Appendix A2.1:
Description of the Tanner Crab Stock Assessment Model, Version 2

Introduction

The "TCSAM02" (Tanner Crab Stock Assessment Model, version 2) modeling framework was developed "from scratch" to eliminate many of the constraints imposed on potential future assessment models by TCSAM2013, the previous assessment model framework (Stockhausen, 2016). Like TCSAM2013, TCSAM02 uses AD Model Builder libraries as the basis for model optimization using a maximum likelihood (or Bayesian) approach. The model code for TCSAM02 is available on GitHub (the 2018 assessment model code is available at "201809AssessmentVersion" while the current (May, 2019) development version is "201905CPT"). TCSAM02 was first used for the Tanner crab assessment in 2017 (Stockhausen, 2017) and will be used until a transition is made to Gmacs (the Generalized Model for Alaska Crab Stocks). Gmacs is intended to be used for all crab stock assessments conducted for the North Pacific Fisheries Management Council (NPFMC), including both lithodid (king crab) and *Chionoecetes* (Tanner and snow crab) stocks, while TCSAM02 is specific to *Chionoecetes* biology (i.e., terminal molt).

TCSAM02 is referred to here as a "modeling framework" because, somewhat similar to Stock Synthesis (Methot and Wetzel, 2013), model structure and parameters are defined "on-the-fly" using control files—rather than editing and re-compiling the underlying code. In particular, the number of fisheries and surveys, as well as their associated data types (abundance, biomass, and /or size compositions) and the number and types of time blocks defined for every model parameter, are defined using control files in TCSAM02 and have not been pre-determined. Priors can be placed on any model parameter. New data types (e.g., growth data) can also be included in the model optimization that could not be fit with TCSAM2013. Additionally, status determination and OFL calculations can be done directly within a TCSAM02 model run, rather having to run a separate "projection model".

Model Description

A. General population dynamics

TCSAM02 is a stage/size-based population dynamics model. Population abundance at the start (July 1) of year y in the model, $n_{y,x,m,s,z}$, is characterized by sex x (male, female), maturity state m (immature, mature), shell condition s (new shell, old shell), and size z (carapace width, CW). Changes in abundance due to natural mortality, molting and growth, maturation, shell aging, fishing mortality and recruitment are tracked on an annual basis. Because the principal crab fisheries occur during the winter, the model year runs from July 1 to June 30 of the following calendar year.

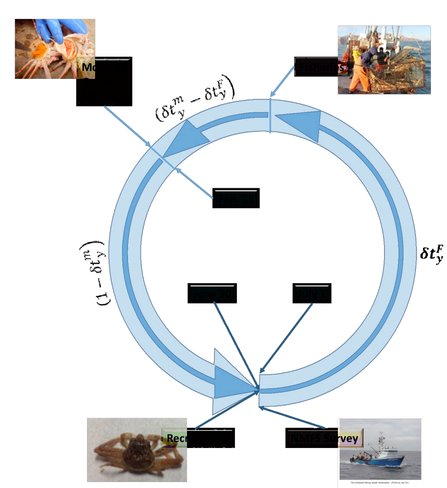


Fig. 1. Timing of annual events in TCSAM02 when fisheries occur before molting/growth/mating.

The order of calculation steps to project population abundance from year y to y+1 depends on the assumed timing of the fisheries (δt_y^F) relative to molting/growth/mating (δt_y^m) in year y. The steps when the fisheries occur before molting/growth/mating ($\delta t_y^F \leq \delta t_y^m$) are outlined below first (Steps A1.1-A1.4), followed by the steps when molting/growth/mating occurs after the fisheries ($\delta t_y^m < \delta t_y^F$; Steps A2.1-A2.4).

A1. Calculation sequence when $\boldsymbol{\delta t_y^F \leq \delta t_y^m}$

Step A1.1: Survival prior to fisheries
Natural mortality is applied to the population from the start of the model year (July 1) until just prior to prosecution of pulse fisheries for year *y* at δt_y^F. The numbers surviving to δt_y^F in year *y* are given by:

$n_{y,x,m,s,z}^1 = e^{-M_{y,x,m,s,z} \cdot \delta t_y^F} \cdot n_{y,x,m,s,z}$	A1.1

where *M* represents the annual rate of natural mortality in year *y* on crab classified as *x*, *m*, *s*, *z*.

Step A1.2: Prosecution of the fisheries
The directed and bycatch fisheries are modeled as simultaneous pulse fisheries occurring at δt_y^F in year *y*. The numbers that remain after the fisheries are prosecuted are given by:

$n_{y,x,m,s,z}^2 = e^{-F_{y,x,m,s,z}^T} \cdot n_{y,x,m,s,z}^1$	A1.2

where $F_{y,x,m,s,z}^T$ represents the total fishing mortality (over all fisheries) on crab classified as *x*, *m*, *s*, *z* in year *y*.

Step A1.3: Survival after fisheries to time of molting/growth/mating
Natural mortality is again applied to the population from just after the fisheries to the time just before molting/growth/mating occurs for year *y* at δt_y^m (generally Feb. 15). The numbers surviving to δt_y^m in year *y* are given by:

$n_{y,x,m,s,z}^3 = e^{-M_{y,x,m,s,z} \cdot (\delta t_y^m - \delta t_y^F)} \cdot n_{y,x,m,s,z}^2$	A1.3

where, as above, *M* represents the annual rate of natural mortality in year *y* on crab classified as *x*, *m*, *s*, *z*.

Step A1.4: Molting, growth, and maturation
The changes in population structure due to molting, growth and maturation of immature (new shell) crab, as well as the change in shell condition for mature new shell (MAT, NS) crab to mature old shell (MAT, OS) crab due to aging, are given by:

$n_{y,x,MAT,NS,z}^4 = \phi_{y,x,z} \cdot \sum_{z'} \Theta_{y,x,z,z'} \cdot n_{y,x,IMM,NS,z'}^3$	A1.4a
$n_{y,x,IMM,NS,z}^4 = (1 - \phi_{y,x,z}) \cdot \sum_{z'} \Theta_{y,x,z,z'} \cdot n_{y,x,IMM,NS,z'}^3$	A1.4b
$n_{y,x,MAT,OS,z}^4 = n_{y,x,MAT,OS,z}^3 + n_{y,x,MAT,NS,z}^3$	A1.4c

where $\Theta_{y,x,z,z'}$ is the growth transition matrix in year *y* for an immature new shell (IMM, NS) crab of sex *x* and pre-molt size *z'* to post-molt size *z* and $\phi_{y,x,z}$ is the probability that a just-molted crab of sex *x* and post-molt size *z* has undergone its terminal molt to maturity (MAT). All crab that molted remain new shell (NS) crab. Additionally, all mature crab that underwent terminal molt to maturity the previous year are assumed to change shell condition from new shell to old shell (A1.4c). Note that the numbers of immature old shell (IMM, OS) crab are identically zero in the current model because immature crab are assumed to molt each year until they undergo the terminal molt to maturity; consequently, the "missing" equation for *m=IMM*, *s=OS* is unnecessary.

Step A1.5: Survival to end of year, recruitment, and update to start of next year
Finally, the population abundance at the start of year *y*+1, due to natural mortality on crab from just after the time of molting/growth/mating in year *y* until the end of the model year (June 30) and recruitment ($R_{y,x,z}$) at the end of year *y* of immature new shell (IMM, NS) crab by sex *x* and size *z*, is given by:

$n_{y+1,x,m,s,z} = \begin{cases} e^{-M_{y,x,IMM,NS,z}\cdot(1-\delta t_y^m)} \cdot n_{y,x,IMM,NS,z}^4 + R_{y,x,z} & m = IMM, s = NS \\ e^{-M_{y,x,m,s,z}\cdot(1-\delta t_y^m)} \cdot n_{y,x,m,s,z}^4 & otherwise \end{cases}$	A1.5

A2. Calculation sequence when $\delta t_y^m < \delta t_y^F$

Step A2.1: Survival prior to molting/growth/mating
As in the previous sequence, natural mortality is first applied to the population from the start of the model year (July 1), but this time until just prior to molting/growth/mating in year *y* at δt_y^m (generally Feb. 15). The numbers surviving at δt_y^m in year *y* are given by:

$n_{y,x,m,s,z}^1 = e^{-M_{y,x,m,s,z}\cdot\delta t_y^m} \cdot n_{y,x,m,s,z}$	A2.1

where *M* represents the annual rate of natural mortality in year *y* on crab classified as *x*, *m*, *s*, *z*.

Step A2.2: Molting, growth, and maturation
The changes in population structure due to molting, growth and maturation of immature new shell (IMM, NS) crab, as well as the change in shell condition for mature new shell (MAT, NS) crab to mature old shell (MAT, OS) crab due to aging, are given by:

$n_{y,x,MAT,NS,z}^2 = \phi_{y,x,z} \cdot \sum_{z'} \Theta_{y,x,z,z'} \cdot n_{y,x,IMM,NS,z'}^1$	A2.2a
$n_{y,x,IMM,NS,z}^2 = (1 - \phi_{y,x,z}) \cdot \sum_{z'} \Theta_{y,x,z,z'} \cdot n_{y,x,IMM,NS,z'}^1$	A2.2b
$n_{y,x,MAT,OS,z}^2 = n_{y,x,MAT,OS,z}^1 + n_{y,x,MAT,NS,z}^1$	A2.2c

where $\Theta_{y,x,z,z'}$ is the growth transition matrix in year *y* for an immature new shell (IMM, NS) crab of sex *x* and pre-molt size *z'* to post-molt size *z* and $\phi_{y,x,z}$ is the probability that a just-molted crab of sex *x* and post-molt size *z* has undergone its terminal molt to maturity. Additionally, mature new shell (MAT, NS) crab that underwent their terminal molt to maturity the previous year are assumed to change shell condition from new shell to old shell (A2.2c). Again, the numbers of immature old shell crab are identically zero because immature crab are assumed to molt each year until they undergo the terminal molt to maturity.

Step A2.3: Survival after molting/growth/mating to prosecution of fisheries
Natural mortality is again applied to the population from just after molting/growth/mating to the time at which the fisheries occur for year *y* (at δt_y^F). The numbers surviving at δt_y^F in year *y* are then given by:

$n^3_{y,x,m,s,z} = e^{-M_{y,x,m,s,z} \cdot (\delta t^F_y - \delta t^m_y)} \cdot n^2_{y,x,m,s,z}$	A2.3

where, as above, *M* represents the annual rate of natural mortality in year *y* on crab classified as *x*, *m*, *s*, *z*.

Step A2.4: Prosecution of the fisheries
The directed fishery and bycatch fisheries are modeled as pulse fisheries occurring at δt^F_y in year *y*. The numbers that remain after the fisheries are prosecuted are given by:

$n^4_{y,x,m,s,z} = e^{-F^T_{y,x,m,s,z}} \cdot n^3_{y,x,m,s,z}$	A2.4

where $F^T_{y,x,m,s,z}$ represents the total fishing mortality (over all fisheries) on crab classified as *x*, *m*, *s*, *z* in year *y*.

Step A2.5: Survival to end of year, recruitment, and update to start of next year
Finally, population abundance at the start of year *y*+1 due to natural mortality on crab from just after prosecution of the fisheries in year *y* until the end of the model year (June 30) and recruitment of immature new (IMM, NS) shell crab at the end of year *y* ($R_{y,x,z}$) and are given by:

$n_{y+1,x,m,s,z} = \begin{cases} e^{-M_{y,x,IMM,NS,z} \cdot (1-\delta t^F_y)} \cdot n^4_{y,x,IMM,NS,z} + R_{y,x,z} & m = IMM, s = NS \\ e^{-M_{y,x,m,s,z} \cdot (1-\delta t^F_y)} \cdot n^4_{y,x,m,s,z} & otherwise \end{cases}$	A2.5

B. Parameter specification

Because parameterization of many model processes (e.g., natural mortality, fishing mortality) in TCSAM02 is fairly flexible, it is worthwhile discussing how model processes and their associated parameters are configured in TCSAM02 before discussing details of the model processes themselves. Each type of model process has a set of (potentially estimable) model parameters and other information associated with it, but different "elements" of a model process can be defined that apply, for example, to different segments of the population and/or during different time blocks. In turn, several "elements" of a model parameter associated with a model process may also be defined (and applied to different elements of the process). At least one combination of model parameters and other information associated with a model process must be defined—i.e., one process element must be defined.

Model processes and parameters are configured in a "ModelParametersInfo" file, one of the three control files required for a model run (the others are the "ModelConfiguration" file and the "ModelOptions" file). As an example of the model processes and parameter specification syntax, Text Box 1 presents the part of a "ModelParametersInfo" file concerned with specifying fishing processes in the directed Tanner crab fishery.

In Text Box 1, the keyword "fisheries" identifies the model process in question. The first section, following the "PARAMETER_COMBINATIONS" keyword (up to the first set of triple blue dots), specifies the indices associated with fishing process parameters (pHM, pLnC, pDC1, pDC2, pDC3, pDC4, pDevsLnC, pLnEffX, pLgtRet), selectivity and retention functions (idxSelFcn, idxRetFcn), and effort averaging time period (effAvgID) that apply to a single fishing process element. In this example, the indices for the selectivity and retention functions, as well as those for the effort averaging time period, constitute the "other information" specified for each fishing process element. Each fishing process element in turn applies to a specific fishery (FISHERY=1 indicates the directed fishery, in this case), time block (specified by YEAR_BLOCK), and components of the model population (specified by SEX, MATURITY STATE, and SHELL CONDITION). Using indices to identify which parameters and selectivity and retention functions apply to a given combination of fishery/time block/sex/maturity state/shell condition allows one to "share" individual parameters and selectivity and retention functions across different fishery/time block/sex/maturity state/shell condition combinations.

The second section (following the "PARAMETERS" keyword) determines the characteristics for each of the fishing process parameters, organized by parameter name (note: the parameters associated with the different selectivity and retention functions are specified in a different section of the ModelParametersInfo file). Here, each parameter name corresponds to an ADMB "param_init_bounded_number_vector" in the model code—the exception being pDevsLnC, which corresponds to an ADMB "param_init_bounded_vector_vector".

Each row under a "non-devs" parameter name in the fisheries section (e.g., pLnC) specifies the index used to associate an element of the parameter with the fishing processes defined in the PARAMETER_COMBINATIONS section, as well as characteristics of the element in the associated ADMB number_vector (upper and lower bounds, initial value, and initial estimation phase), various flags for initialization ("jitter", "resample"), definition of an associated prior probability distribution, and a label. Each row under a "devs" parameter name (e.g., pDevsLnC) specifies much the same information for the associated ADMB devs vector, with the "read" flag replacing the "initial value" entry. If "read?" is TRUE, then a vector of initial values is read from the file after all "info" rows for the devs parameter have been read. The "jitter" flag (if set to TRUE) provides the ability to change the initial value for an element of a non-devs parameter using a randomly selected value based on the element's upper and lower bounds. For a devs parameter, an element with jitter set to TRUE is initialized using a vector of randomly-generated numbers (subject to being a devs vector within the upper and lower bounds). The "resample" flag was intended to specify an alternative method to providing randomly-generated initial values (based on an element's prior probability distribution, rather than its upper and lower bounds), but this has not yet been fully implemented.

Some model processes apply only to specific segments of the population (e.g., growth only applies to immature, new shell crab). In general, though, a model process element can be defined to apply to any segment of the population (by specifying SEX, MATURITY STATE, and SHELL CONDITION appropriately) and range of years (by specifying YEAR_BLOCK). In turn, an element of a parameter may be "shared" across multiple processes by specifying the element's index in multiple rows of a PARAMETERS_COMBINATION block.

```
#------------------------------
# Fishery parameters
#------------------------------
fisheries #process name
PARAMETER_COMBINATIONS
42  #number of rows defining parameter combinations for all fisheries
#Directed Tanner Crab Fishery (TCF)
#                                           |MATURITY|SHELL|                                |pDevs| pLn | pLgt| idx  | idx  |  eff  |
#id  FISHERY  YEAR_BLOCK             SEX    | STATE  |COND |  pHM  pLnC pDC1 pDC2 pDC3 pDC4 | LnC | EffX| Ret |SelFcn|RetFcn| AvgID | label
1       1     [-1:1964]              MALE      ALL     ALL     1     1    0     0    0    0      0     0    0     9      5       0     TCF:_M_T1
2       1     [1965:1984;1987:1990]  MALE      ALL     ALL     1     2    0     0    0    0      1     0    0     9      5       0     TCF:_M_T2
3       1     [1991:1996]            MALE      ALL     ALL     1     2    0     0    0    0      1     0    0    10      6       0     TCF:_M_T3
4       1     [2005:2009]            MALE      ALL     ALL     1     2    0     0    0    0      1     0    1    11      7       0     TCF:_M_T4
5       1     [2013:-1]              MALE      ALL     ALL     1     2    0     0    0    0      1     0    1    12      8       0     TCF:_M_T5
6       1     [-1:1964]              FEMALE    ALL     ALL     1     1    0     1    0    0      0     0    0    13      0       0     TCF:_F_T1
7       1     [1965:1984;1987:1996]  FEMALE    ALL     ALL     1     2    0     1    0    0      1     0    0    13      0       0     TCF:_F_T2
8       1     [2005:2009;2013:-1]    FEMALE    ALL     ALL     1     2    0     1    0    0      1     0    0    14      0       0     TCF:_F_T3

…
PARAMETERS
pHM #handling mortality (0-1)
3   #number of parameters
#   |   limits    |       | initial | start |         |-       priors          -|
#id |lower   upper|jitter?| value   | phase |resample?| wgt| type| params| consts| label
1      0       1    OFF     0.321      -1       OFF      1   none   none     none    handling_mortality_for_crab_pot_fisheries

…
pLnC #base (ln-scale) capture rate (mature males)
9    #number of parameters
#   |   limits    |       |  initial    | start |         |-       priors          -|
#id |lower   upper|jitter?|   value     | phase |resample?| wgt| type| params| consts| label
 1    -15      15     OFF    -2.995732274    -1       OFF      1   none  none     none      TCF:_base_capture_rate,_pre-1965_(=0.05)
 2    -15      15     ON     -1.164816291     1       OFF      1   none  none     none      TCF:_base_capture_rate,_1965+

…
pDC1 #main temporal ln-scale capture rate offset
0    #number of parameters
pDC2 #ln-scale capture rate offset for female crabs
6    #number of parameters
#   |   limits    |       |  initial    | start |         |-       priors           -|
#id |lower  upper |jitter?|   value     | phase |resample?|  wgt  type  params  consts| label
1    -5.0    5.0     ON    -2.058610432    1       OFF       1.0  none   none    none   TCF:_female_offset

…
pDevsLnC #annual ln-scale capture rate deviations
6        #number of parameter vectors
#   | index |              index                   |       |   limits     |        |initial |start |         |-      priors               -|
#id |  type |              block                   | read? |lower   upper | jitter?| value  |phase |resample?|  wgt | type | params | consts |label
 1     YEAR   [1965:1984;1987:1996;2005:2009;2013:-1]  FALSE   -15     15       ON       0       1      OFF       2.0  normal    0 1     none    TCF:_T2345
…
```

Text Box 1. Abbreviated example of process and parameter specifications in a “ModelParametersInfo” file for fishing mortality in TCSAM02. Only parameter combinations and parameters relevant to the directed fishery are shown. Input values are in black text, comments are in green, triple blue dots indicate additional input lines not shown.

C. Model processes: natural mortality

The natural mortality rate applied to crab of sex *x*, maturity state *m*, shell condition *s*, and size *z* in year *y*, $M_{y,x,m,s,z}$, can be specified using one of two parameterizations. The first parameterization option uses a ln-scale parameterization with an option to include an inverse- size dependence using Lorenzen's approach:

$lnM_{y,x,m,s} = \mu^0_{y,x,m,s} + \sum_{i=1}^{4} \delta\mu^i_{y,x,m,s}$	C.1a
$M_{y,x,m,s,z} = \begin{cases} \exp(lnM_{y,x,m,s}) & if\ Lorenzen\ option\ is\ not\ selected \\ \exp(lnM_{y,x,m,s}) \cdot \frac{z_{base}}{z} & if\ Lorenzen\ option\ is\ selected \end{cases}$	C.1b C.1c

where the μ^0 and the $\delta\mu^i$'s are (potentially) estimable parameters defined for time block *T*, sex *S* (MALE, FEMALE, or ANY), maturity *M* (IMMATURE, MATURE, or ANY), and shell condition *S* (NEWSHELL, OLDSHELL, or ANY), and $\{y,x,m,s\}$ falls into the set $\{T,X,M,S\}$. In Eq. C.1c, z_{base} denotes the specified reference size (mm CW) for the inverse-size dependence.

The second parameterization option uses an arithmetic parameterization in order to provide backward compatibility with the 2016 assessment model based on TCSAM2013. In TCSAM2013, the natural mortality rate $M_{y,x,m,s,z}$ was parameterized using:

$M_{y,x,m=IMM,s,z} = M^{base} \cdot \delta M_{IMM}$	C.2a
$M_{y,x,m=MAT,s,z} = \begin{cases} M^{base} \cdot \delta M_{x,MAT} & otherwise \\ M^{base} \cdot \delta M_{x,MAT} \cdot \delta M^T_{x,MAT} & 1980 \le y \le 1984 \end{cases}$	C.2b

where M^{base} was a fixed value (0.23 yr^{-1}), δM_{IMM} was a multiplicative factor applied for all immature crab, the $\delta M_{x,MAT}$ were sex-specific multiplicative factors for mature crab, and the $\delta M^T_{x,MAT}$ were additional sex-specific multiplicative factors for mature crab during the 1980-1984 time block (which has been identified as a period of enhanced natural mortality on mature crab, the mechanisms for which are not understood). While it would be possible to replicate Eq.s C.2a and C.2b using ln-scale parameters, TCSAM2013 also placed informative arithmetic-scale priors on some of these parameters—and this could not be duplicated on the ln-scale. Consequently, the second option uses the following parameterization, where the parameters (and associated priors) are defined on the arithmetic-scale:

$lnM_{y,x,m,s} = \ln[\mu^0_{y,x,m,s}] + \sum_{i=1}^{4} \ln[\delta\mu^i_{y,x,m,s}]$	C.3a

A system of equations identical to C.2a-b can be achieved under the following assignments:

$\mu^0_{\{y,x,m,s\}\in\{T=ALL,X=ALL,M=ALL,S=ALL\}} = M^{base}$	C.4a
$\delta\mu^1_{\{y,x,m,s\}\in\{T=ALL,X=ALL,M=IMM,S=ALL\}} = \delta M_{IMM}$	C.4e
$\delta\mu^1_{\{y,x,m,s\}\in\{T=ALL,X=x,M=MAT,S=ALL\}} = \delta M_{x,MAT}$	C.4f
$\delta\mu^2_{\{y,x,m,s\}\in\{T=1980-1984,X=x,M=MAT,S=ALL\}} = \delta M^T_{x,MAT}$	C.4g

where unassigned $\delta\mu^{i}_{y,x,m,s}$ are set equal to 1. Pending further model testing using alternative model configurations, the TCSAM2013 option is standard.

It is worth noting explicitly that, given the number of potential parameters above that could be used, extreme care must be taken when defining a model to achieve a set of parameters that are not confounded and are, at least potentially, estimable.

D. Model processes: growth

Because Tanner crab are assumed to undergo a terminal molt to maturity, in TCSAM02 only immature crab experience growth. Annual growth of immature crab is implemented as using two options, the first based on a formulation used in Gmacs and the second (mainly for purposes of backward compatibility) based on that used in TCSAM2013. In TCSAM02, growth can vary by time block and sex, so it is expressed by sex-specific transition matrices for time block t, $\Theta_{t,x,z,z'}$, that specify the probability that crab of sex x in pre-molt size bin z' grow to post-molt size bin z at molting.

In the Gmacs-like approach (the standard approach as of May, 2017), the sex-specific growth matrices are given by:

$\Theta_{t,x,z,z'} = c_{t,x,z'} \cdot \int_{z-bin/2}^{z+bin/2} \Gamma\left(\frac{z'' - \bar{z}_{t,x,z'}}{\beta_{t,x}}\right) dz''$	Sex-specific (x) transition matrix for growth from pre-molt z' to post-molt z, with $z \geq z'$	D.1a
$c_{t,x,z'} = \left[\int_{z'}^{\infty} \Gamma\left(\frac{z'' - \bar{z}_{t,x,z'}}{\beta_{t,x}}\right) dz''\right]^{-1}$	Normalization constant so $1 = \sum_{z} \Theta_{t,x,z,z'}$	D.1b
$\bar{z}_{t,x,z'} = e^{a_{t,x}} \cdot z'^{b_{t,x}}$	Mean size after molt, given pre-molt size z'	D.1c

where the integral represents a cumulative gamma distribution across the post-molt (z) size bin. This approach may have better numerical stability properties than the TCSAM2013 approach below.

The TCSAM2013 approach is an approximation to the Gmacs approach, where the sex-specific growth matrices $\Theta_{t,x,z,z'}$ are given by

$\Theta_{t,x,z,z'} = c_{t,x,z'} \cdot \Delta_{z,z'}{}^{\alpha_{t,x,z'}-1} \cdot e^{-\frac{\Delta_{z,z'}}{\beta_{t,x}}}$	Sex-specific (x) transition matrix for growth from pre-molt z' to post-molt z, with $z \geq z'$	D.2a
$c_{t,x,z'} = \left[\sum_{z'} \Delta_{z,z'}{}^{\alpha_{t,x,z'}-1} \cdot e^{-\frac{\Delta_{z,z'}}{\beta_{t,x}}}\right]^{-1}$	Normalization constant so $1 = \sum_{z} \Theta_{t,x,z,z'}$	D.2b
$\Delta_{z,z'} = z - z'$	Actual growth increment	D.2c
$\alpha_{t,x,z'} = \left[\bar{z}_{t,x,z'} - z'\right]/\beta_{t,x}$	Mean molt increment, scaled by $\beta_{t,x}$	D.2d
$\bar{z}_{t,x,z'} = e^{a_{t,x}} \cdot z'^{b_{t,x}}$	Mean size after molt, given pre-molt size z'	D.2e

In both approaches, the $a_{t,x}$, $b_{t,x}$, and $\beta_{t,x}$ are arithmetic-scale parameters with imposed bounds. $\Theta_{t,x,z,z'}$ is used to update the numbers-at-size for immature crab, $n_{y,x,z}$, from pre-molt size z' to post-molt size z using:

$n^{+}_{y,x,z} = \sum_{z'} \Theta_{t,x,z,z'} \cdot n_{y,x,z'}$	numbers at size of immature crab after growth	D.3

where *y* falls within time block *t* (see also Eq.s A1.4a-b and A2.2a-b).

Priors using normal distributions are imposed on $a_{t,x}$ and $b_{t,x}$ in TCSAM2013, with the values of the hyper-parameters hard-wired in the model code. While priors may be defined for the associated parameters here, these are identified by the user in the model input files and are not hard-wired in the model code.

E. Model processes: maturity (terminal molt)

Maturation of immature crab in TCSAM02 is based on a similar approach to that taken in TCSAM2013, except that the sex- and size-specific probabilities of terminal molt for immature crab, $\phi_{t,x,z}$ (where size *z* is post-molt size), can vary by time block. After molting and growth, the numbers of (new shell) crab at post-molt size *z* remaining immature, $n^{+}_{y,x,IMM,NS,z}$, and those maturing, $n^{+}_{x,MAT,NS,z}$, are given by:

$n^{+}_{y,x,IMM,NS,z} = \left(1 - \phi_{t,x,z}\right) \cdot n_{y,x,IMM,NS,z}$ $n^{+}_{y,x,MAT,NS,z} = \phi_{t,x,z} \cdot n_{y,x,IMM,NS,z}$	crab remaining immature crab maturing (terminal molt)	E.1a E.1b

where *y* falls in time block *t* and $n_{y,x,IMM,NS,z}$ is the number of immature, new shell crab of sex *x* at post-molt size *z*.

The sex- and size-specific probabilities of terminal molt, $\phi_{t,x,z}$, are related to logit-scale model parameters $p^{mat}_{t,x,z}$ by:

$\phi_{t,FEM,z} = \begin{cases} \dfrac{1}{1 + e^{p^{mat}_{t,FEM,z}}} & z \le z^{mat}_{t,FEM} \\ 1 & z > z^{mat}_{t,FEM} \end{cases}$	female probabilities of maturing at post-molt size *z*	E.2a
$\phi_{t,MALE,z} = \begin{cases} \dfrac{1}{1 + e^{p^{mat}_{t,MALE,z}}} & z \le z^{mat}_{t,MALE} \\ 1 & z > z^{mat}_{t,MALE} \end{cases}$	male probabilities of maturing at post-molt size *z*	E.2b

where the $z^{mat}_{t,x}$ are constants specifying the minimum pre-molt size at which to assume all immature crab will mature upon molting. The $z^{mat}_{t,x}$ are used here pedagogically; in actuality, the user specifies the *number* of logit-scale parameters to estimate (one per size bin starting with the first bin) for each sex, and this determines the $z^{mat}_{t,x}$ used above. This parameterization is similar to that implemented in TCSAM2013 for the 2016 assessment model.

Second difference penalties are applied to the parameter estimates in TCSAM2013's objective function to promote relatively smooth changes in these parameters with size. Similar penalties (smoothness, non-decreasing) can be applied in TCSAM02.

F. Model processes: recruitment

Recruitment in TCSAM02 consists of immature new shell crab entering the population at the end of the model year (June 30). Recruitment in TCSAM02 has a similar functional form to that used in TCSAM2013, except that the sex ratio at recruitment is not fixed at 1:1 and multiple time blocks can be specified. In TCSAM2013, two time blocks were defined: "historical" (model start to 1974) and "current" (1975-present), with "current" recruitment starting in the first year of NMFS survey data. In TCSAM02, recruitment in year *y* of immature new shell crab of sex *x* at size *z* is specified as

$R_{y,x,z} = \dot{R}_y \cdot \ddot{R}_{y,x} \cdot \dddot{R}_{y,z}$	recruitment of immature, new shell crab by sex and size bin	F.1

where $\dot{R}_y$ represents total recruitment in year *y* and $\ddot{R}_{y,x}$ represents the fraction of sex *x* crab recruiting, and $\dddot{R}_{y,z}$ is the size distribution of recruits, which is assumed identical for males and females.

Total recruitment in year *y*, $\dot{R}_y$, is parameterized as

$\dot{R}_y = e^{pLnR_t + \delta R_{t,y}} \quad y \in t$	total recruitment in year *y*	F.2

where *y* falls within time block *t*, $pLnR_t$ is the ln-scale mean recruitment parameter for *t*, and $\delta R_{t,y}$ is an element of a "devs" parameter vector for *t* (constrained such that the elements of the vector sum to zero over the time block).

The fraction of crab recruiting as sex *x* in year *y* in time block *t* is parameterized using the logistic model

$\ddot{R}_{y,x} = \begin{cases} \dfrac{1}{1+e^{pLgtRx_t}} & x = MALE \\ 1 - \ddot{R}_{y,MALE} & x = FEMALE \end{cases} \quad y \in t$	sex-specific fraction recruiting in year *y*	F.3

where $pLgtRx_t$ is a logit-scale parameter determining the sex ratio in time block *t*.

The size distribution for recruits in time block *t*, $\dddot{R}_{t,z}$, is assumed to be a gamma distribution and is parameterized as

$\dddot{R}_{t,z} = c^{-1} \cdot \Delta_z^{\frac{\alpha_t}{\beta_t}-1} \cdot e^{-\frac{\Delta_z}{\beta_t}}$	size distribution of recruiting crab	F.4
$c_t = \sum_z \Delta_z^{\frac{\alpha_t}{\beta_t}-1} \cdot e^{-\frac{\Delta_z}{\beta_t}}$	normalization constant so that $1 = \sum_z \dddot{R}_{t,z}$	F.5
$\Delta_z = z + \delta z/2 - z_{min}$	offset from minimum size bin	F.6
$\alpha_t = e^{pLnRa_t}$	gamma distribution location parameter	F.7
$\beta_t = e^{pLnRb_t}$	gamma distribution shape parameter	F.8

where $pLnRa_t$ and $pLnRb_t$ are the ln-scale location and shape parameters and the constant δz is the size bin spacing.

A final time-blocked parameter, $pLnRCV_t$, is associated with the recruitment process representing the ln-scale coefficient of variation (cv) in recruitment variability in time block *t*. These parameters are used to apply priors on the recruitment "devs" in the model likelihood function.

G. Selectivity and retention functions

Selectivity and retention functions in TCSAM02 are specified independently from the fisheries and surveys to which they are subsequently applied. This allows a single selectivity function to be "shared" among multiple fisheries and/or surveys, as well as among multiple time block/sex/maturity state/shell condition categories, if so desired.

Currently, the following functions are available for use as selectivity or retention curves in a model:

$S_z = \left\{1 + e^{-\beta \cdot (z - z_{50})}\right\}^{-1}$	standard logistic	G.1
$S_z = \left\{1 + e^{-\beta \cdot (z - \exp(lnZ_{50}))}\right\}^{-1}$	logistic w/ alternative parameterization	G.2
$S_z = \left\{1 + e^{-\ln(19) \cdot \frac{(z - z_{50})}{\Delta z_{95-50}}}\right\}^{-1}$	logistic w/ alternative parameterization	G.3
$S_z = \left\{1 + e^{-\ln(19) \cdot \frac{(z - z_{50})}{\exp(ln\Delta z_{95-50})}}\right\}^{-1}$	logistic w/ alternative parameterization	G.4
$S_z = \left\{1 + e^{-\ln(19) \cdot \frac{(z - \exp(lnZ_{50}))}{\exp(ln\Delta z_{95-50})}}\right\}^{-1}$	logistic w/ alternative parameterization	G.5
$S_z = \frac{1}{1 + e^{-\beta_a \cdot (z - z_{a50})}} \cdot \frac{1}{1 + e^{\beta_d \cdot (z - z_{d50})}}$	double logistic	G.6
$S_z = \frac{1}{1 + e^{-\ln(19) \cdot \frac{(z - z_{a50})}{\Delta z_{a(95-50)}}}} \cdot \frac{1}{1 + e^{\ln(19) \cdot \frac{(z - z_{d50})}{\Delta z_{d(95-50)}}}}$	double logistic with alt. parameterization	G.7
$S_z = \frac{1}{1 + e^{-\ln(19) \cdot \frac{(z - z_{a50})}{\exp(ln\Delta z_{a(95-50)})}}} \cdot \frac{1}{1 + e^{\ln(19) \cdot \frac{(z - z_{d50})}{\exp(ln\Delta z_{d(95-50)})}}}$ $where\ z_{d50} = [z_{a50} + \exp(ln\Delta z_{a(95-50)}) + \exp(ln\Delta z_{d(95-50)})]$	double logistic with alt. parameterization	G.8
$S_z = \frac{1}{1 + e^{-\ln(19) \cdot \frac{(z - \exp(lnz_{a50}))}{\exp(ln\Delta z_{a(95-50)})}}} \cdot \frac{1}{1 + e^{\ln(19) \cdot \frac{(z - z_{d50})}{\exp(ln\Delta z_{d(95-50)})}}}$ $where\ z_{d50} = [\exp(lnz_{a50}) + \exp(ln\Delta z_{a(95-50)}) + \exp(ln\Delta z_{d(95-50)})]$	double logistic with alt. parameterization	G.9
$S_z = \frac{1}{1 + e^{-\beta_a \cdot (z - z_{a50})}} \cdot \frac{1}{1 + e^{\beta_d \cdot (z - [z_{a50} + \exp(lnZ_{d50-a50})])}}$	double logistic with alt. parameterization	G.10

A double normal selectivity function (requiring 6 parameters to specify) has also been implemented as an alternative to the double logistic functions. In the above functions, all symbols (e.g., β, z_{50}, Δz_{95-50}) represent parameter values, except "*z*" which represents crab size.

Selectivity parameters are defined independently of the functions themselves, and subsequently assigned. It is thus possible to "share" parameters across multiple functions. The "parameters" used in selectivity functions are further divided into mean parameters across a time block and annual deviations within a time block. To accommodate the 6-parameter double normal equation, six "mean" parameter sets (*pS1*, *pS2*,…, *pS6*) and six associated sets of "devs" parameter vectors (*pDevsS1*, *pDevsS2*,…, *pDevsS6*) are defined to specify the parameterization of individual selectivity/retention functions. Thus, for example, z_{50} in eq. F1 is actually expressed as $z_{50,y} = \bar{z}_{50} + \delta z_{50,y}$ in terms of model parameters *pS1* and $pDevsS1_y$, where $\bar{z}_{50} = pS1$ is the mean size-at-50%-selected over the time period and $\delta z_{50,y} = pDevsS1_y$ is the annual deviation.

Finally, three different options to normalize individual selectivity curves are provided: 1) no normalization, 2) specifying a fully-selected size, and 3) re-scaling such that the maximum value of the re-scaled function is 1. A normalization option must be specified in the model input files for each defined selectivity/retention curve.

H. Fisheries

Unlike TCSAM2013, which explicitly models 4 fisheries that catch Tanner crab (one as a directed fishery, three as bycatch), there is no constraint in TCSAM02 on the number of fisheries that can be incorporated in the model. All fisheries are modeled as "pulse" fisheries occurring at the same time.

TCSAM02 uses the Gmacs approach to modeling fishing mortality (also implemented in TCSAM2013). The total (retained + discards) fishing mortality rate, $F_{f,y,x,m,s,z}$, in fishery *f* during year *y* on crab in state *x*, *m*, *s*, and *z* (i.e., sex, maturity state, shell condition, and size) is related to the associated fishery capture rate $\phi_{f,y,x,m,s,z}$ by

$F_{f,y,x,m,s,z} = \left[h_{f,t} \cdot \left(1 - \rho_{f,y,x,m,s,z}\right) + \rho_{f,y,x,m,s,z}\right] \cdot \phi_{f,y,x,m,s,z}$	fishing mortality rate	H.1

where $h_{f,t}$ is the handling (discard) mortality for fishery *f* in time block t (which includes year *y*) and $\rho_{f,y,x,m,s,z}$ is the fraction of crabs in state *x*, *m*, *s*, *z* that were caught and retained (i.e., the retention function). The retention function is assumed to be identically 0 for females in a directed fishery and for both sexes in a bycatch fishery.

In TCSAM2013, the same retention function (in each of two time blocks) was applied to male crab regardless of maturity state or shell condition. Additionally, full retention of large males was assumed, such that the retention function essentially reached 1 at large sizes. In TCSAM02, different retention functions can be applied based on maturity state and/or shell condition, and "max retention" is now an (potentially) estimable logit-scale parameter. Thus, in TCSAM02, the retention function $\rho_{f,y,x,m,s,z}$ is given by

$\rho_{f,y,x,m,s,z} = \dfrac{1}{1 + e^{\rho_{f,t,x,m,s}}} \cdot R_{f,y,x,m,s,z}$	retention function	H.2

where *f* corresponds to the directed fishery, *y* is in time block *t*, *x*=MALE, $\rho_{f,t,x,m,s}$ is the corresponding logit-scale "max retention" parameter, and $R_{f,y,x,m,s,z}$ is the associated selectivity/retention curve.

If $n_{y,x,m,s,z}$ is the number of crab classified as *x*, *m*, *s*, *z* in year *y* just prior to the prosecution of the fisheries, then

$c_{f,y,x,m,s,z} = \dfrac{\phi_{f,y,x,m,s,z}}{F^T_{y,x,m,s,z}} \cdot \left[1 - e^{-F^T_{y,x,m,s,z}}\right] \cdot n_{y,x,m,s,z}$	number of crab captured	H.3

is the number of crab classified in that state that were *captured* by fishery *f*, where $F^T_{y,x,m,s,z} = \sum_f F_{f,y,x,m,s,z}$ represents the total (across all fisheries) fishing mortality on those crab. The number of crab retained in fishery *f* classified as *x*, *m*, *s*, *z* in year *y* is given by

$r_{f,y,x,m,s,z} = \dfrac{\rho_{f,y,x,m,s,z} \cdot \phi_{f,y,x,m,s,z}}{F^T_{y,x,m,s,z}} \cdot \left[1 - e^{-F^T_{y,x,m,s,z}}\right] \cdot n_{y,x,m,s,z}$	number of retained crab	H.4

while the number of discarded crab, $d_{f,y,x,m,s,z}$, is given by

$d_{f,y,x,m,s,z} = \frac{(1-\rho_{f,y,x,m,s,z}) \cdot \phi_{f,y,x,m,s,z}}{F^T_{y,x,m,s,z}} \cdot \left[1 - e^{-F^T_{y,x,m,s,z}}\right] \cdot n_{y,x,m,s,z}$	number of discarded crab	H.5

and the discard mortality, $dm_{f,y,x,m,s,z}$, is

$dm_{f,y,x,m,s,z} = \frac{h_{f,y} \cdot (1-\rho_{f,y,x,m,s,z}) \cdot \phi_{f,y,x,m,s,z}}{F^T_{y,x,m,s,z}} \cdot \left[1 - e^{-F^T_{y,x,m,s,z}}\right] \cdot n_{y,x,m,s,z}$	discard mortality (numbers)	H.6

The capture rate $\phi_{f,y,x,m,s,z}$ (*not* the fishing mortality rate $F_{f,y,x,m,s,z}$) is modeled as a function separable into separate year and size components such that

$\phi_{f,y,x,m,s,z} = \phi_{f,y,x,m,s} \cdot S_{f,y,x,m,s,z}$	fishing capture rate	H.7

where $\phi_{f,y,x,m,s}$ is the fully-selected capture rate in year *y* and $S_{f,y,x,m,s,z}$ is the size-specific selectivity.

The fully-selected capture rate $\phi_{f,y,x,m,s}$ for *y* in time block *t* is parameterized in the following manner:

$\phi_{f,y,x,m,s} = \exp\left(\overline{lnC}_{f,t,x,m,s} + pDevsC_{f,y,x,m,s}\right)$	H.8

where the $pDevsC_{f,y,x,m,s}$ are elements for year *y* in time block *t* of a "devs" vectors representing annual variations from the ln-scale mean fully-selected capture rate $\overline{lnC}_{f,t,x,m,s}$. The latter is expressed in terms of model parameters as

$\overline{lnC}_{f,t,x,m,s} = pLnC_{f,t,x,m,s} + \sum_{i=1}^{4} \delta C^i_{f,t,x,m,s}$	H.9

where the $pLnC_{f,t,x,m,s}$ is the mean ln-scale capture rate (e.g., for mature males) and the $\delta C^i_{f,t,x,m,s}$ are ln-scale offsets.

I. Surveys

If $n_{y,x,m,s,z}$ is the number of crab classified as *x*, *m*, *s*, *z* in year *y* just prior to the prosecution of a survey, then the survey abundance, $a_{v,y,x,m,s,z}$, of crab classified in that state by survey *v* is given by

$a_{v,y,x,m,s,z} = q_{v,y,x,m,s,z} \cdot n_{y,x,m,s,z}$	survey abundance	I.1

where $q_{v,y,x,m,s,z}$ is the size-specific survey catchability on this component of the population.

The survey catchability $q_{v,y,x,m,s,z}$ is decomposed in the usual fashion into separate time block and size components such that, for *y* in time block *t*:

$q_{v,y,x,m,s,z} = q_{v,t,x,m,s} \cdot S_{v,t,x,m,s,z}$	survey catchability	I.2

where $q_{v,t,x,m,s}$ is the fully-selected catchability in time block *t* and $S_{v,t,x,m,s,z}$ is the size-specific survey selectivity.

The fully-selected catchability $q_{v,t,x,m,s}$ is parameterized in a fashion similar to that for fully-selected fishery capture rates (except that annual "devs" are not included) in the following manner:

$q_{v,t,x,m,s} = \exp\left(pLnQ_{v,t,x,m,s} + \sum_{i=1}^{4} \delta Q^{i}_{v,t,x,m,s}\right)$	I.3

where the $pLnQ_{v,t,x,m,s}$ is the mean ln-scale catchability (e.g., for mature males) and the $\delta Q^{i}_{v,t,x,m,s}$ are ln-scale offsets.

J. Model fitting: objective function equations

The TCSAM02 model is fit by minimizing an objective function, $\mathcal{O}$, with additive components consisting of: 1) negative log-likelihood functions based on specified prior probability distributions associated with user-specified model parameters, and 2) several negative log-likelihood functions based on input data components, of the form:

$\mathcal{O} = -2\sum_{p} \lambda_p \cdot \ln(\wp_p) - 2\sum_{l} \lambda_l \cdot \ln(\mathcal{L}_l)$	model objective function	J.1

where $\wp_p$ represents the *p*th prior probability function, $\mathcal{L}_l$ represents the *l*th likelihood function, and the λ's represent user-adjustable weights for each component.

Prior Probability Functions

Prior probability functions can be associated with each model parameter or parameter vector by the user in the model input files (see Section L below for examples on specifying priors).

Likelihood Functions

The likelihood components included in the model's objective function are based on normalized size frequencies and time series of abundance or biomass from fishery or survey data. Survey data optionally consists of abundance and/or biomass time series for males, females, and/or all crab (with associated survey cv's), as well as size frequencies by sex, maturity state, and shell condition. Fishery data consists of similar data types for optional retained, discard, and total catch components.

Size frequency components

Likelihood components involving size frequencies are based on multinomial sampling:

$\ln(\mathcal{L}) = \sum_{y} n_{y,c} \cdot \sum_{z} \{p^{obs}_{y,c,z} \cdot \ln(p^{mod}_{y,c,z} + \delta) - p^{obs}_{y,c,z} \cdot \ln(p^{obs}_{y,c,z} + \delta)\}$	multinomial log-likelihood	J.2

where the *y*'s are years for which data exists, "*c*" indicates the population component classifiers (i.e., sex, maturity state, shell condition) the size frequency refers to, $n_{y,c}$ is the classifier-specific effective sample size for year y, $p^{obs}_{y,c,z}$ is the observed size composition in size bin *z* (i.e., the size frequency normalized to sum to 1 across size bins for each year), $p^{mod}_{y,c,z}$ is the corresponding model-estimated size composition, and δ is a small constant. The manner in which the observed and estimated size frequencies for each data component are aggregated (e.g., over shell condition) prior to normalization is specified by the user in the model input files. Data can be entered in input files at less-aggregated levels of than will be used in the model; it will be aggregated in the model to the requested level before fitting occurs.

Aggregated abundance/biomass components
Likelihood components involving aggregated (over size, at least) abundance and or biomass time series can be computed using one of three potential likelihood functions: the normal, the lognormal, and the "norm2". The likelihood function used for each data component is user-specified in the model input files.

The ln-scale normal likelihood function is

$\ln(\mathcal{L}^N)_c \;= -\frac{1}{2}\sum_y \left\{ \frac{\left[a_{y,c}^{obs} - a_{y,c}^{mod}\right]^2}{\sigma_{y,c}^2} \right\}$	normal log-likelihood	J.3

where $a_{y,c}^{obs}$ is the observed abundance/biomass value in year y for aggregation level c, $a_{y,c}^{mod}$ is the associated model estimate, and $\sigma_{y,c}^2$ is the variance associated with the observation.

The ln-scale lognormal likelihood function is

$\ln(\mathcal{L}^{LN})_c \;= -\frac{1}{2}\sum_y \left\{ \frac{\left[ln\left(a_{y,c}^{obs} + \delta\right) - ln\left(a_{y,c}^{mod} + \delta\right)\right]^2}{\sigma_{y,c}^2} \right\}$	lognormal log-likelihood	J.4

where $a_{y,c}^{obs}$ is the observed abundance/biomass value in year y for aggregation level c, $a_{y,c}^{mod}$ is the associated model estimate, and $\sigma_{y,c}^2$ is the ln-scale variance associated with the observation.

For consistency with TCSAM2013, a third type, the "norm2", may also be specified

$\ln(\mathcal{L}^{N2})_x \;= -\frac{1}{2}\sum_y \left[a_{y,x}^{obs} - a_{y,x}^{mod}\right]^2$	"norm2" log-likelihood	J.5

This is equivalent to specifying a normal log-likelihood with $\sigma_{y,x}^2 \equiv 1.0$. This is the standard likelihood function applied in TCSAM2013 to fishery catch time series.

Growth data
Growth (molt increment) data can be fit as part of a TCSAM02 model. Multiple datasets can be fit at the same time. The likelihood for each dataset (L_d) is based on the same gamma distribution used in the growth model:

$\mathrm{L}_d = -\sum_{i \in d} ln\left\{\Gamma\left(\frac{\tilde{z}_i - \bar{z}_{y_i,x_i,z_i}}{\beta_{y_i,x_i}}\right)\right\}$	gamma log-likelihood	J.6

where z_i and $\tilde{z}_i$ are the pre-molt and post-molt sizes for individual i (of sex x_i collected in year y_i) in dataset d, respectively, $\bar{z}_{y_i,x_i,z_i}$ is the predicted mean post-molt size for individual i, and β_{y_i,x_i} is the scale factor for the gamma distribution corresponding to individual i.

Maturity ogive data
Annual maturity ogive data, the observed proportions-at-size of mature crab in a given year, can also be fit as part of a TCSAM02 model. This data consists of proportions of mature crab observed within a size bin, as well as the total number of observations for that size bin. The proportions are assumed to represent the fraction of new shell mature crab (i.e., having gone through terminal molt within the previous growth season) to all new shell crab within the size bin in that year. Multiple datasets can be fit at the same time. The likelihood for each observation is based on a binomial distribution with sample size equal to the

number of observations within the corresponding size bin, so the likelihood for each dataset (L_m) is given by:

$\mathrm{L}_m = \sum_{y,z} n_{y,z} \cdot \{p_{y,z}^{obs} \cdot \ln(p_{y,z}^{mod} + \delta) + (1 - p_{y,z}^{obs}) \cdot \ln(1 - p_{y,z}^{mod} + \delta)\}$	binomial log-likelihood	J.7

where y is a year, z is a size bin, $n_{y,z}$ is the total number of classified crab in size bin z in year y, $p_{y,z}^{obs}$ is the observed ratio of mature, new shell males to total new shell males in size bin z in year y, $p_{y,z}^{obs}$ is the corresponding model-predicted ratio, and δ is a small constant to prevent trying to calculate ln(0).

Effort data
In both TCSAM2013 and TCSAM02, fishery-specific effort data is used to predict annual fully-selected fishery capture rates for Tanner crab bycatch in the snow crab and Bristol Bay red king crab fisheries in the period before at-sea observer data is available (i.e., prior to 1991), based on the assumed relationship

$$F_{f,y} = q_f \cdot E_{f,y}$$

where $F_{f,y}$ is the fully-selected capture rate in fishery f in year y, q_f is the estimated catchability in fishery f, and $E_{f,y}$ is the reported annual, fishery-specific effort (in pots). In TCAM2013, the fishery q's are estimated directly from the ratio of fishery mean F to mean E over the time period (t_f) when at-sea observer data is available from which to estimate the $F_{f,y}$'s as parameters:

$$q_f = \frac{\sum_{y \in t_f} F_{f,y}}{\sum_{y \in t_f} E_{f,y}}.$$

Note that, in this formulation, the fishery q's are not parameters (i.e., estimated via maximizing the likelihood) in the model. In TCSAM2013, the time period over which q is estimated for each fishery is hard-wired. This approach is also available as an option in TCSAM02, although different time periods for the averaging can be specified in the model options file.

A second approach to effort extrapolation in which the fishery q's are fully-fledged parameters estimated as part of maximizing the likelihood is provided in TCSAM02 as an option, as well. In this case, the effort data is assumed to have a lognormal error distribution and the following negative log-likelihood components are included in the overall model objective function:

$$L_f = \sum_y \frac{\left(\ln(E_{f,y} + \delta) - \ln\left(\frac{F_{f,y}}{q_f} + \delta\right)\right)^2}{2 \cdot \sigma_f^2}$$

where σ_f^2 is the assumed ln-scale variance associated with the effort data and δ is a small value so that the arguments of the ln functions do not go to zero.

Aggregation fitting levels
A number of different ways to aggregate input data and model estimates prior to fitting likelihood functions have been implemented in TCSAM02. These include:

Abundance/Biomass by	Size Conpositions by	extended by
total	total	x
x		x, m
x, mature only	x	--
x, m		m
x, s		s
x, m, s	x, m	--
		s
	x, s	
	x, m, s	

where x, m, s refer to sex, maturity state and shell condition and missing levels are aggregated over. For size compositions that are "extended by" x, m, s, or $\{x, m\}$, this involves appending the size compositions corresponding to each combination of "extended by" factor levels, renormalizing the extended composition to sum to 1, and then fitting the extended composition using a multinomial likelihood.

K. Devs vectors

For TCSAM02 to accommodate arbitrary numbers of fisheries and time blocks, it is necessary to be able to define arbitrary numbers of "devs" vectors. This is currently not possible using the ADMB C++ libraries, so TCSAM02 uses an alternative implementation of devs vectors from that implemented in ADMB. For the 2017 assessment, an n-element "devs" vector was implemented using an n-element bounded parameter vector. with the final element of the "devs" vector defined as $-\sum_{n-1} v_i$, where v_i was the ith value of the parameter (or devs) vector, so that the sum over all elements of the devs vector was identically 0. Penalties were placed on the final element of the devs vector to ensure it was bounded in the same manner as the parameter vector. However, this approach was problematic when initializing the model with the values for the n-1 elements that defined the n-element devs vector, the value of the n-th element ($-\sum_{n-1} v_i$) was not guaranteed to satisfy the bounds placed on the vector. Thus, this approach was revised to allow specification of all n element values (the $v_n = -\sum_{n-1} v_i$ constraint was removed) while the likelihood penalty was changed to ensure the sum of the elements was 0. The new approach also has the advantage that it more closely follows the one used in ADMB to define "devs" vectors. Test runs with both approaches showed no effect on convergence to the MLE solution.

L. Priors for model parameters

A prior probability distribution can be specified for any element of model parameter. The following distributions are available for use as priors:

indicator	**parameters**	**constants**	**description**
none	none	none	no prior applied
ar1_normal	μ, σ	none	random walk with normal deviates
cauchy	x_0, γ	none	Cauchy pdf
chisquare	υ	none	χ^2 pdf
constant	min, max	none	uniform pdf
exponential	λ	none	exponential pdf
gamma	r, μ	none	gamma pdf
invchisquare	υ	none	inverse χ^2 pdf

invgamma	r, μ	none	inverse gamma pdf
invgaussian	μ, λ	none	inverse Gaussian pdf
lognormal	median, CV	none	lognormal pdf
logscale_normal	median, CV	none	normal pdf on ln-scale
normal	μ, σ	none	normal pdf
scaled_invchisquare	υ, s	none	inverse χ^2 scaled pdf
scaledCV_invchisquare	υ, CV	none	inverse χ^2 pdf, scaled by CV
t	υ	none	t distribution
truncated_normal	μ, σ	min, max	truncated normal pdf

M. Parameters and other information determined outside the model

Several nominal model parameters are not estimated in the model, rather they are fixed to values determined outside the model. These include Tanner crab handling mortality rates for discards in the crab fisheries (32.1%), the groundfish trawl fisheries (80%), and the groundfish pot fisheries (50%), as well the base rate for natural mortality (0.23 yr^{-1}). Sex- and maturity-state-specific parameters for individual weight-at-size have also been determined outside the model, based on fits to data collected on the NMFS EBS bottom trawl survey (Daly et al., 2016). Weight-at-size, $w_{x,m,z}$, is given by

$$w_{x,m,z} = a_{x,m} \cdot z^{b_{x,m}}$$

where

sex	**maturity state**	$\boldsymbol{a_{x,m}}$	$\boldsymbol{b_{x,m}}$
male	all states	0.000270	3.022134
female	immature	0.000562	2.816928
	mature	0.000441	2.898686

and size is in mm CW and weight is in kg.

N. OFL calculations and stock status determination

Overfishing level (OFL) calculations and stock status determination for Tanner crab are based on Tier 3 considerations for crab stocks as defined by the North Pacific Fishery Management Council (NPFMC; NPFMC 2016). Tier 3 considerations require life history information such as natural mortality rates, growth, and maturity but use proxies based on a spawner-per-recruit approach for F_{MSY}, B_{MSY}, and MSY because there is no reliable stock-recruit relationship. Equilibrium recruitment is assumed to be equal to the average recruitment over a selected time period (1982-present for Tanner crab). For Tier 3 stocks, the proxy for B_{MSY} is defined as 35% of longterm (equilibrium) mature male biomass (MMB) for the unfished stock (B_0). The proxy F_{MSY} for Tier 3 stocks is then the directed fishing mortality rate that results in $B_{35\%}$ (i.e., $F_{35\%}$), while the MSY proxy is the longterm total (retained plus discard) catch mortality resulting from fishing at F_{MSY}. The OFL calculation for the upcoming year is based on a sloping

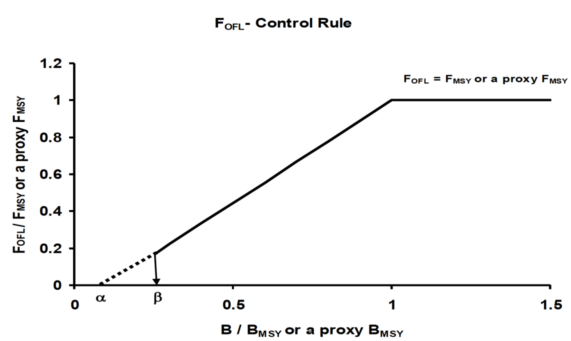


Fig. 2. The F_{OFL} harvest control rule.

harvest control rule for F_{OFL} (Fig. 2), the directed fishing mortality rate that results in the OFL. If the "current" MMB (projected to Feb. 15 of the upcoming year under the F_{OFL}) is above B_{MSY} ($B_{35\%}$), then F_{OFL}=F_{MSY}=$F_{35\%}$. If the current MMB is between $\beta \cdot B_{MSY}$ and B_{MSY}, then F_{OFL} is determined from the slope of the control rule. In either of these cases, the OFL is simply the projected total catch mortality under directed fishing at F_{OFL}. If current MMB is less than $\beta \cdot B_{MSY}$, then no directed fishing is allowed (F_{OFL}=0) and the OFL is set to provide for stock rebuilding with bycatch in non-directed fisheries. Note that if current MMB is less than B_{MSY}, then the process of determining F_{OFL} is generally an iterative one.

Stock status is determined by comparing "current" MMB with the Minimum Stock Size Threshold (MSST), which is defined as 0.5*x*B_{MSY}: if "current" MMB is below the MSST, then the stock is overfished—otherwise, it is not overfished.

N.1 Equilibrium conditions

Both OFL calculations and stock status determination utilize equilibrium considerations, both equilibrium under unfished conditions (to determine B_0 and $B_{35\%}$) and under fished conditions (to determine $F_{35\%}$). For Tier 3 stocks, because there is no reliable stock-recruit relationship, analytical solutions can be found for equilibrium conditions for any fishing mortality conditions. These solutions are described below (the notation differs somewhat from that used in previous sections).

N.1.1 Population states

The Tanner crab population on July 1 can be characterized by abundance-at-size in four population states:

in– immature new shell crab
io– immature old shell crab
mn – mature new shell crab
mo – mature old shell crab

where each of these states represents a vector of abundance-at-size (i.e., a vector subscripted by size).

N.1.2 Population processes

The following processes then describe the dynamics of the population over a year:

S_1 – survival from start of year to time of molting/growth of immature crab, possibly including fishing mortality (a diagonal matrix)
S_2 – survival after time of molting/growth of immature crab to end of year, possibly including fishing mortality (a diagonal matrix)
Φ – probability of an immature crab molting (pr(molt|*z*), where *z* is pre-molt size; a diagonal matrix) (pr(molt|*z*) is assumed to be 1 in TCSAM02).
Θ – probability that a molt was terminal (pr(molt to maturity|*z*, molt), where *z* is post-molt size; a diagonal matrix)
T – size transition matrix (a non-diagonal matrix)
1 – identity matrix
R –number of recruits by size (a vector)

The matrices above are doubly–subscripted, and *R* is singly-subscripted, by size. Additionally, the matrices above (except for the identity matrix) can also be subscripted by population state (*in*, *io*, *mn*, *mo*) for generality. For example, survival of immature crab may differ between those that molted and those that skipped.

N.1.3 Population dynamics

The following equations then describe the development of the population from the beginning of one year to the beginning of the next:

$$in^{+} = R + S_{2in} \cdot \{(1-\Theta_{in}) \cdot T_{in} \cdot \Phi_{in} \cdot S_{1in} \cdot in + T_{io} \cdot (1-\Theta_{io}) \cdot \Phi_{io} \cdot S_{1io} \cdot io\} \quad \text{(N.1)}$$
$$io^{+} = S_{2io} \cdot \{(1-\Phi_{in}) \cdot S_{1in} \cdot in + (1-\Phi_{io}) \cdot S_{1io} \cdot io\} \quad \text{(N.2)}$$
$$mn^{+} = S_{2mn} \cdot \{\Theta_{in} \cdot T_{in} \cdot \Phi_{in} \cdot S_{1in} \cdot in + \Theta_{io} \cdot T_{io} \cdot \Phi_{io} \cdot S_{1io} \cdot io\} \quad \text{(N.3)}$$
$$mo^{+} = S_{2mo} \cdot \{S_{1mn} \cdot mn + S_{1mo} \cdot mo\} \quad \text{(N.4)}$$

where "+" indicates year+1 and all recruits (*R*) are assumed to be new shell.

N.1.4 Equilibrium equations

The equations reflecting equilibrium conditions (i.e., $in^{+} = in$, etc.) are simply:

$$in = R + S_{2in} \cdot \{(1-\Theta_{in}) \cdot T_{in} \cdot \Phi_{in} \cdot S_{1in} \cdot in + (1-\Theta_{io}) \cdot T_{io} \cdot \Phi_{io} \cdot S_{1io} \cdot io\} \quad \text{(N.5)}$$
$$io = S_{2io} \cdot \{(1-\Phi_{in}) \cdot S_{1in} \cdot in + (1-\Phi_{io}) \cdot S_{1io} \cdot io\} \quad \text{(N.6)}$$
$$mn = S_{2mn} \cdot \{\Theta_{in} \cdot T_{in} \cdot \Phi_{in} \cdot S_{1in} \cdot in + \Theta_{io} \cdot T_{io} \cdot \Phi_{io} \cdot S_{1io} \cdot io\} \quad \text{(N.7)}$$
$$mo = S_{2mo} \cdot \{S_{1mn} \cdot mn + S_{1mo} \cdot mo\} \quad \text{(N.8)}$$

where *R* above is now the equilibrium (longterm average) number of recruits-at-size vector.

N.1.5 Equilibrium solution

The equilibrium solution can be obtained by rewriting the above equilibrium equations as:

$$in = R + A \cdot in + B \cdot io \quad \text{(N.9)}$$
$$io = C \cdot in + D \cdot io \quad \text{(N.10)}$$
$$mn = E \cdot in + F \cdot io \quad \text{(N.11)}$$
$$mo = G \cdot mn + H \cdot mo \quad \text{(N.12)}$$

where *A*, *B*, *C*, *D*, *E*, *F*, *G*, and *H* are square matrices. Solving for *io* in terms of *in* in eq. 10, one obtains

$$io = \{1-D\}^{-1} \cdot C \cdot in \quad \text{(N.13)}$$

Plugging eq. 13 into 9 and solving for *in* yields

$$in = \{1 - A - B \cdot [1-D]^{-1} \cdot C\}^{-1} \cdot R \quad \text{(N.14)}$$

Equations 13 for *io* and 14 for *in* can simply be plugged into eq. 11 to yield *mn:*

$$mn = E \cdot in + F \cdot io \quad \text{(N.15)}$$

while eq. 12 can then be solved for *mo*, yielding:

$$mo = \{1-H\}^{-1} \cdot G \cdot mn \quad \text{(N.16)}$$

where (for completeness):

$$A = S_{2in} \cdot (1-\Theta_{in}) \cdot T_{in} \cdot \Phi_{in} \cdot S_{1in} \quad \text{(N.17)}$$
$$B = S_{2in} \cdot (1-\Theta_{io}) \cdot T_{io} \cdot \Phi_{io} \cdot S_{1io} \quad \text{(N.18)}$$
$$C = S_{2io} \cdot (1-\Phi_{in}) \cdot S_{1in} \quad \text{(N.19)}$$
$$D = S_{2io} \cdot (1-\Phi_{io}) \cdot S_{1io} \quad \text{(N.20)}$$
$$E = S_{2mn} \cdot \Theta_{in} \cdot T_{in} \cdot \Phi_{in} \cdot S_{1in} \quad \text{(N.21)}$$

$$F = S_{2mn} \cdot \Theta_{io} \cdot T_{io} \cdot \Phi_{io} \cdot S_{1io} \tag{N.22}$$
$$G = S_{2mo} \cdot S_{1mn} \tag{N.23}$$
$$H = S_{2mo} \cdot S_{1mo} \tag{N.24}$$

Note that Θ, the size-specific conditional probability of a molt being the terminal molt-to-maturity, is defined above on the basis of post-molt, not pre-molt, size. This implies that whether or not a molt is terminal depends on the size a crab grows into, not the size it at which it molted. An alternative approach would be to assume that the conditional probability of terminal molt is determined by pre-molt size. This would result in an alternative set of equations, but these can be easily obtained from the ones above by simply reversing the order of the terms involving T and Θ (e.g., the term $(1-\Theta_{in}) \cdot T_{in}$ becomes $T_{in} \cdot (1-\Theta_{in})$).

N.2 OFL calculations

Because a number of the calculations involved in determining the OFL are iterative in nature, the OFL calculations do not involve automatically-differentiated (AD) variables. Additionally, they are only done after model convergence or when evaluating an MCMC chain. The steps involved in calculating the OFL are outlined as follows:

1. The initial population numbers-at-sex/maturity state/shell condition/size for the upcoming year are copied to a non-AD array.
2. Mean recruitment is estimated over a pre-determined time frame (currently 1982-present).
3. The arrays associated with all population rates in the final year are copied to non-AD arrays for use in the upcoming year.
4. Calculate the average selectivity and retention functions for all fisheries over the most recent 5-year period.
5. Determine the average maximum capture rates for all fisheries over the most recent 5-year period.
6. Using the equilibrium equations, calculate B_0 for unfished stock (B35% = $0.35*B_0$).
7. Using the equilibrium equations, iterate on the maximum capture rate for males in the directed fishery to find the one ($F_{35\%}$) that results in the equilibrium MMB = $B_{35\%}$.
8. Calculate "current" MMB under directed fishing at $F=F_{35\%}$ by projecting initial population (1) to Feb. 15.
   a. If current MMB > $B_{35\%}$, $F_{OFL} = F_{35\%}$. The associated total catch mortality is OFL.
   b. Otherwise
      i. set directed F based on the harvest control rule and the ratio of the calculated current MMB to $B_{35\%}$
      ii. recalculate current MMB
      iii. iterate i-iii until current MMB doesn't change between iterations. Then $F_{OFL} = F\ (< F_{35\%})$ and the OFL is the associated total (retained plus discard) catch mortality.

References

Fournier, D.A., H.J. Skaug, J. Ancheta, J. Ianelli, A. Magnusson, M.N. Maunder, A. Nielsen, and J. Sibert. 2012. AD Model Builder: using automatic differentiation for statistical inference of highly parameterized complex nonlinear models. Optim. Methods Softw. 27:233-249.

Methot, R.D. and C.R. Wetzel. 2013. Stock synthesis: A biological and statistical framework for fish stock assessment and fishery management. Fish. Res. 142: 86-99.

NPFMC. 2016. Introduction. In: Stock Assessment and Fishery Evaluation Report for the King and Tanner Crab Fisheries of the Bering Sea and Aleutian Islands: 2016 Crab SAFE. North Pacific Fishery Management Council. Anchorage, AK. pp. 1-40.

Rugolo, L.J. and B.J. Turnock. 2011. Length-Based Stock Assessment Model of eastern Bering Sea Tanner Crab. Report to Subgroup of NPFMC Crab Plan Team. 61p.

Rugolo, L.J. and B.J. Turnock. 2012a. Length-Based Stock Assessment Model of eastern Bering Sea Tanner Crab. Report to Subgroup of NPFMC Crab Plan Team. 69p.

Rugolo L,J. and B.J. Turnock. 2012b. 2012 Stock Assessment and Fishery Evaluation Report for the Tanner Crab Fisheries of the Bering Sea and Aleutian Islands Regions. In: Stock Assessment and Fishery Evaluation Report for the King and Tanner Crab Fisheries of the Bering Sea and Aleutian Islands: 2012 Crab SAFE. North Pacific Fishery Management Council. Anchorage, AK. pp. 267-416.

Stockhausen, W.T., B.J. Turnock and L. Rugolo. 2013. 2013 Stock Assessment and Fishery Evaluation Report for the Tanner Crab Fisheries of the Bering Sea and Aleutian Islands Regions. In: Stock Assessment and Fishery Evaluation Report for the King and Tanner Crab Fisheries of the Bering Sea and Aleutian Islands: 2013 Crab SAFE. North Pacific Fishery Management Council. Anchorage, AK. pp. 342-449.

Stockhausen, W.T. 2014. 2014 Stock Assessment and Fishery Evaluation Report for the Tanner Crab Fisheries of the Bering Sea and Aleutian Islands Regions. In: Stock Assessment and Fishery Evaluation Report for the King and Tanner Crab Fisheries of the Bering Sea and Aleutian Islands: 2014 Crab SAFE. North Pacific Fishery Management Council. Anchorage, AK. pp. 324-545.

Stockhausen, W.T. 2015. 2015 Stock Assessment and Fishery Evaluation Report for the Tanner Crab Fisheries of the Bering Sea and Aleutian Islands Regions. In: Stock Assessment and Fishery Evaluation Report for the King and Tanner Crab Fisheries of the Bering Sea and Aleutian Islands: 2015 Crab SAFE. North Pacific Fishery Management Council. Anchorage, AK. pp. 293-440.

Stockhausen, W.T. 2016. 2016 Stock Assessment and Fishery Evaluation Report for the Tanner Crab Fisheries of the Bering Sea and Aleutian Islands Regions. In: Stock Assessment and Fishery Evaluation Report for the King and Tanner Crab Fisheries of the Bering Sea and Aleutian Islands: 2016 Crab SAFE. North Pacific Fishery Management Council. Anchorage, AK. pp. 251-446.

Appendix 3.1: A Check on Total Catch Expansions using ADFG Measure Pot Data

William T. Stockhausen (AFSC)

11 April, 2019

Fishery and observer sampling effort in the crab fisheries

Total annual fishing effort (potlifts) in the the directed Tanner crab fisheries, the snow crab fishery, and the BBRKC fishery from fish ticket data is used to expand the observed numbers of crab sampled by at-sea observers to estimates of total at-sea catch (and bycatch) of Tanner crab prior to sorting for retention and discard. Annual fishery effort and at-sea observer sampling effort was provided for the crab fisheries by Ben Daly (ADFG, Aug. 10, 2018, "Stockhausen Tanner crab data requests_18_BD_fixedeffort.xlsx"). The following table reports the annual fishing effort (potlifts) and at-sea sampling effort (numbers of "summary" pots and measure" pots) for each of the principal crab fisheries in which Tanner crab are taken, either as targets or incidentally as bycatch. "Summary pots" are randomly-selected pots that an at-sea observer samples for species composition and abundance, recording the number of male crab categorized as "sublegal"",”legal retained"", "legal not retained"", or”legal unknown"" (retention status), as well as the total number of female crab and species. "Measure pots" represent a subset of "sample pots" for which detailed biological measurements are taken on each crab caught. For males, these measurements include shell condition and size (carapace width). Additional information is recorded for females, including maturity state (for Tanner and snow crab), clutch size and condition, and egg color.

Table 1: Total annual effort (potlifts) and at-sea observer 'summary pot' and 'measure pot' sampling in the directed fisheries. The expansion factor is the number of potlifts divided by the number of measure pots sampled. The directed fisheries were closed 1997/98-2004/05, 2010/11-2012/13 and 2016/17.

	TCF (East 166W)				TCF (West 166W)			
year	potlifts	summary pots	measure pots	expansion	potlifts	summary pots	measure pots	expansion
1990	493820	0	0	0.00	479	0	0	0.00
1991	360864	359	356	1013.66	140050	176	175	800.29
1992	508922	816	570	892.85	166670	206	204	817.01
1993	286620	896	887	323.13	40100	112	112	358.04
1994	228254	411	203	1124.40	21282	6	4	5320.50
1995	201988	382	211	957.29	46454	39	20	2322.70
1996	64989	100	45	1444.20	8533	34	6	1422.17
1997	0	0	0	0.00	0	0	0	0.00
1998	0	0	0	0.00	0	0	0	0.00
1999	0	0	0	0.00	0	0	0	0.00
2000	0	0	0	0.00	0	0	0	0.00
2001	0	0	0	0.00	0	0	0	0.00
2002	0	0	0	0.00	0	0	0	0.00
2003	0	0	0	0.00	0	0	0	0.00
2004	0	0	0	0.00	0	0	0	0.00
2005	0	0	0	0.00	6346	160	139	45.65
2006	15273	280	158	96.66	4517	141	72	62.74
2007	26441	773	478	55.32	7268	103	76	95.63
2008	19401	607	378	51.33	2336	77	46	50.78
2009	6635	354	188	35.29	0	0	0	0.00
2010	0	0	0	0.00	0	0	0	0.00
2011	0	0	0	0.00	0	0	0	0.00
2012	0	0	0	0.00	0	0	0	0.00
2013	16613	265	145	114.57	23062	309	164	140.62
2014	72768	939	492	147.90	68695	874	470	146.16
2015	130302	1442	783	166.41	84933	898	525	161.78
2016	0	0	0	0.00	0	0	0	0.00

2017	11	0	0	0.00	19284	329	183	105.38

Table 2: Total annual effort (potlifts) and at-sea observer 'measure pot' sampling in the BBRKC (RKF) and snow crab (SCF) fisheries. The expansion factor is the number of potlifts divided by the number of measure pots sampled.

	RKF				SCF			
year	potlifts	summary pots	measure pots	expansion	potlifts	summary pots	measure pots	expansion
1990	262761	140	138	1904.07	1382908	0	0	0.0
1991	227555	272	267	852.27	1278502	2308	2127	601.1
1992	206815	290	281	736.00	969209	1217	1188	815.8
1993	254389	558	556	457.53	716524	1151	1119	640.3
1994	697	0	0	0.00	507603	2479	711	713.9
1995	547	0	0	0.00	520685	1530	418	1245.7
1996	77081	84	33	2335.79	754140	1394	406	1857.5
1997	91085	604	586	155.44	930794	1733	547	1701.6
1998	145689	399	387	376.46	945533	2132	613	1542.5
1999	151212	178	171	884.28	182634	1506	400	456.6
2000	104056	673	671	155.08	191200	173	56	3414.3
2001	66947	494	466	143.66	326977	722	239	1368.1
2002	72514	487	485	149.51	153862	1316	457	336.7
2003	134515	731	725	185.54	123709	872	176	702.9
2004	97621	536	534	182.81	75095	847	172	436.6
2005	116320	1855	1841	63.18	117375	3010	672	174.7
2006	72404	1214	1202	60.24	86328	1118	350	246.7
2007	113948	1918	1911	59.63	140857	1731	506	278.4
2008	139937	1849	1831	76.43	163537	1657	552	296.3
2009	119261	1950	1939	61.51	137292	1646	477	287.8
2010	132183	1891	1864	70.91	147478	2142	617	239.0
2011	45784	696	692	66.16	270602	2235	654	413.8
2012	38842	437	433	89.70	225627	2877	834	270.5
2013	46589	657	657	70.91	225245	2664	751	299.9
2014	57725	520	520	111.01	279183	2196	663	421.1
2015	48763	413	413	118.07	202526	1857	530	382.1
2016	33608	413	413	81.38	118548	1374	396	299.4
2017	49169	803	803	61.23	114673	1093	322	356.1

The “expansion” factor in the previous tables is the ratio of the annual number of pots fished in a given fishery to the number of measure pots sampled by observers. This factor provides a simple means of scaling observed numbers of crab (in any category: male, female, new shell, old shell, etc) in the measure pots to estimates of the total number of crab caught, under the assumption that the measure pots represent a random sample of all pots fished. Thus, expanded catch abundance A_y in year y is estimated using the formula

$$A_y = f \cdot a_y = (N_y/M_y) \cdot a_y$$

where f $(= N_y/M_y)$ is the expansion factor for year y, a_y is the number of observed crab in the sampled measure pots, N_y is the total number of potlifts in the fishery, and M_y is the number of sampled measure pots. Similarly, expanded catch biomass B_y in year y is estimated using the formula

$$B_y = f \cdot b_y = (N_y/M_y) \cdot b_y$$

where f has the same value as in the previous equation and b_y is the biomass of observed crab in the sampled measure pots (length-weight regressions from Lang et al., 2018, are used to estimate individual crab weights based on measured carapace width).

Measure pot data: 1990/91-2017/18

Measure pot data from at-sea crab observer sampling for Tanner crab during 1990/91-2017/18 was provided by Ben Daly (ADFG; June 15, 2018, “Buck data dumps.7z”) for the directed Tanner crab, snow crab, and BBRKC fisheries. The dataset was reformatted and standardized. Subsequently, the directed Tanner crab fisheries will be referred to collectively as “TCF”, the snow crab fishery as “SCF”, and the BBRKC fishery as “RKF”. Additionally, year “YYYY” will refer to crab fishery year “YYYY/YY+1” (i.e., “2012” refers to crab fishery year “2012/13”).

Sample sizes

Sample sizes for Tanner crab in the measure pots are given in the following tables:

Table 3: Sample sizes for measure pots and measured Tanner crab in the directed fisheries ('TCF'). Note that the directed fisheries were closed in 1997/98-2004/05, 2010/11-2012/13 and 2016/17.

	East 166W						West 166W					
year	vessels	trips	measure pots	non-empty pots	males	females	vessels	trips	measure pots	non-empty pots	males	females
1990	0	0	0	0	0	0	1	1	0	3	51	34
1991	19	21	356	353	21650	3937	14	19	175	172	9655	1670
1992	15	21	570	569	42263	5707	16	19	204	202	12577	3048
1993	15	15	887	883	36065	9417	7	7	112	111	4326	1054
1994	9	9	203	197	5786	2016	1	1	4	4	135	128
1995	10	10	211	206	5183	2914	3	3	20	20	409	205
1996	2	2	45	43	220	168	2	2	6	6	133	0
1997	0	0	0	0	0	0	0	0	0	0	0	0
1998	0	0	0	0	0	0	0	0	0	0	0	0
1999	0	0	0	0	0	0	0	0	0	0	0	0
2000	0	0	0	0	0	0	0	0	0	0	0	0
2001	0	0	0	0	0	0	0	0	0	0	0	0
2002	0	0	0	0	0	0	0	0	0	0	0	0
2003	0	0	0	0	0	0	0	0	0	0	0	0
2004	0	0	0	0	0	0	0	0	0	0	0	0
2005	0	0	0	0	0	0	4	5	139	139	19762	1108
2006	11	11	158	156	12706	1573	2	2	72	70	11538	2859
2007	15	19	478	473	51526	2416	6	6	76	74	10448	903
2008	8	11	378	376	25568	536	4	4	46	45	3815	118
2009	10	11	188	186	17293	147	0	0	0	0	0	0
2010	0	0	0	0	0	0	0	0	0	0	0	0
2011	0	0	0	0	0	0	0	0	0	0	0	0
2012	0	0	0	0	0	0	0	0	0	0	0	0
2013	5	5	145	143	7629	314	5	5	164	164	9663	400
2014	14	15	492	492	51223	287	9	10	470	468	34457	913
2015	17	18	783	782	61791	714	16	18	525	524	58147	913
2016	0	0	0	0	0	0	0	0	0	0	0	0
2017	0	0	0	0	0	0	13	13	183	183	18831	1726

Table 4: Sample sizes for measure pots and measured Tanner crab in the snow crab ('SCF') and BBRKC ('RKF') fisheries.

	RKF						SCF					
year	vessels	trips	measure pots	non-empty pots	males	females	vessels	trips	measure pots	non-empty pots	males	females
1990	18	18	138	123	1630	44	26	74	0	1527	14083	479
1991	24	24	267	239	2292	93	28	52	2127	983	11727	686
1992	17	17	281	251	2058	105	26	29	1188	743	6284	859
1993	16	16	556	551	7360	1196	23	23	1119	554	6974	1549
1994	0	0	0	0	0	0	19	21	711	312	3172	1549
1995	0	0	0	0	0	0	15	21	418	267	1943	429
1996	4	4	33	28	114	5	13	14	406	425	3269	662
1997	19	19	586	375	1063	41	32	37	547	762	3993	657
1998	25	25	387	200	459	20	30	39	613	425	1912	324
1999	19	19	171	105	207	14	21	21	400	213	976	82
2000	30	31	671	361	845	44	25	26	56	293	1237	74
2001	32	32	466	247	456	40	29	31	239	523	3115	160
2002	32	32	485	380	754	51	29	32	457	218	984	119
2003	32	35	725	357	555	46	26	27	176	128	688	152
2004	32	35	534	301	487	44	24	25	172	83	833	707
2005	24	26	1841	618	985	70	30	36	672	502	9822	368
2006	21	22	1202	458	752	68	27	36	350	315	10441	1256
2007	20	21	1911	527	1364	91	30	37	506	464	13873	728
2008	20	20	1831	822	3805	121	29	31	552	486	8462	723
2009	21	23	1939	791	2886	70	26	31	477	464	11085	474
2010	14	16	1864	339	582	28	26	33	617	581	12077	250
2011	16	17	692	140	324	4	24	48	654	576	9456	189
2012	13	13	433	186	618	48	26	35	834	693	11031	270
2013	18	18	657	432	2117	62	20	37	751	631	12935	356
2014	13	13	520	434	3112	33	24	35	663	607	24880	804
2015	13	13	413	333	2181	186	23	32	530	463	19843	234
2016	17	18	413	369	3316	246	24	27	396	325	16396	264
2017	17	21	803	635	3843	86	17	18	322	243	5598	109

Total catch abundance and biomass estimates

Estimates of total catch abundance were obtained from the measure pot data by scaling the observed numbers of crab by the appropriate expansion factors from Tables 1 and 2. Estimates of total catch biomass were obtained by estimating an individual weight for each measured crab using its reported size, sex, and standard size-weight regressions based on NMFS EBS Shelf Bottom Trawl Survey data (Lang et al., 2018), and then scaling the observed weights by the expansion factors from Tables 1 and 2.

Table 5: Estimates of total Tanner crab catch abundance and biomass from measure pots in the directed Tanner crab fishery east of 166W longitude.

	female				male			
	observed		expanded		observed		expanded	
	count	weight	abundance	biomass	count	weight	abundance	biomass
year	–	kg	thousands	t	–	kg	thousands	t
1991	3,937	1,018	3,990	1,031	21,650	16,491	21,945	16,716
1992	5,707	1,374	5,095	1,226	42,263	31,041	37,734	27,715
1993	9,417	2,375	3,042	767	36,065	27,401	11,653	8,854
1994	2,016	450	2,266	506	5,786	4,517	6,505	5,079
1995	2,914	669	2,789	641	5,183	3,761	4,961	3,600
1996	168	40	242	58	220	153	317	221
2006	1,573	410	152	39	12,706	9,566	1,228	924
2007	2,416	583	133	32	51,526	34,144	2,850	1,888
2008	536	127	27	6	25,568	21,852	1,312	1,121
2009	147	39	5	1	17,293	18,460	610	651
2013	314	89	35	10	7,629	6,335	874	725
2014	287	68	42	10	51,223	35,905	7,576	5,310
2015	714	175	118	29	61,791	41,936	10,282	6,978

Table 6: Estimates of total Tanner crab catch abundance and biomass from measure pots in the directed Tanner crab fishery west of 166W longitude.

	female				male			
	observed		expanded		observed		expanded	
	count	weight	abundance	biomass	count	weight	abundance	biomass
year	–	kg	thousands	t	–	kg	thousands	t
1990	34	8	0	0	51	37	0	0
1991	1,670	368	1,336	295	9,655	7,306	7,726	5,846
1992	3,048	677	2,490	553	12,577	8,914	10,275	7,283
1993	1,054	226	377	81	4,326	3,030	1,548	1,085
1994	128	25	681	135	135	72	718	383
1995	205	42	476	98	409	243	949	564
1996	0	0	0	0	133	93	189	133
2005	1,108	234	50	10	19,762	13,451	902	614
2006	2,859	599	179	37	11,538	7,873	723	493
2007	903	186	86	17	10,448	6,702	999	640
2008	118	24	5	1	3,815	2,708	193	137
2013	400	90	56	12	9,663	6,713	1,358	944
2014	913	211	133	30	34,457	21,086	5,036	3,081
2015	913	187	147	30	58,147	34,668	9,406	5,608
2017	1,726	366	181	38	18,831	12,956	1,984	1,365

Table 7: Estimates of total Tanner crab catch abundance and biomass from measure pots in the combined directed Tanner crab fisheries.

	female				male			
	observed		expanded		observed		expanded	
	count	weight	abundance	biomass	count	weight	abundance	biomass
year	–	kg	thousands	t	–	kg	thousands	t
1990	34	8	0	0	51	37	0	0
1991	5,607	1,386	5,327	1,327	31,305	23,797	29,672	22,563
1992	8,755	2,051	7,585	1,780	54,840	39,956	48,009	34,998
1993	10,471	2,602	3,420	848	40,391	30,431	13,202	9,939
1994	2,144	476	2,947	642	5,921	4,589	7,224	5,463
1995	3,119	712	3,265	739	5,592	4,004	5,911	4,165
1996	168	40	242	58	353	247	506	355
2005	1,108	234	50	10	19,762	13,451	902	614
2006	4,432	1,009	331	77	24,244	17,440	1,952	1,418
2007	3,319	770	219	50	61,974	40,846	3,849	2,529
2008	654	151	33	7	29,383	24,560	1,506	1,259
2009	147	39	5	1	17,293	18,460	610	651
2013	714	180	92	23	17,292	13,048	2,232	1,669
2014	1,200	280	175	41	85,680	56,992	12,612	8,392
2015	1,627	362	266	59	119,938	76,604	19,689	12,587
2017	1,726	366	181	38	18,831	12,956	1,984	1,365

Table 8: Estimates of total Tanner crab bycatch abundance and biomass from measure pots in the BBRKC fishery.

	female				male			
	observed		expanded		observed		expanded	
	count	weight	abundance	biomass	count	weight	abundance	biomass
year	–	kg	thousands	t	–	kg	thousands	t
1990	44	10	83	20	1,630	1,747	3,103	3,326
1991	93	26	79	22	2,292	2,309	1,953	1,968
1992	105	26	77	19	2,058	1,837	1,514	1,352
1993	1,196	317	547	145	7,360	6,770	3,367	3,097
1996	5	1	11	3	114	118	266	277
1997	41	10	6	1	1,063	1,037	165	161
1998	20	5	7	1	459	405	172	152
1999	14	3	12	3	207	147	183	130
2000	44	11	6	1	845	599	131	93
2001	40	10	5	1	456	341	65	49
2002	51	12	7	1	754	566	112	84
2003	46	11	8	2	555	376	102	69
2004	44	10	8	1	487	317	89	57
2005	70	15	4	0	985	659	62	41
2006	68	23	4	1	752	487	45	29
2007	91	24	5	1	1,364	1,016	81	60
2008	121	33	9	2	3,805	3,692	290	282
2009	70	19	4	1	2,886	3,068	177	188
2010	28	8	1	0	582	458	41	32
2011	4	1	0	0	324	269	21	17
2012	48	15	4	1	618	481	55	43
2013	62	17	4	1	2,117	1,843	150	130
2014	33	8	3	0	3,112	2,752	345	305
2015	186	49	21	5	2,181	1,813	257	214
2016	246	54	20	4	3,316	2,383	269	193
2017	86	23	5	1	3,843	3,049	235	186

Table 9: Estimates of total Tanner crab bycatch abundance and biomass from measure pots in the snow crab fishery.

	female				male			
	observed		expanded		observed		expanded	
	count	weight	abundance	biomass	count	weight	abundance	biomass
year	–	kg	thousands	t	–	kg	thousands	t
1990	479	81	0	0	14,083	9,077	0	0
1991	686	133	412	80	11,727	7,004	7,048	4,210
1992	859	159	700	130	6,284	2,684	5,126	2,189
1993	1,549	270	991	173	6,974	2,929	4,465	1,876
1994	1,549	234	1,105	167	3,172	1,206	2,264	861
1995	429	72	534	89	1,943	821	2,420	1,023
1996	662	122	1,229	227	3,269	1,636	6,072	3,039
1997	657	114	1,117	195	3,993	2,074	6,794	3,529
1998	324	61	499	95	1,912	919	2,949	1,418
1999	82	15	37	7	976	420	445	192
2000	74	15	252	51	1,237	565	4,223	1,930
2001	160	28	218	38	3,115	1,450	4,261	1,983
2002	119	20	40	6	984	381	331	128
2003	152	27	106	19	688	273	483	192
2004	707	110	308	48	833	276	363	120
2005	368	67	64	11	9,822	5,234	1,715	914
2006	1,256	250	309	61	10,441	5,900	2,575	1,455
2007	728	144	202	40	13,873	7,093	3,861	1,974
2008	723	107	214	31	8,462	3,939	2,506	1,166
2009	474	78	136	22	11,085	5,694	3,190	1,639
2010	250	47	59	11	12,077	6,556	2,886	1,567
2011	189	35	78	14	9,456	5,573	3,912	2,306
2012	270	45	73	12	11,031	6,184	2,984	1,673
2013	356	63	106	19	12,935	6,543	3,879	1,962
2014	804	142	338	60	24,880	12,374	10,476	5,210
2015	234	48	89	18	19,843	10,688	7,582	4,084
2016	264	57	79	17	16,396	8,853	4,908	2,650
2017	109	19	38	6	5,598	3,037	1,993	1,081

Comparison with ADFG results

Ben Daly (ADFG) provided a summary table of estimated catch (abundance and biomass) of Tanner crab in the directed Tanner crab, snow crab and BBRKC fisheries for 1991/92-2017/18. His values provide checks on the simple approach used here to expand from observed catch to estimated total catch. The estimates are compared in the following tables:

Table 10: Comparison of estimates of total male Tanner crab catch abundance and biomass in the directed fisheries from measure pots.

	East 166W				West 166W			
	abundance		biomass		abundance		biomass	
	ADFG	this	ADFG	this	ADFG	this	ADFG	this
year	thousands	thousands	t	t	thousands	thousands	t	t
1990	0	0	0	0	0	0	0	0
1991	25,791	21,945	19,596	16,716	8,210	7,726	6,220	5,846
1992	40,384	37,734	29,660	27,715	10,335	10,275	7,347	7,283
1993	13,437	11,653	10,209	8,854	2,346	1,548	1,643	1,085
1994	8,907	6,505	6,958	5,079	666	718	357	383
1995	6,083	4,961	4,415	3,600	1,093	949	650	564
1996	327	317	228	221	101	189	71	133
2005	0	0	0	0	1,003	902	684	614
2006	1,503	1,228	1,132	924	848	723	579	493
2007	2,681	2,850	1,779	1,888	1,059	999	679	640
2008	1,377	1,312	1,177	1,121	167	193	119	137
2009	622	610	664	651	0	0	0	0
2013	898	874	746	725	1,342	1,358	933	944
2014	7,570	7,576	5,306	5,310	4,998	5,036	3,057	3,081
2015	10,264	10,282	6,761	6,978	9,441	9,406	5,467	5,608
2017	0	0	0	0	3,069	1,984	2,112	1,365

Table 11: Comparison of estimates of total female Tanner crab bycatch abundance and biomass in the directed fisheries from measure pots.

	East 166W				West 166W			
	abundance		biomass		abundance		biomass	
	ADFG	this	ADFG	this	ADFG	this	ADFG	this
year	thousands	thousands	t	t	thousands	thousands	t	t
1990	0	0	0	0	0	0	0	0
1991	5,611	3,990	1,445	1,031	2,001	1,336	440	295
1992	5,244	5,095	1,103	1,226	2,718	2,490	599	553
1993	3,429	3,042	860	767	634	377	136	81
1994	3,276	2,266	729	506	567	681	112	135
1995	4,057	2,789	924	641	683	476	140	98
1996	237	242	56	58	0	0	0	0
2005	0	0	0	0	112	50	23	10
2006	187	152	48	39	344	179	72	37
2007	121	133	29	32	71	86	14	17
2008	28	27	6	6	7	5	1	1
2009	8	5	2	1	0	0	0	0
2013	42	35	12	10	51	56	11	12
2014	36	42	8	10	133	133	30	30
2015	119	118	28	29	148	147	29	30
2017	0	0	0	0	281	181	59	38

Table 12: Comparison of estimates of total male Tanner crab bycatch abundance and biomass in the snow crab fishery from measure pots.

	abundance		biomass	
	ADFG	this	ADFG	this
year	thousands	thousands	t	t
1990	11,946	0	7,081	0
1991	13,995	7,048	8,360	4,210
1992	5,822	5,126	2,487	2,189
1993	6,841	4,465	2,874	1,876
1994	3,513	2,264	1,345	861
1995	2,422	2,420	1,021	1,023
1996	3,916	6,072	1,960	3,039
1997	3,696	6,794	1,963	3,529
1998	1,424	2,949	655	1,418
1999	336	445	131	192
2000	641	4,223	312	1,930
2001	1,196	4,261	545	1,983
2002	407	331	167	128
2003	172	483	64	192
2004	419	363	134	120
2005	2,182	1,715	1,162	914
2006	2,696	2,575	1,527	1,455
2007	3,641	3,861	1,861	1,974
2008	2,363	2,506	1,100	1,166
2009	3,034	3,190	1,559	1,639
2010	2,676	2,886	1,453	1,567
2011	3,633	3,912	2,141	2,306
2012	2,790	2,984	1,564	1,673
2013	3,640	3,879	1,841	1,962
2014	10,716	10,476	5,330	5,210
2015	7,455	7,582	3,919	4,084
2016	4,899	4,908	2,575	2,650
2017	2,052	1,993	1,113	1,081

Table 13: Comparison of estimates of total female Tanner crab bycatch abundance and biomass in the snow crab fishery from measure pots.

	abundance		biomass	
	ADFG	this	ADFG	this
year	thousands	thousands	t	t
1990	628	0	105	0
1991	752	412	144	80
1992	883	700	162	130
1993	2,314	991	400	173
1994	1,288	1,105	194	167
1995	727	534	120	89
1996	659	1,229	119	227
1997	536	1,117	92	195
1998	435	499	80	95
1999	62	37	11	7
2000	27	252	6	51
2001	118	218	20	38
2002	71	40	13	6
2003	46	106	7	19
2004	256	308	39	48
2005	90	64	16	11
2006	429	309	85	61
2007	263	202	52	40
2008	169	214	24	31
2009	97	136	15	22
2010	49	59	9	11
2011	72	78	13	14
2012	63	73	10	12
2013	90	106	15	19
2014	295	338	50	60
2015	87	89	16	18
2016	78	79	16	17
2017	39	38	7	6

Table 14: Comparison of estimates of total male Tanner crab bycatch abundance and biomass in the BBRKC fishery from measure pots.

	abundance		biomass	
	ADFG	this	ADFG	this
year	thousands	thousands	t	t
1990	3,470	3,103	3,722	3,326
1991	1,954	1,953	1,970	1,968
1992	1,474	1,514	1,316	1,352
1993	3,403	3,367	3,130	3,097
1996	258	266	269	277
1997	163	165	160	161
1998	131	172	115	152
1999	111	183	75	130
2000	93	131	66	93
2001	56	65	42	49
2002	83	112	61	84
2003	81	102	54	69
2004	77	89	49	57
2005	61	62	41	41
2006	45	45	29	29
2007	81	81	60	60
2008	288	290	279	282
2009	175	177	186	188
2010	40	41	31	32
2011	21	21	17	17
2012	54	55	42	43
2013	148	150	128	130
2014	345	345	305	305
2015	256	257	204	214
2016	252	269	175	193
2017	227	235	180	186

Table 15: Comparison of estimates of total female Tanner crab bycatch abundance and biomass in the BBRKC fishery from measure pots.

	abundance		biomass	
	ADFG	this	ADFG	this
year	thousands	thousands	t	t
1990	144	83	35	20
1991	94	79	27	22
1992	76	77	19	19
1993	567	547	149	145
1996	9	11	2	3
1997	6	6	1	1
1998	6	7	1	1
1999	8	12	2	3
2000	5	6	1	1
2001	3	5	0	1
2002	6	7	1	1
2003	7	8	1	2
2004	7	8	1	1
2005	4	4	0	0
2006	4	4	1	1
2007	5	5	1	1
2008	9	9	2	2
2009	4	4	1	1
2010	1	1	0	0
2011	0	0	0	0
2012	4	4	1	1
2013	4	4	1	1
2014	3	3	0	0
2015	21	21	5	5
2016	19	20	4	4
2017	5	5	1	1

The estimates of total catch abundance and biomass estimates from ADFG and the calculations presented here are also compared graphically in the following plots:

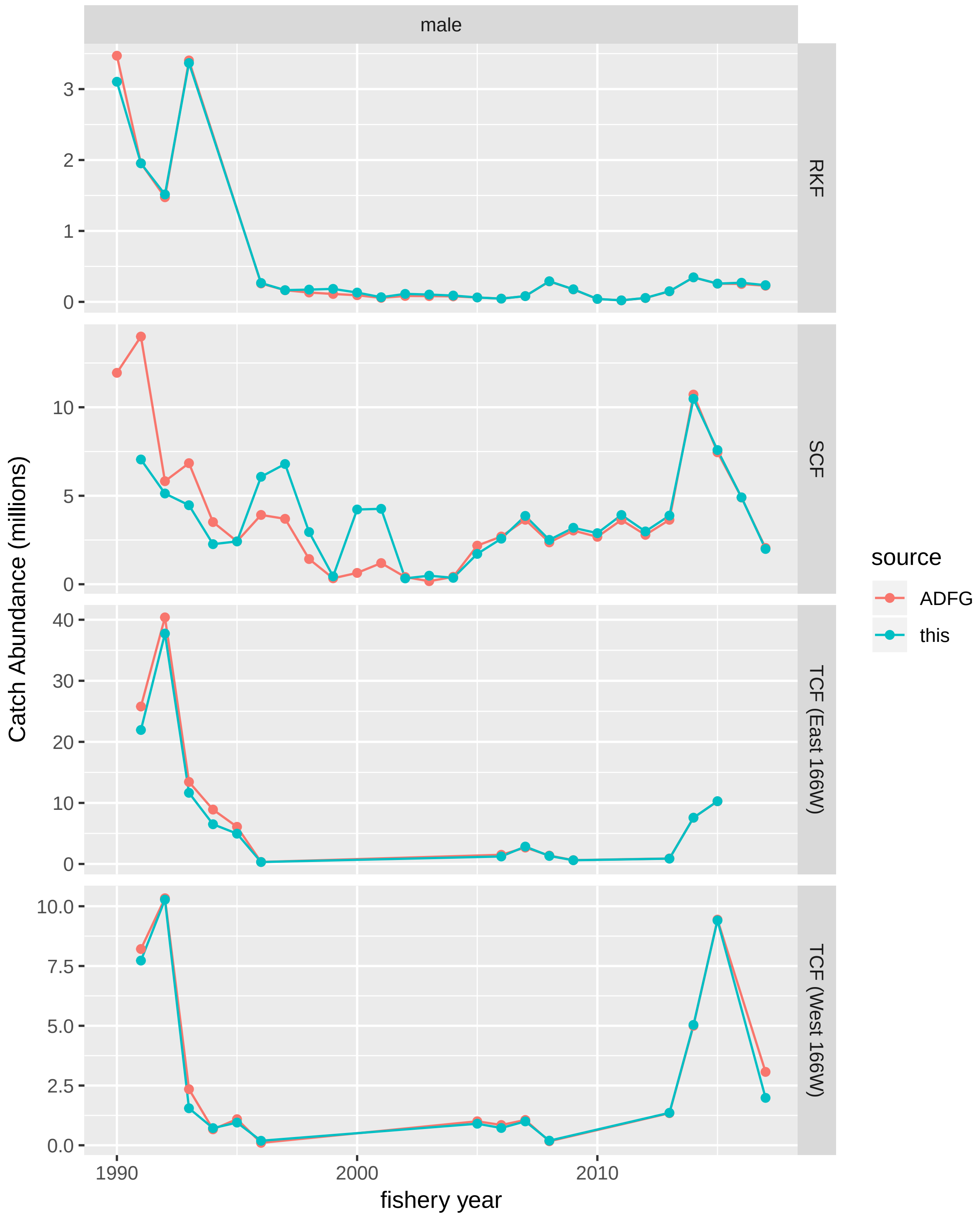


Figure 1: Comparison of total catch abundance estimatesfor males from this analysis and ADFG.

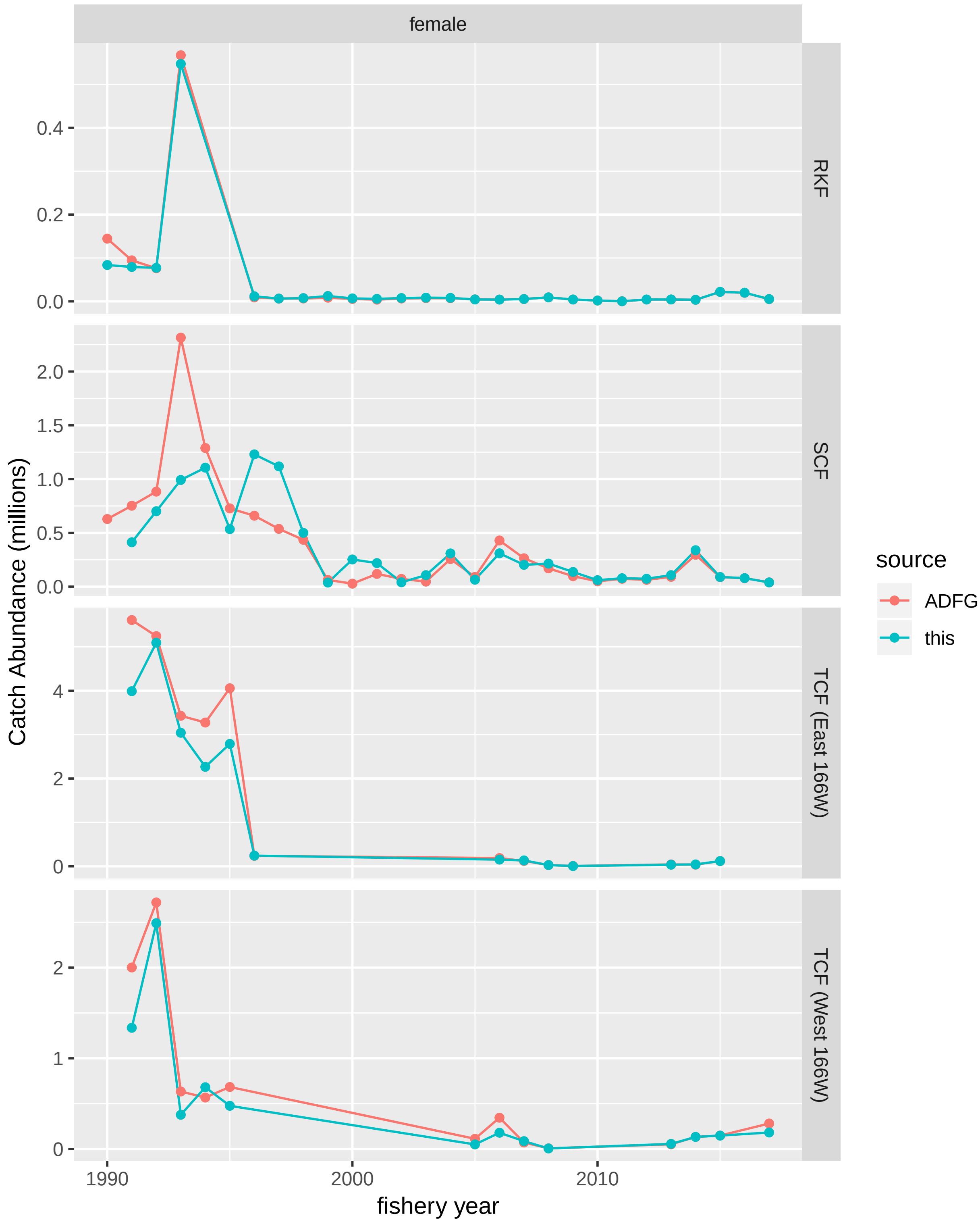


Figure 2: Comparison of total catch abundance estimatesfor females from this analysis and ADFG.

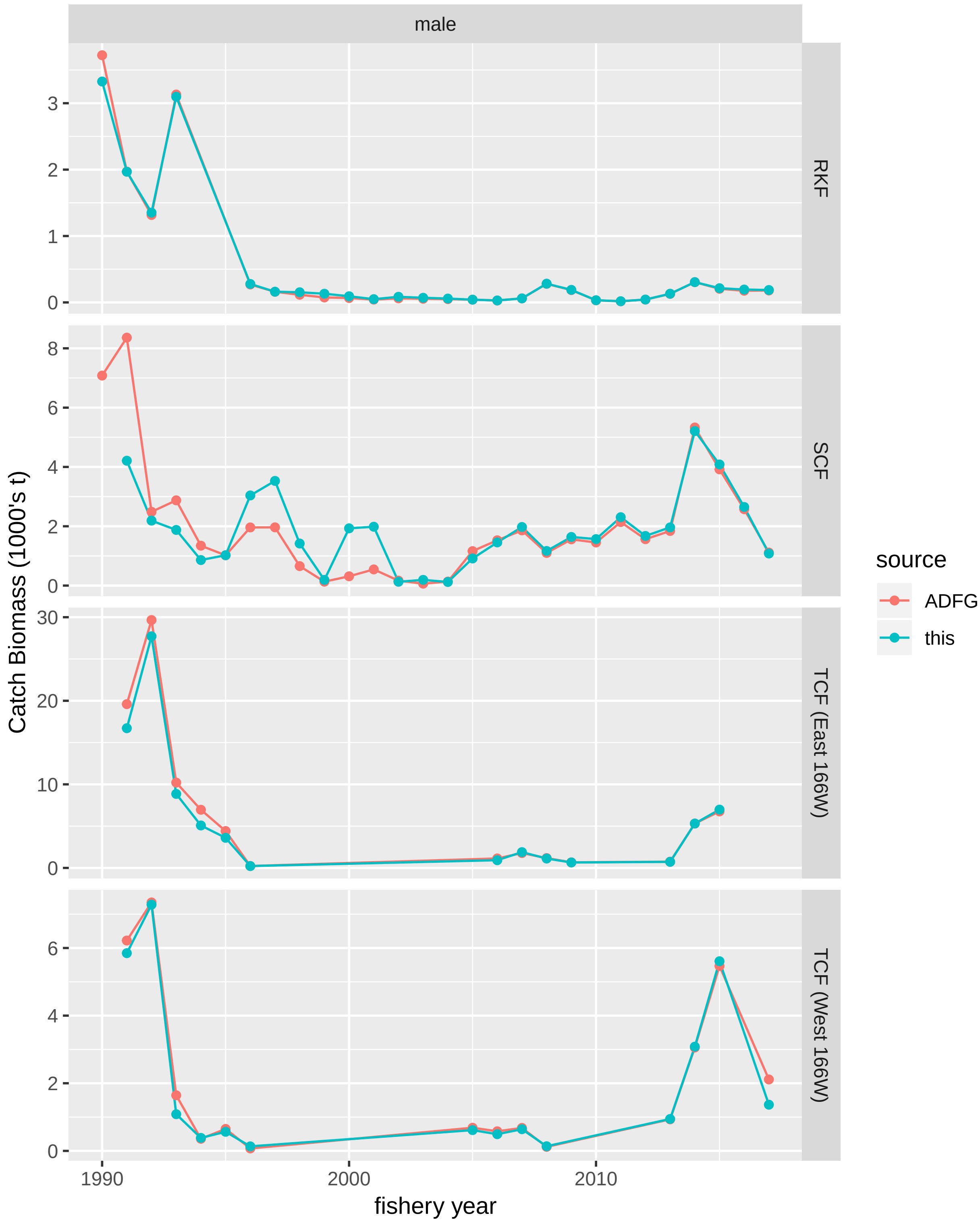


Figure 3: Comparison of total catch biomass estimatesfor males from this analysis and ADFG.

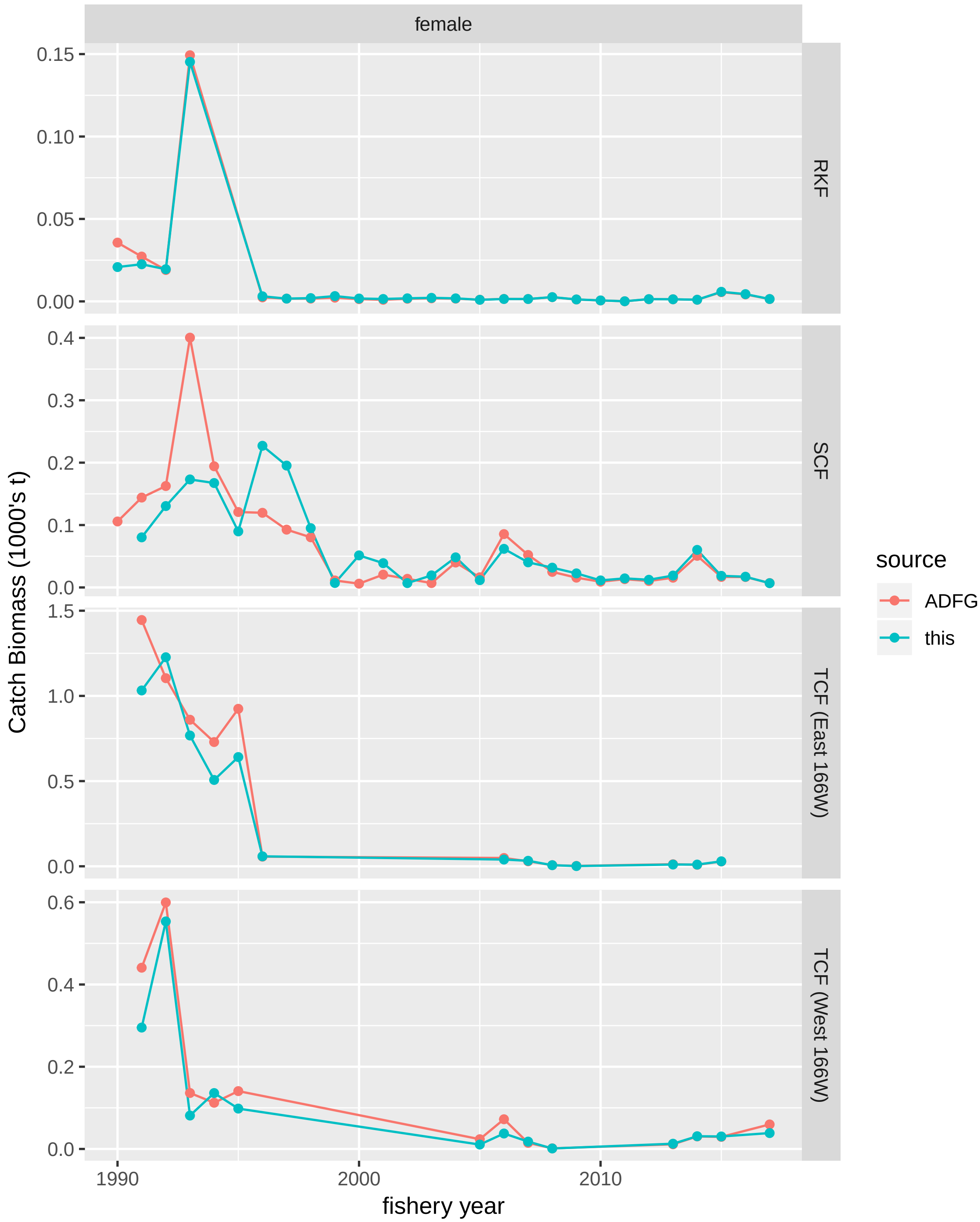


Figure 4: Comparison of total catch biomass estimatesfor females from this analysis and ADFG.

Conclusions

Clearly there are some discrepancies between the methods, particularly for bycatch in the snow crab fishery. At this point, it is unclear what the source of these discrepancies is (and whether it is a single source or multiple sources). However, ADFG is either 1) using a somewhat different approach to expanding estimates of total catch abundance and biomass than the simple one outlined and used here; 2) using different values for total fishery effort and/or measure pot sampling effort than those used here; 3) using different measure pot data than those used here; or 4) some combination of the first three possibilities. It seems unlikely that either 2) or 3) is the case (although this would be worth confirming) and more likely that their approach is somewhat more complicated (e.g., accounting for crab with missing information) than the one applied here. In this case, it would be worthwhile to understand **exactly** what procedures are used.

References

Lang, C. A., J. I. Richar, and R. J. Foy. 2019. The 2018 eastern Bering Sea continental shelf and northern Bering Sea trawl surveys: Results for commercial crab species. U.S. Dep. Commer., NOAA Tech. Memo. NMFS-AFSC-386, 220 p.

Appendix 4.1: Estimating Growth Outside the Assessment Model

William T. Stockhausen

18 April, 2019

Contents

Tanner crab growth data **2**

Mean growth **3**

Comparison with previous assessment models **5**

List of Tables

1 Estimated growth parameters for the EBS molt increment data with post-molt size as a power law of pre-molt size. 3

2 Comparison of the estimated mean growth parameters from the TMB model and the 2016 and 2018 assessment models. 4

List of Figures

1 Tanner crab molt increment data, by year and sex. 2

2 Tanner crab growth data, by sex. Colored lines indicate mean growth by sex as determined by the assessment model or the TMB model. 5

Tanner crab growth data

Figure 1 shows molt increment data collected in the eastern Bering Sea (EBS). during 2015, 2016, and 2017 through cooperative research conducted by the AFSC/NMFS and the Bering Sea Research Foundation (BSFRF).

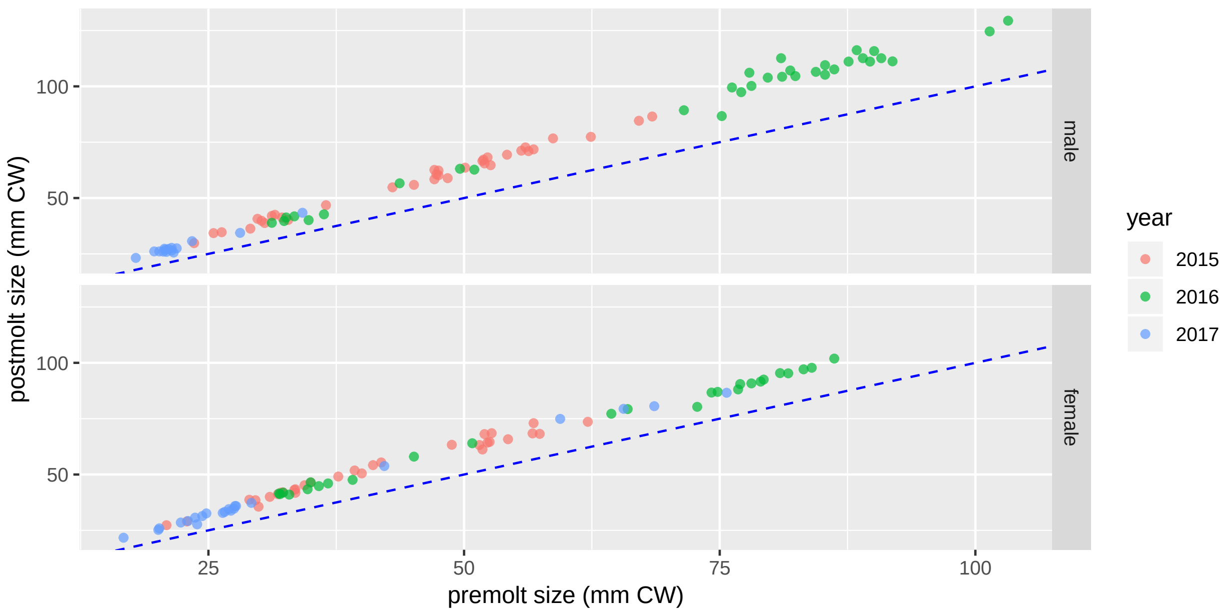


Figure 1: Tanner crab molt increment data, by year and sex.

Mean growth

The 2018 assessment model described mean post-molt size $\overline{z}_{pst}$ as a power function of pre-molt size z_{pre}, paramterized as:

$$\overline{z}_{pst} = z_{pst}^{A} \cdot exp\{\frac{log(\overline{z}_{pst}^{B}/\overline{z}_{pst}^{A})}{log(z_{pre}^{B}/z_{pre}^{A})} \cdot log(z_{pre}/z_{pre}^{A})\}$$

where z_{pst}^{A} is the estimated mean post-molt size at pre-molt size z_{pre}^{A} and z_{pst}^{B} is the estimated mean post-molt size at pre-molt size z_{pre}^{B}.

In the assessment model, the actual post-molt size z_{pst} for a crab, given that it was in size bin z_i prior to molting, is described using a γ distribution, with the probability that the post-molt crab falls into the jth size bin z_j given by:

$$p(z_j|z_i) = \int_{\alpha_i(z_j)-\frac{\delta\alpha}{2}}^{\alpha_i(z_j)+\frac{\delta\alpha}{2}} \gamma(\alpha - \overline{\alpha_i}) \cdot d\alpha$$

where $\alpha_i(z) = \frac{z-z_i}{\beta}$ represents the scaled molt increment, $\overline{\alpha_i} = \frac{\overline{z}_{pst}-z_i}{\beta}$ is the scaled mean molt increment for pre-molt size bin z_i, $\delta\alpha = \frac{\delta z}{\beta}$ is the scaled size bin width, and β is the scale factor. The largest model size bin, z_{max}, functions as an accumulator bin, so it is handled slightly differently: the probability of a post-molt crab ending up in the largest size bin is simply the probability of it ending up at any larger size than its lower cutpoint:

$$p(z_{max}|z_i) = \int_{\alpha_i(z_{max})-\frac{\delta\alpha}{2}}^{\inf} \gamma(\alpha - \overline{\alpha_i}) \cdot d\alpha = 1 - \int_{0}^{\alpha_i(z_{max})-\frac{\delta\alpha}{2}} \gamma(\alpha - \overline{\alpha_i}) \cdot d\alpha$$

The assessment model also allows one to limit potential growth to a maximum number of size bins, n_{max}, in which case $p(z_j|z_i)$ is set to 0 for $j-i > n_{max}$ and normalized to sum to 1 for $j-i \leq n_{max}$.

Here, we fit the sex-specific molt increment data using a similar approach and likelihood component to those in the assessment model, but implemented as a stand-alone model using TMB and without constraining potential growth.

Table 1: Estimated growth parameters for the EBS molt increment data with post-molt size as a power law of pre-molt size.

model	sex	parameter	value	ref. size
TMB	male	pGrA	31.97074	25
TMB	male	pGrB	101.39992	80
TMB	female	pGrA	32.75113	25
TMB	female	pGrB	93.91184	80

Sex-specific parameters from the 2016 and 2018 assessment models reflecting estimated mean growth are compared with the TMB results in Table 2.

Table 2: Comparison of the estimated mean growth parameters from the TMB model and the 2016 and 2018 assessment models.

sex	parameter	2016AM	2018AM	TMB
female	pGrA	34.77473	34.46397	32.75113
female	pGrB	97.35328	94.79594	93.91184
male	pGrA	34.85163	33.08883	31.97074
male	pGrB	107.96415	106.58443	101.39992

Comparison with previous assessment models

The 2016 assessment model estimated mean growth parameters from based on fits to size composition data alone . Priors were placed on the growth parameters based on a previous analysis by Rugolo and Turnock of molt increment data from Kodiak Island in the Gulf of Alaska. In 2018, molt increment data from the EBS collected in 2015 and 2016 was included in the assessment, in addition to new size composition data. The estimated mean growth curves from both assessment over-predict post-molt size at larger pre-molt sizes for both males and females.

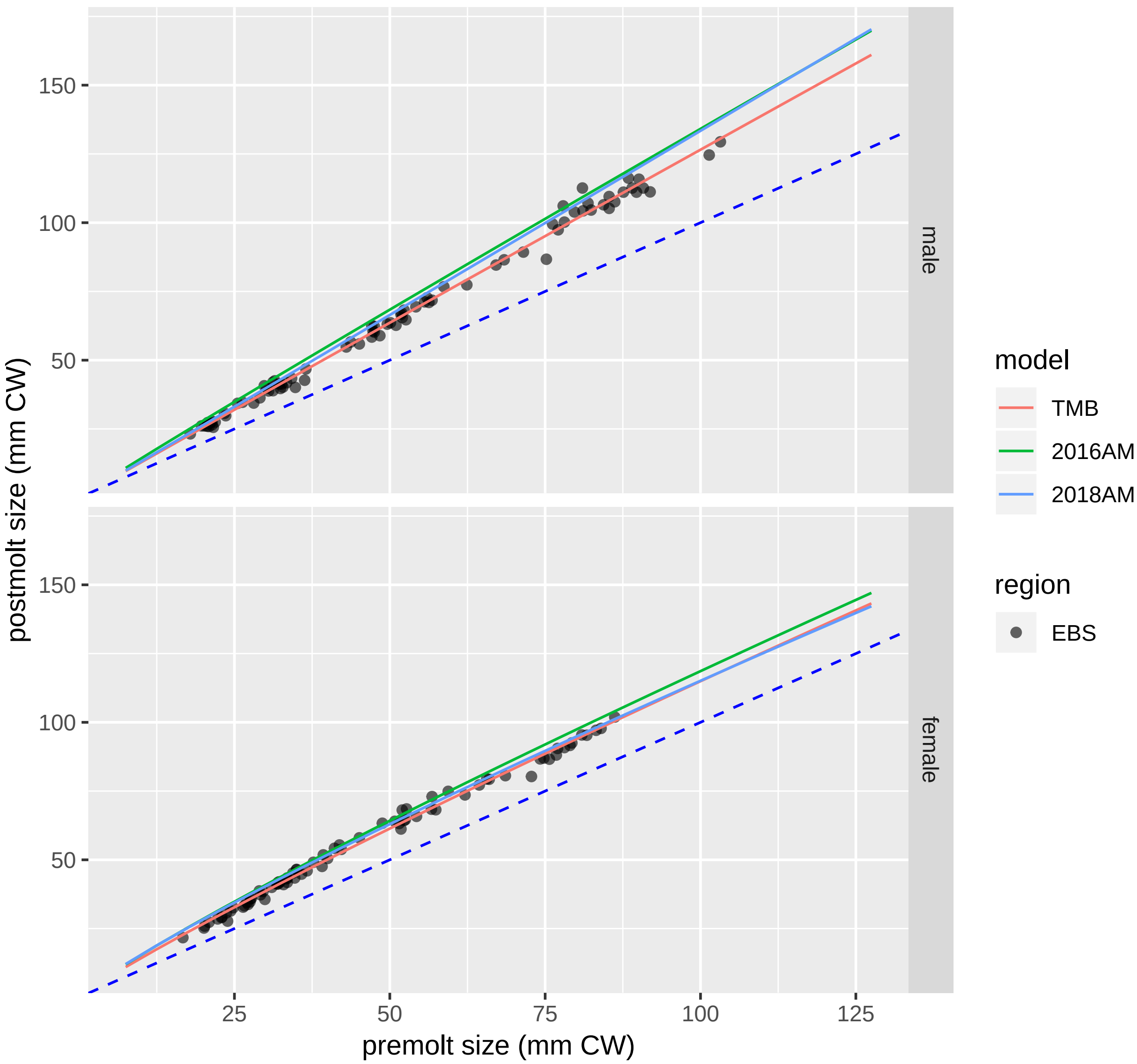


Figure 2: Tanner crab growth data, by sex. Colored lines indicate mean growth by sex as determined by the assessment model or the TMB model.

Appendix 4.2: Tanner Crab Cohort Progression for Scenario 19.0

William Stockhausen

18 April, 2019

Contents

Tanner crab cohort progression **1**
- Model configuration 1
- Recruitment 1
- Natural mortality rates 3
- Molting 4
- Growth 6
- Terminal molt 8
- Cohort progression 10

Tanner crab cohort progression

This report characterizes, based on population processes for an unfished stock as incorporated in the assessment model, the progression of a single-sex cohort of Tanner crab through a succession of years. The progression takes the initial size distribution of immature, new shell crab at recruitment to the assessment model and projects it forward in time on an annual basis, applying size- and life stage-specific model processes for natural mortality, annual molting, growth, terminal molt, and changes in shell condition to the relative abundance of crab by maturity state (m), shell condition (s), and size (z). Plots describing all model processes and the resulting cohort progression are included in the associated sections in this report.

Model configuration

The cohort progression model was run using the following general configuration:

```
molt timing (fraction of year): 0.625
maturity states               : immature, mature
shell conditions              : new shell, old shell
bin size (mm CW)              : 5
min size (mm CW)              : 25
max size (mm CW)              : 185
```

Recruitment

Annual recruitment to the model may be spread across several size bins and may reflect several age classes. All recruitment occurs as immature, new shell crab. Here, recruits to the model in a given year are regarded as a “cohort”. A truncated gamma probability distribution, $\gamma_N(z|\alpha,\beta)$, is used to describe the relative abundance of recruiting crab. $\gamma_N(z_i|\alpha,\beta)$ is typically truncated after a few size bins and the resulting distribution is normalized to sum to 1:

$$\gamma_N(z_i|\alpha,\beta) = \frac{\gamma(z_i|\alpha,\beta)}{\sum_i \gamma(z_i|\alpha,\beta)}$$

where z_i is the mid-point of the ith size bin, α is the location parameter for the gamma distribution, β is the scale parameter, and the sum in the denominator is over the non-truncated size bins.

The following parameters were used to describe the relative size distribution at recruitment for this report:

parameter	value
α	11.5
β	4
max size (mm CW)	50

yielding the distribution shown in Figure 1.

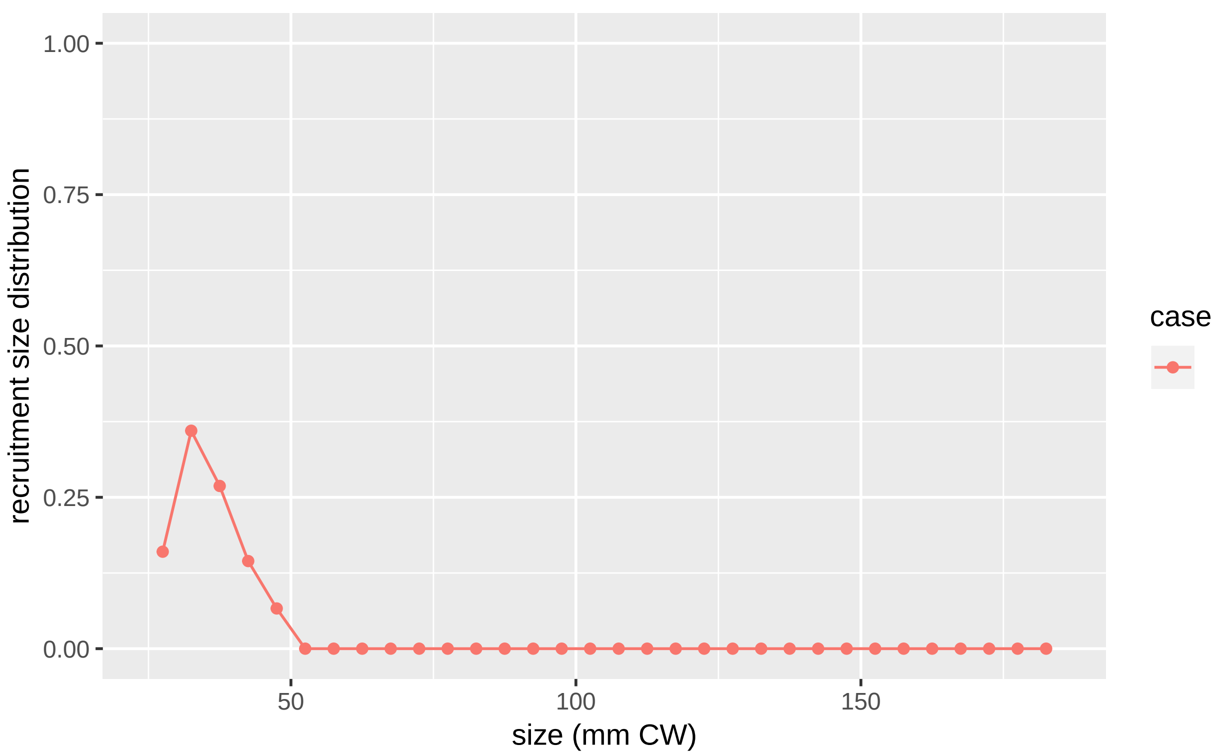


Figure 1. Relative size distribution for recruitment to the cohort progression model.

Natural mortality rates

Natural mortality (M)in the cohort progression model is assumed to be a function of maturity state, such that immature and mature crab may experience different rates of natural mortality (but these rates do not depend on shell condition). These are parameterized using the folllowing multiplicative approach:

$$M_{m,s} = \delta M_m * M_0$$

where M_0 is a baseline value for M and δM_m is the maturity state-specific multiplier.

The following parameters were used to describe M for this report:

parameter	value
M_0	0.23
$\delta M_{immature}$	1.0023896
δM_{mature}	1.1524783

yielding the rates shown in Figure 2.

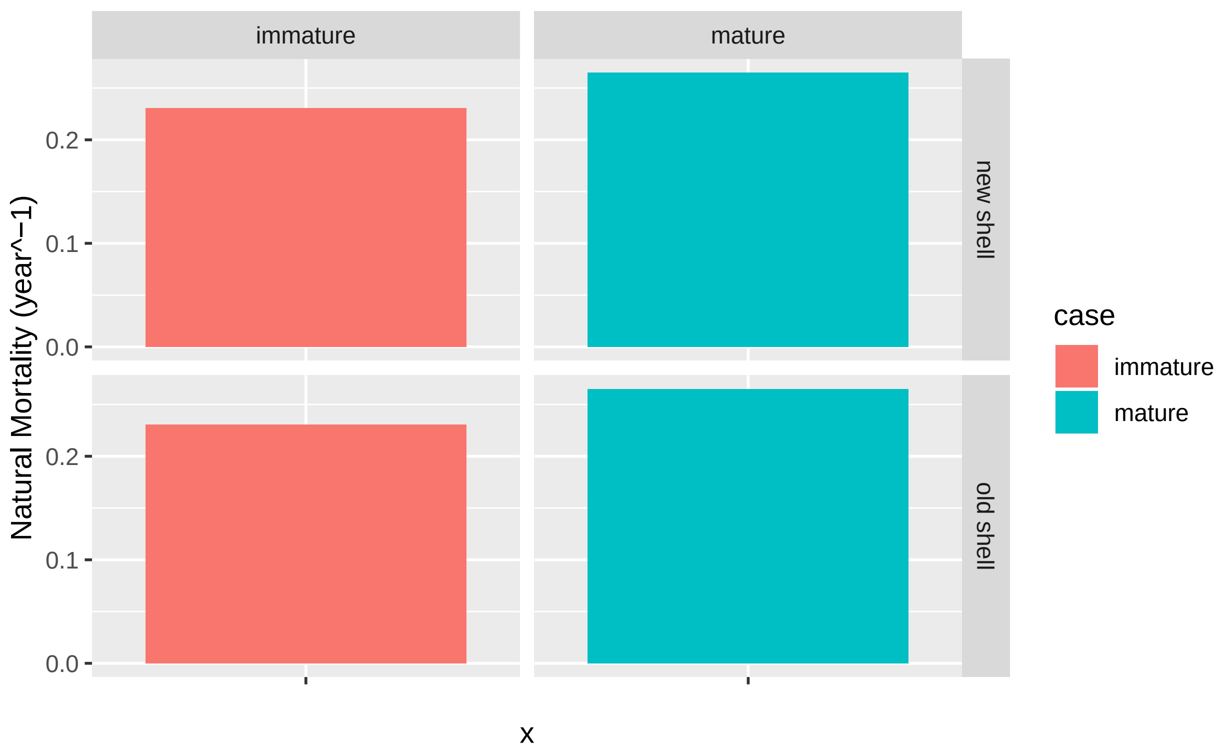


Figure 2. Natural mortality rates by life stage.

Molting

Immature crab in the assessment model are (currently) assumed to molt annually until their terminal molt to maturity. In order to explore the implications of skip molting on cohort progression, the cohort progression model incorporates the ability to specify a size-dependent probbility of molting for immature crab. Crab that don't molt are classified as "old shell" during the following year based on the appearance of their carapace, while crab that do molt are classified as "new shell". The probability that immature crab will undergo a molt, $p^M_{m,s}(z)$, is allowed to be a decreasing logistic function of size (but independent of age) given by:

$$p^M_{i,n}(z_j) = 1 \quad \text{where} \quad z_j < z_{min}$$

$$p^M_{i,n}(z_j) = 1 - \frac{1 - p_{min}}{1 + e^{(z_j - z_{50})/b_{50}}} \quad \text{where} \quad z_j \geq z_{min}$$

$$p^M_{i,o}(z_j) = 1$$

where i indicates "immature", n indicates "new shell", o indicates "old shell", z_j is the midpoint of the jth size bin, p_{min} is the minimum probability of large, immature new shell crab molting, z_{50} is the inflection point of the logistic curve, b_{50} is the scale of the logistic curve, and z_min is the minimum size that immature crab potentially undergo skip molting. The values used for the parameters in this report are:

parameter	value
z_{min}	200
p_{min}	0.5
z_{50}	70
b_{50}	-1

The resulting size-dependent probability of annual molt is shown in Figure 3.

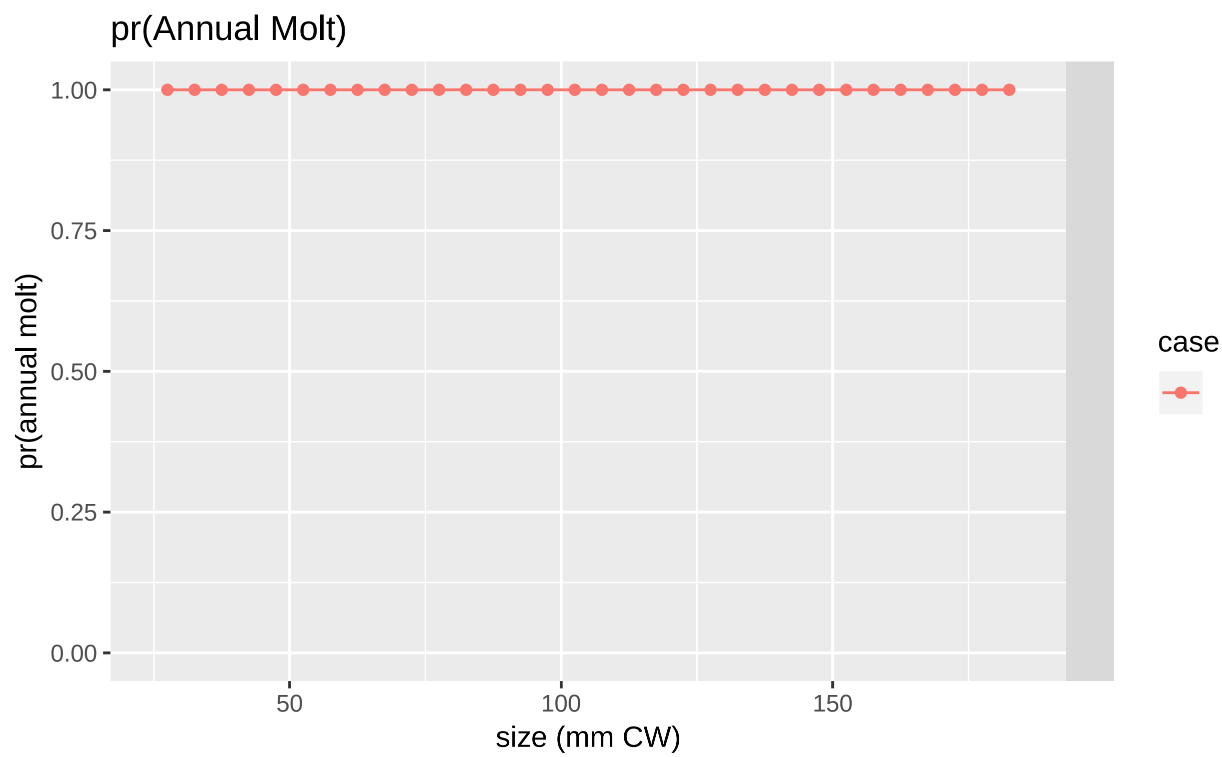


Figure 3. The probability of undergoing an annual molt, by pre-molt size, for immature new shell crab.

Growth

Mean post-molt size $\overline{z}_{pst}$ is modeled as a power function of pre-molt size z_{pre}, paramterized as:

$$\overline{z}_{pst} = z^A_{pst} \cdot exp\{\frac{log(\overline{z}^B_{pst}/\overline{z}^A_{pst})}{log(z^B_{pre}/z^A_{pre})} \cdot log(z_{pre}/z^A_{pre})\}$$

where z^A_{pst} is the estimated mean post-molt size at pre-molt size z^A_{pre} and z^B_{pst} is the estimated mean post-molt size at pre-molt size z^B_{pre}. The actual post-molt size z_{pst} for a crab, given that it was in size bin z_i prior to molting, is described using a γ distribution, with the probability that the post-molt crab falls into the jth size bin z_j given by:

$$p(z_j|z_i) = \int_{\alpha_i(z_j)-\frac{\delta\alpha}{2}}^{\alpha_i(z_j)+\frac{\delta\alpha}{2}} \gamma(\alpha - \overline{\alpha_i}) \cdot d\alpha$$

where $\alpha_i(z) = \frac{z-z_i}{\beta}$ represents the scaled molt increment, $\overline{\alpha_i} = \frac{\overline{z}_{pst}-z_i}{\beta}$ is the scaled mean molt increment for pre-molt size bin z_i, $\delta\alpha = \frac{\delta z}{\beta}$ is the scaled size bin width, and β is the scale factor. The largest model size bin, z_{max}, functions as an accumulator bin, so it is handled slightly differently: the probability of a post-molt crab ending up in the largest size bin is simply the probability of it ending up at any larger size than its lower cutpoint:

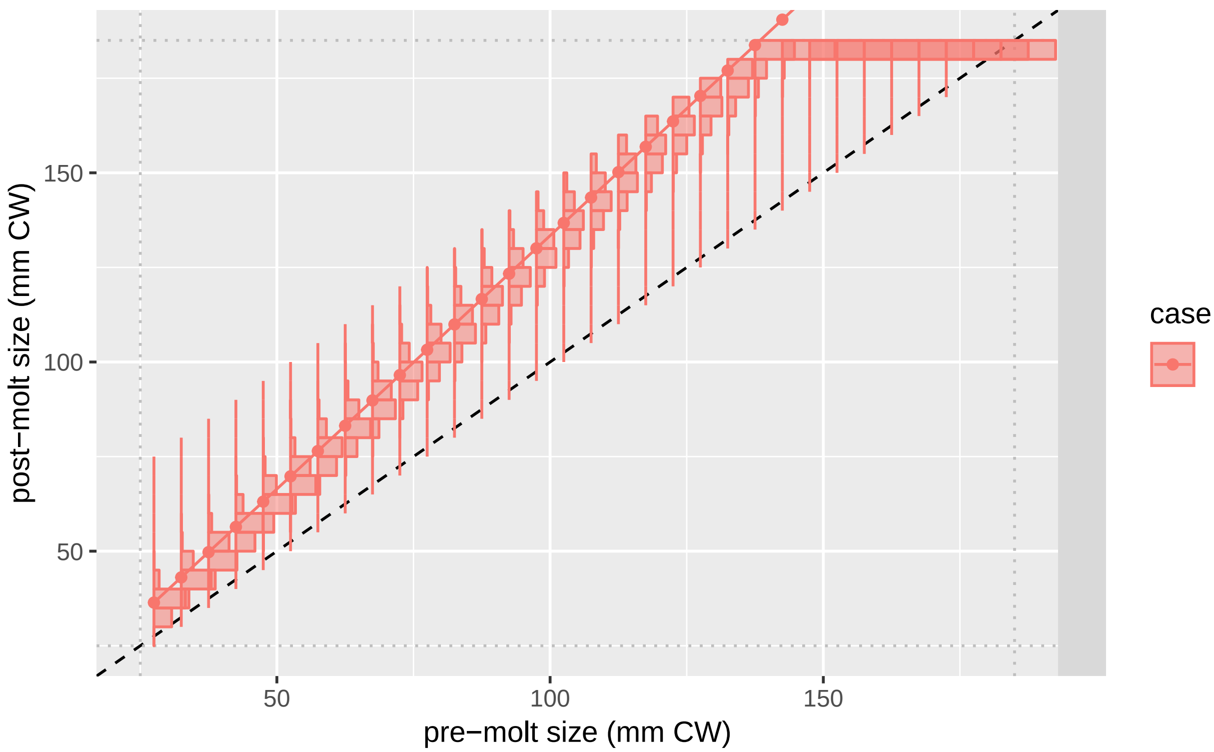
$$p(z_{max}|z_i) = \int_{\alpha_i(z_{max})-\frac{\delta\alpha}{2}}^{\mathrm{inf}} \gamma(\alpha - \overline{\alpha_i}) \cdot d\alpha = 1 - \int_{0}^{\alpha_i(z_{max})-\frac{\delta\alpha}{2}} \gamma(\alpha - \overline{\alpha_i}) \cdot d\alpha$$

The model also allows one to limit potential growth to a maximum number of size bins, n_{max}, in which case $p(z_j|z_i)$ is set to 0 for $j - i > n_{max}$ and normalized to sum to 1 for $j - i \leq n_{max}$.

The values for the parameters used in this report are given in the following table:

parameter	value
$\overline{z}^A_{pst}$	33.0888266
z^A_{pre}	25
$\overline{z}^B_{pst}$	166.9598541
z^B_{pre}	125
β	125
n_{max}	10

The resulting growth probabilities are illustrated in Figure 4.



[[1]]

Figure 4. The pre-molt size-dependent probability of annual growth, given that an immature crab molted.

Terminal molt

The probability that a molt is the terminal molt to maturity, $p^T(z)$ is parameterized in the assessment model on the logit scale as a nonparameteric, smooth function of pre-molt size. Inidividual parameters are estimated on the logit scale for each size bin, with likelihood penalties applied to the second order differences to impose a smoothness constraint on the resulting shape of the function. In addition, the first size bin at which terminal molt *can* occur ($p^T(z_0) > 0$)and the first size bin at which it *must* occur ($p^T(z_1) \equiv 1$) can be set to reduce the number of logit-scale parameters that must be estimated. In the interest of simplicity, it is also possible here to use a logistic function parameterized by size at the inflection point (z_{50}) and scale (b_{50}).

For this report, the nonparametric approach was used. The values for the parameters are given in the following table:

parameter	value
z_0	27.5
z_1	182.5
values	-12.0290, -10.8459, -9.6628, -8.4808, -7.3104, -6.1637, -5.1127, -4.4866, -4.0998, -3.4626, -2.9278, -2.4975, -2.0262, -1.4392, -0.9519, -0.6817, -0.5325, -0.0624, 0.5599, 1.4352, 2.8100, 5.0637, 7.1973, 9.0101, 10.4957, 11.6880, 12.6273, 13.3554, 13.9145, 14.3468, 14.6945, 15.0000

The resulting terminal molt probabilities are illustrated in Figure 5.

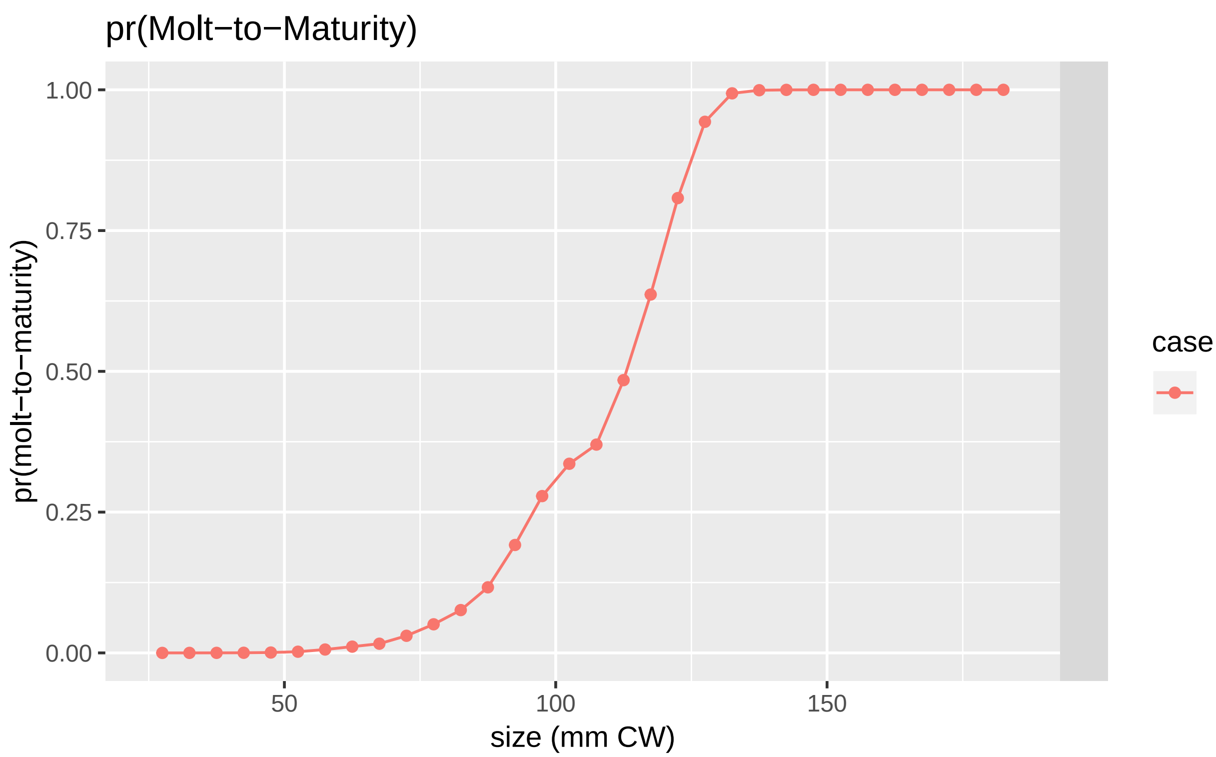


Figure 5. The pre-molt size-dependent probability, given that it molted, that an immature crab underwent its terminal molt to maturity.

Cohort progression

The progression of a cohort through subsequent years following recruitment to the assessment model based on the population processes described previously is illustrated in the remaining figures. Figure 6 documents the progression of the cohort on an absolute scale through time while Figure 7 documents the progression of the cohort on a normalized scale (such that the relative abundance in a year sums to 1 across all life stages and all sizes).

Abundance

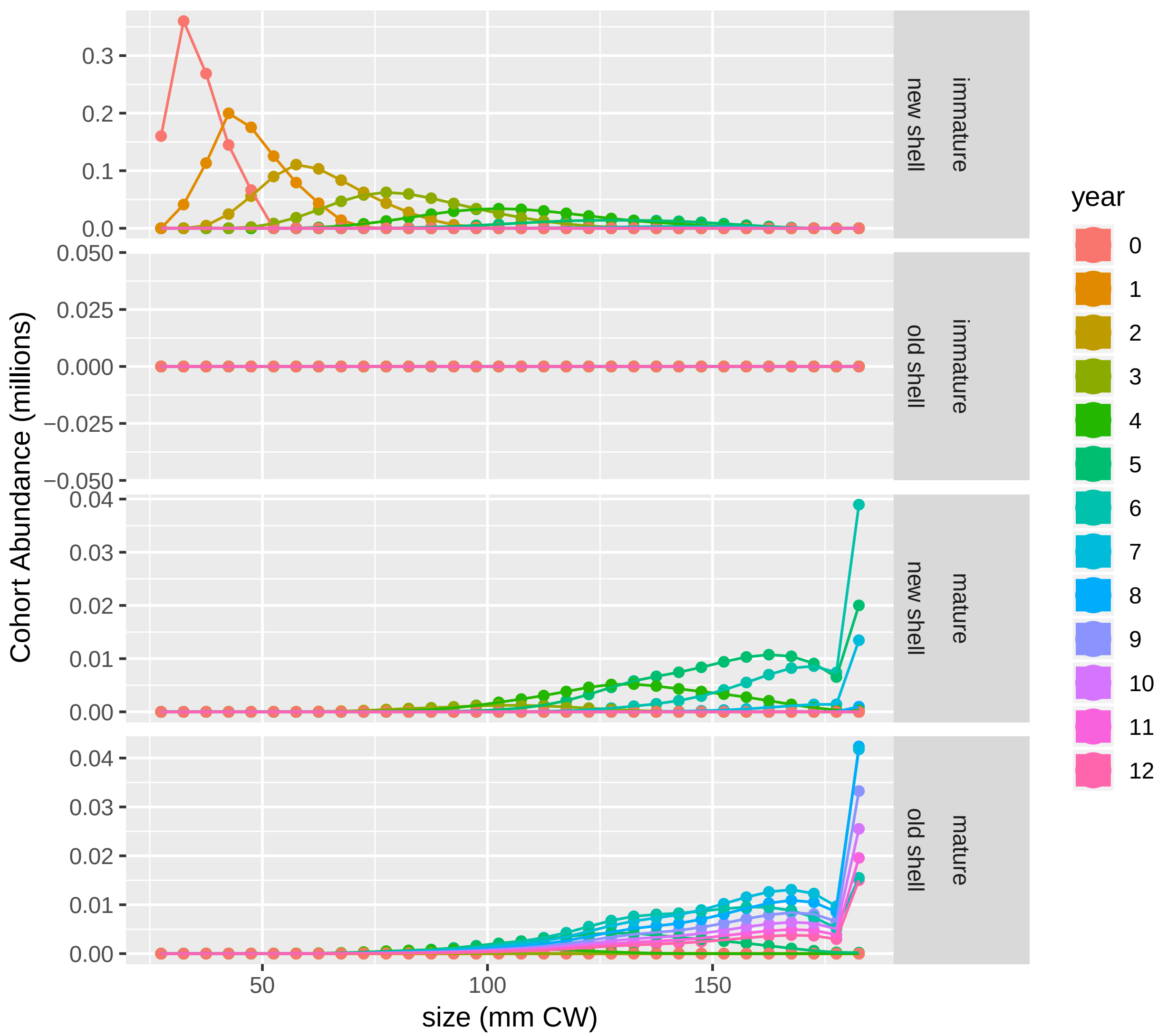


Figure 6. Absolute scale size comps showing cohort progression.

Normalized size compositions

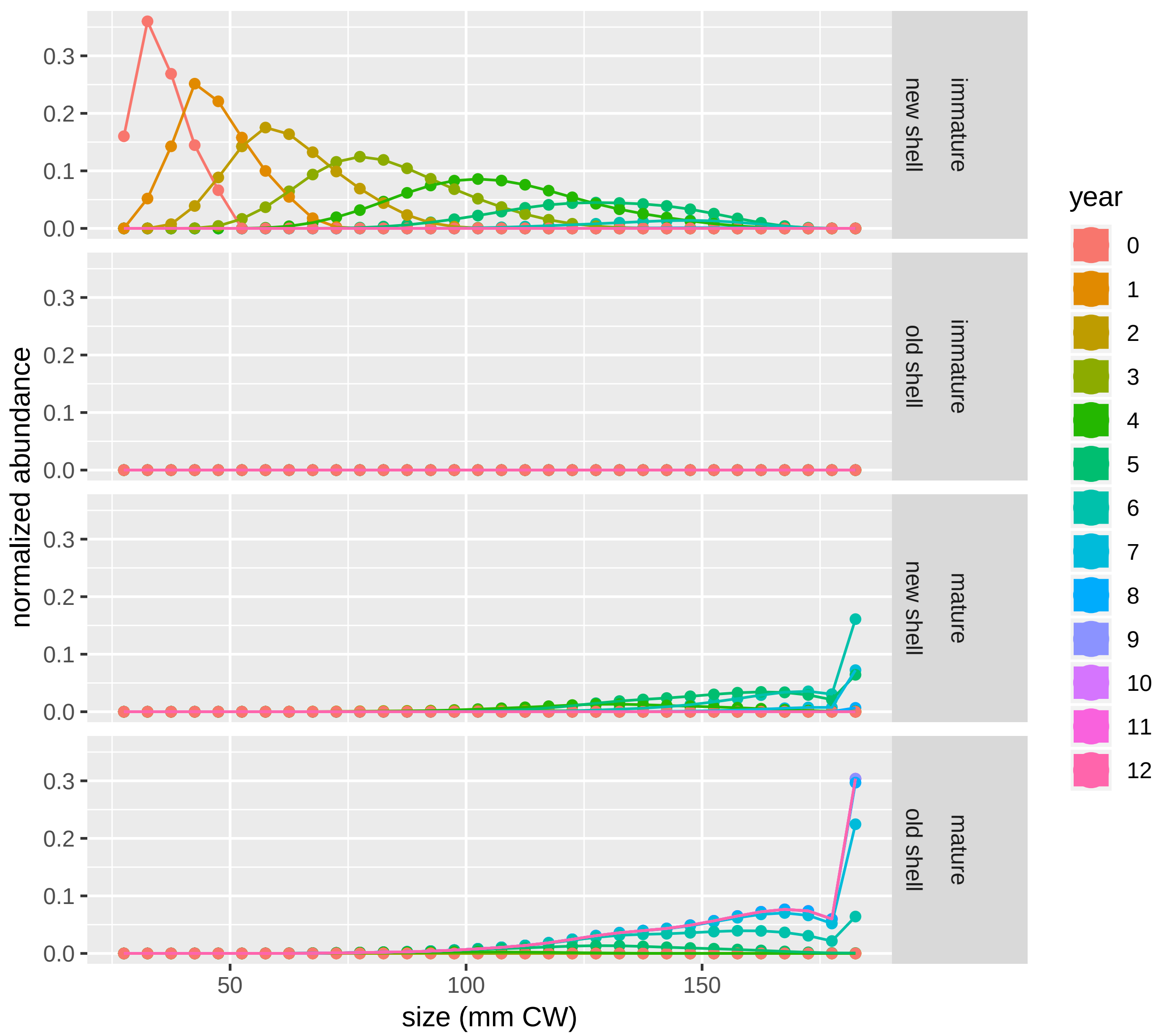


Figure 7. Normalized size comps showing cohort progression.