% of the acoustic backscatter attributed to pollock occurs between 0.5 and 3.0 m. As noted above, the biomass estimates for these comparisons are based on standard design-based estimates from the bottom trawl survey (instead of the "Kotwicki index") so that consistent model comparisons were possible.

Table 6. Acoustic trawl survey biomass estimates for EBS pollock based on the methods of Lauthenberg *et al.* (in press) for different segments of the water column.

	~15m - 3 m estimates	Water column to 0.5	0.5 - 3 m estimates	Proportion increase from 3 m estimates
1994	2,886,235	3,640,106	753,871	26%
1995				
1996	2,310,742	2,955,115	644,373	28%
1997	2,590,929	3,590,695	999,766	39%
1998				
1999	3,344,679	4,202,143	857,464	26%
2000	3,048,718	3,613,940	565,222	19%
2001				
2002	3,622,070	4,330,008	707,938	20%
2003				
2004	3,306,937	4,016,180	709,243	21%
2005				
2006	1,560,174	1,887,421	327,247	21%
2007	1,769,019	2,288,070	519,051	29%
2008	996,939	1,407,479	410,540	41%
2009	923,843	1,323,060	399,217	43%
2010	2,322,643	2,651,176	328,533	14%
2011				
2012	1,842,792	2,298,941	456,149	25%
2013				
2014	3,438,986	4,726,599	1,287,613	37%

Results

Some patterns emerged between model alternatives using survey numbers in place of biomass estimates in the model fitting process. For the bottom trawl survey, model fits to abundances when biomass was tuned (Models 2-4) and tended to show negative residuals, whereas in models 0 and 1 (when bottom trawl survey abundances were used for tuning), the residuals for biomass fits were mostly positive (Fig. 5). For the acoustic trawl survey data, model fits were generally reasonable for numbers and biomass, regardless of which one was actually used in the tuning (Fig. 6). The different model configurations resulted in minor changes to spawning biomass estimates (Fig. 7). Models 3 and 4 appear to have a slightly lower estimates overall.

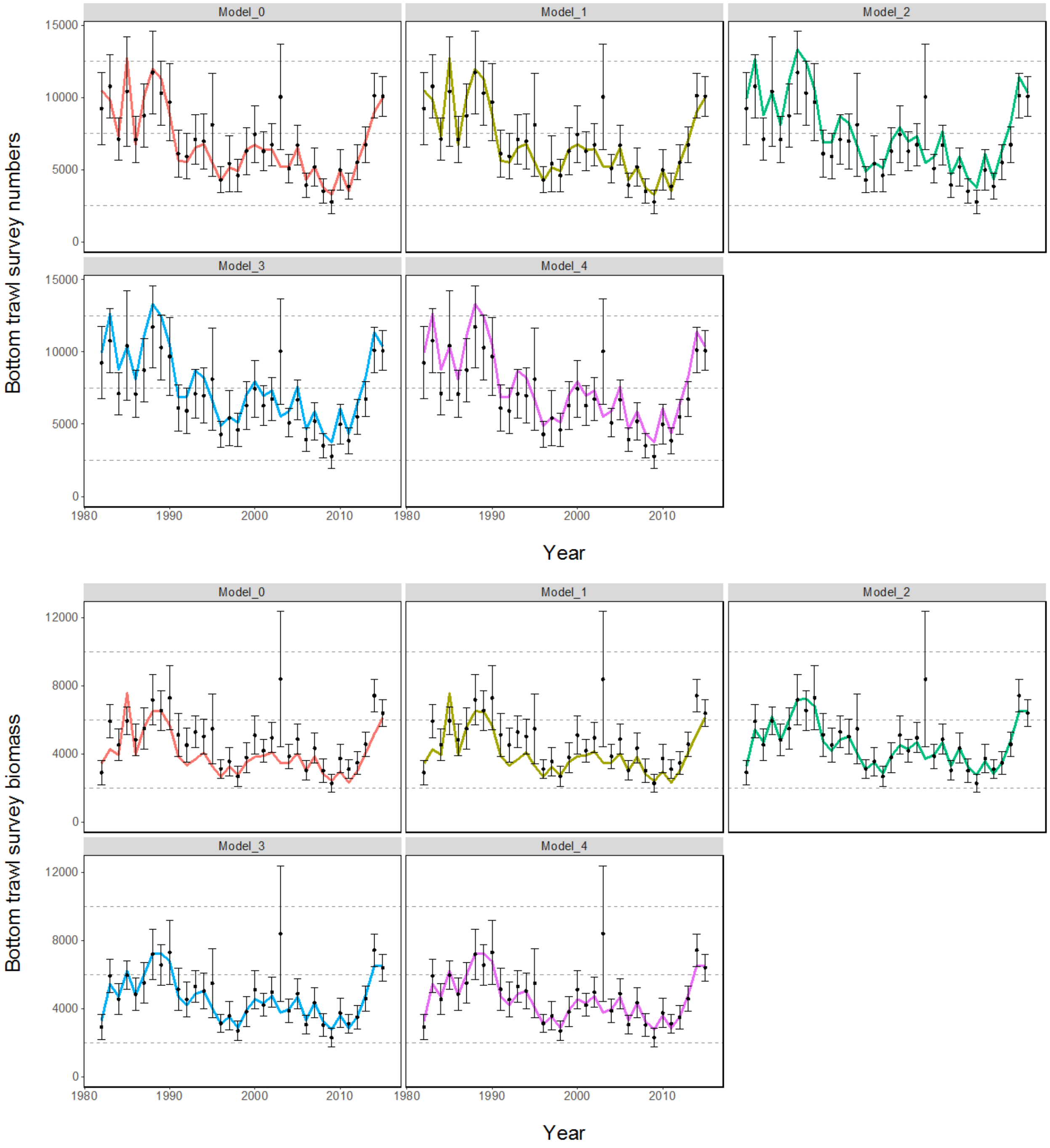

Figure 5. Model fits to design-based **bottom trawl survey** estimates for abundance (top set of panels) and biomass (bottom set) for EBS pollock based on the 2015 model configuration.

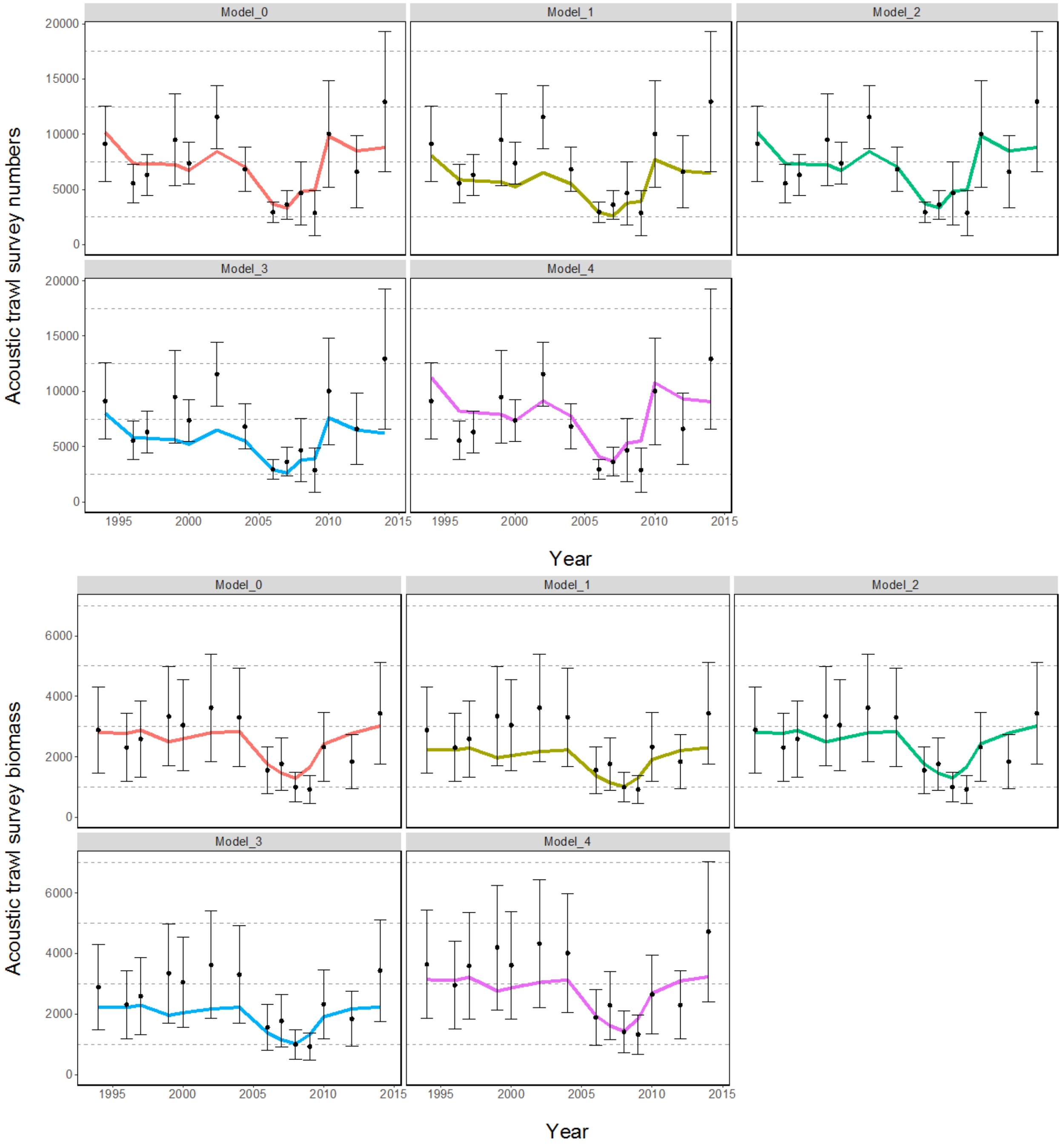

Figure 6. Model fits to **acoustic trawl survey** estimates for abundance (top set of panels) and biomass (bottom set) for EBS pollock based on the 2015 model configuration.

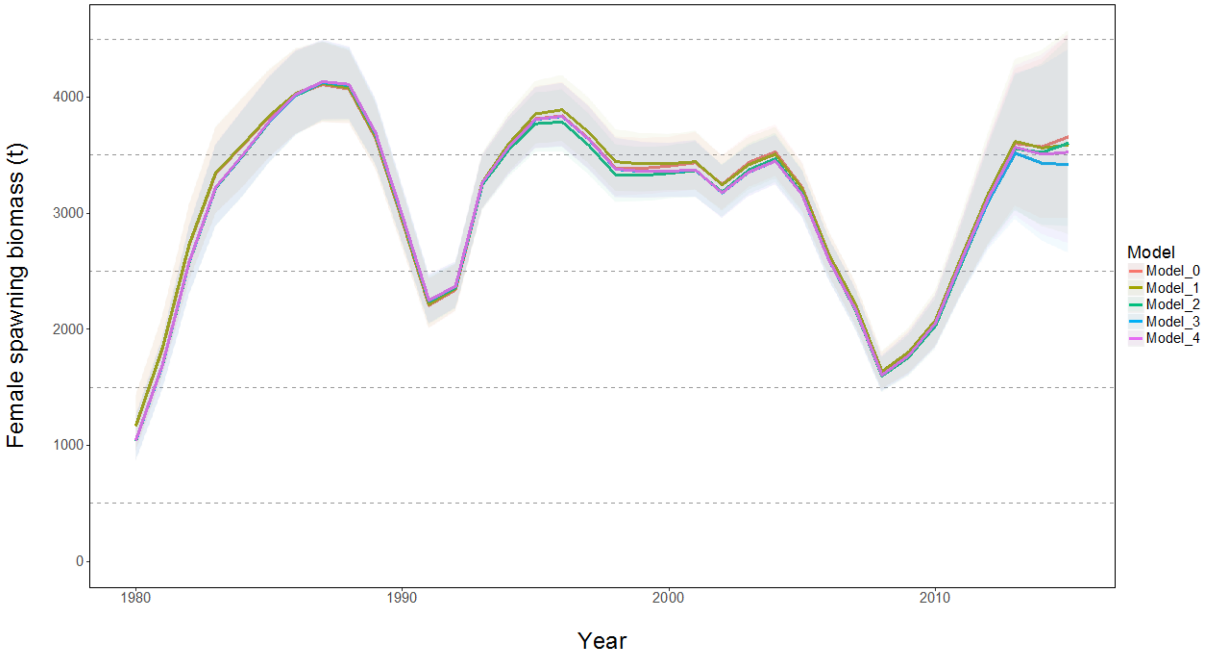

Figure 7. Model fits to acoustic trawl survey estimates for abundance (top set of panels) and biomass (bottom set) for EBS pollock based on the 2015 model configuration.

4. Evaluate whether weightings are appropriate given model fit

Part of the work on this consisted of developing statistics for the CIE panel during the review.

5. Consider components of variability of the F_{msy} estimation and the effect of the prior

The reviews note a number of concerns regarding the pdf of F_{msy} estimate. In an effort to clarify which sources of variability can contribute to the pdf a set of alternatives were developed as follows:

Model	Description
Model 0	The 2015 model used for management advice
Model 1	Fix mean weight-at-age (instead of propagating process-error uncertainty)
Model 2	Fix selectivity to mean
Model 3	Fix both selectivity and mean weight
Model 4	Set steepness to have prior variance alone (omit influence of “data”)

Note that the point of developing these models was to evaluate how the pdf of F_{msy} changes due to different model specifications. In particular, Model 4 could be viewed as an extreme case of uncertainty since this configuration assumes the only information on steepness comes from “expert advice” based on the prior distribution specified.

Results
To come…

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Lauthenberg et al. (In press). To come