Archival tag methods: Estimating habitat preferences and utilization distribution

Julie Nielsen, Kevin Siwicke, James Thorson

Goal: Unified & practical movement analysis

1. Conservation of numbers
   - Isolate movement from other demography
2. Calculate habitat utilization as stationary distribution
   - Easy to convert output to species distribution model
3. Identical parameters in Lagrangian and Eulerian contexts
   - Joint likelihood for population and individual data; OR
   - Fit a model to tags, and then simulate movement for densities
4. Parsimonious and flexible parameterization
   - Use covariates to allow time-variation without extra parameters
5. Computational efficiency
   - Euler approximation and uniformization to avoid large-tailed movement
6. Scale-free parameters
   - Correct for discretization choice
7. Continuous-time calculation
   - Calculate at any intermediate time within intervals

Goal: Unified & practical movement analysis

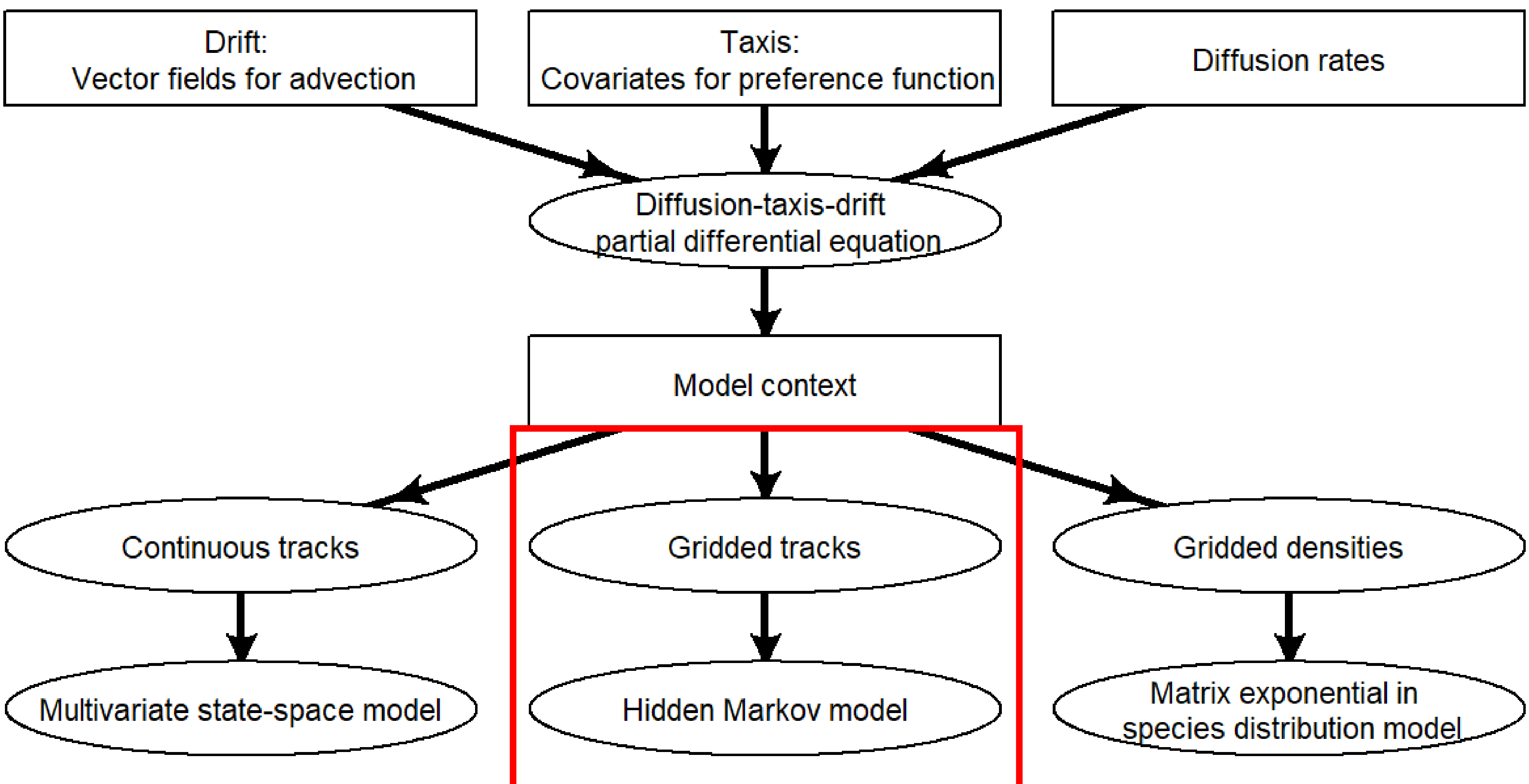
- Specify partial differential equation

$$\frac{\partial}{\partial t} d(s,t) = \underbrace{D\nabla^2 d(s,t)}_{\text{diffusion}} - \underbrace{\mathbf{v}(s)\cdot\nabla d(s,t)}_{\text{drift}} - \underbrace{\nabla h(s)\cdot\nabla d(s,t)}_{\text{taxis}}$$

- Lagrangian:
  - Calculate distribution after time Δ_t starting at known point S_0
  - Use as predicted distribution in a state-space model
- Eulerian
  - Discretize space and define Continuous Time Markov Chain (CTMC)
  - Computation:
    - Exact: Matrix exponential
    - Approximation: Euler method or "Uniformization"

$$\frac{\partial}{\partial t} d(s,t) = \underbrace{D\nabla^2 d(s,t)}_{\text{diffusion}} - \underbrace{\mathbf{v}(s) \cdot \nabla d(s,t)}_{\text{drift}} - \underbrace{\nabla h(s) \cdot \nabla d(s,t)}_{\text{taxis}}$$

Decision tree for movement models



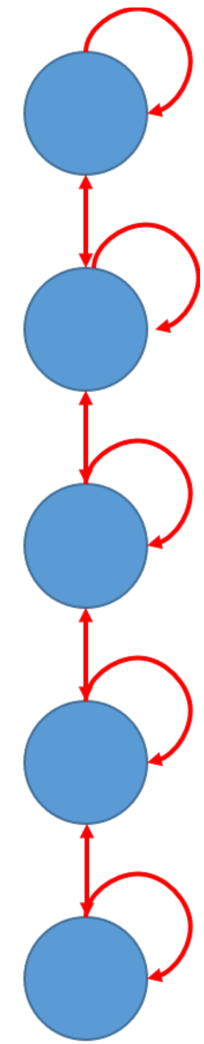
Continuous-time Markov Chain

Different forms of movement

$$\frac{d}{dt}\mathbf{b} = \mathbf{b}^T\mathbf{M}$$

$$\mathbf{M} = \begin{matrix} \mathbf{1-p} & \mathbf{p} & & & \\ \mathbf{p} & \mathbf{1-2p} & \mathbf{p} & & \\ & \mathbf{p} & \mathbf{1-2p} & \mathbf{p} & \\ & & \mathbf{p} & \mathbf{1-2p} & \mathbf{p} \\ & & & \mathbf{p} & \mathbf{1-p} \end{matrix}$$

Diffusion



Continuous-time Markov Chain

Review: Solving differential equations

- Say you know a rate:

$$\frac{\mathrm{d}}{\mathrm{d}t} b(t) = \alpha b(t)$$

- How do you solve for change after some time Δt?

$$b(t + \Delta t) = b(t) e^{\alpha \Delta t}$$

- This is the definition of the exponential function

Continuous-time Markov Chain

Review: simultaneous differential equations

- Say you know a rate:

$$\frac{\delta}{\delta t}\mathbf{b}(t) = \mathbf{b}(t)\mathbf{A}$$

- The a general solution is:

$$\mathbf{b}(t + \Delta_t) = \mathbf{b}(t)\mathbf{B}$$
$$\mathbf{B} = \mathrm{e}^{\mathbf{A}\Delta_\mathrm{t}}$$

where $\mathrm{e}^{\mathbf{A}\Delta_\mathrm{t}}$ is the matrix exponential of $\mathbf{A}\Delta_\mathrm{t}$

Continuous-time Markov Chain

Three implementations for matrix exponential:

1. Just use software versions
   - expm::expm() in R
   - expm() in TMB

2. Euler approximation

$$e^{\mathbf{A}} = \left(\mathbf{I} + \frac{\mathbf{A}}{N}\right)^N$$

   - Where N is the number of sub-intervals, and we linearize in each

3. Uniformization

$$e^{\mathbf{A}} = \frac{\mathbf{A}^0}{0!} + \frac{\mathbf{A}^1}{1!} + \frac{\mathbf{A}^2}{2!} + \frac{\mathbf{A}^3}{3!} + \cdots$$

So:

$$\mathbf{b}^T e^{\mathbf{A}} = \mathbf{b}^T + \mathbf{b}^T \mathbf{A} + \frac{\mathbf{b}^T \mathbf{A}^2}{2!} + \frac{\mathbf{b}^T \mathbf{A}^3}{3!} + O(\dots)$$

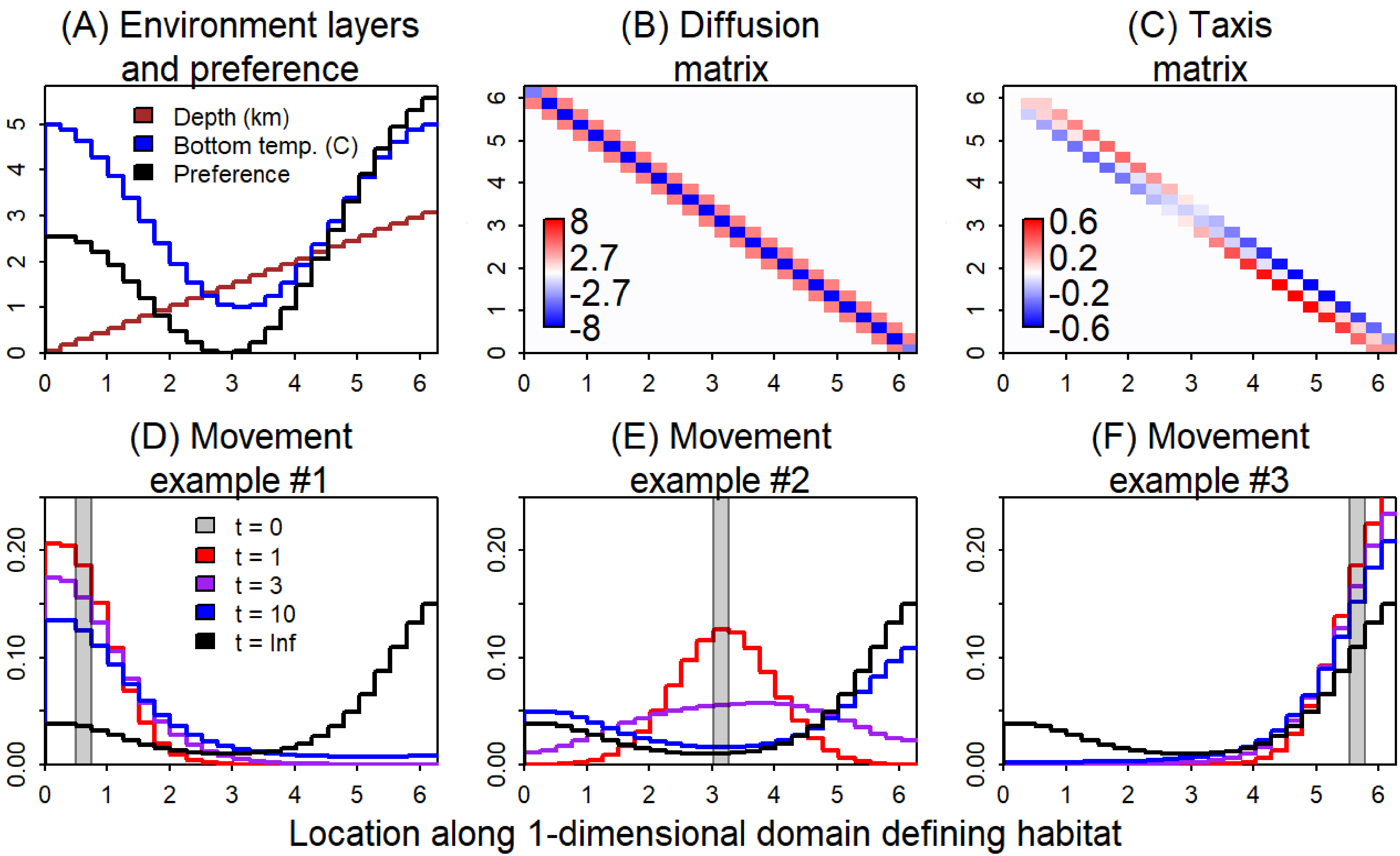
So:

$$\mathbf{b}^T e^{\mathbf{A}} = \sum_{n=\mathbf{0}}^{N} \mathbf{b}_n^T + O(\dots)$$

$$\mathbf{b}_n^T = \begin{cases} \mathbf{b}^T & \text{if } n = 0 \\ \dfrac{\mathbf{b}_{n-1}^T \mathbf{A}}{n} & \text{if } n > 0 \end{cases}$$

1D example of diffusion-taxis movement

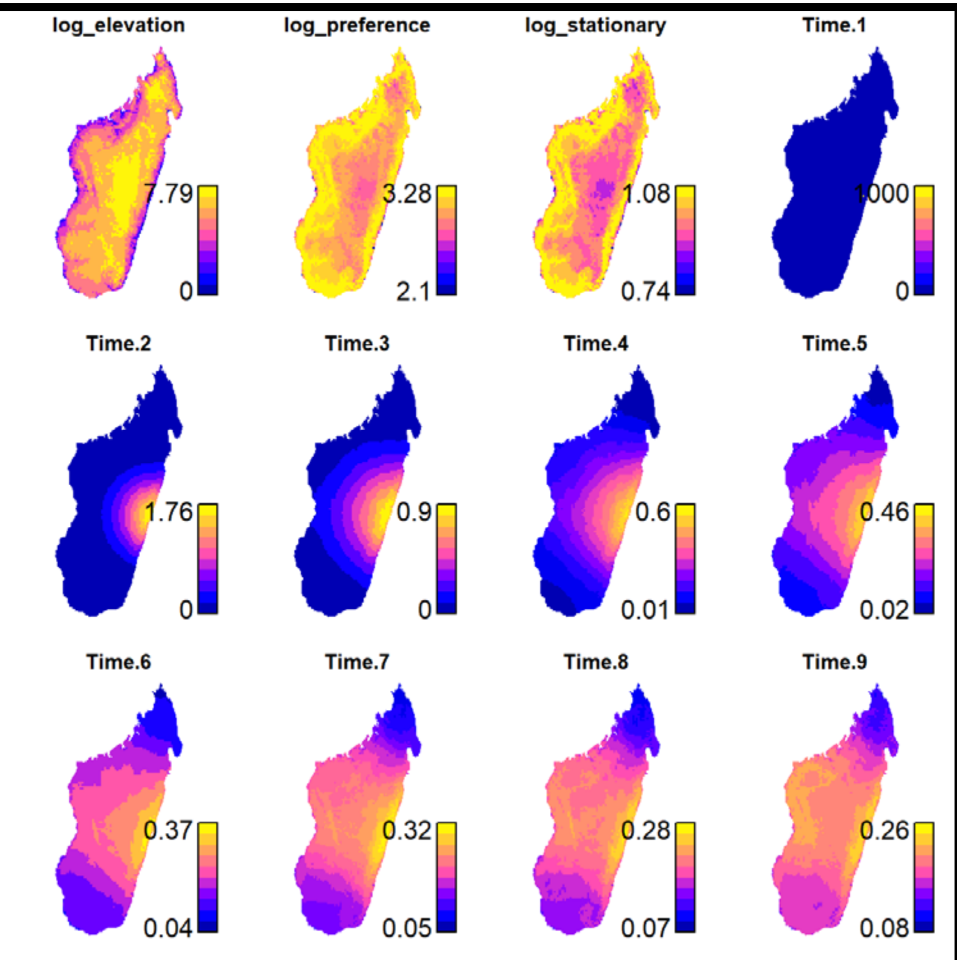
- Example with $n_g = 25$ and $h_g = 0.5T_g + D_g$



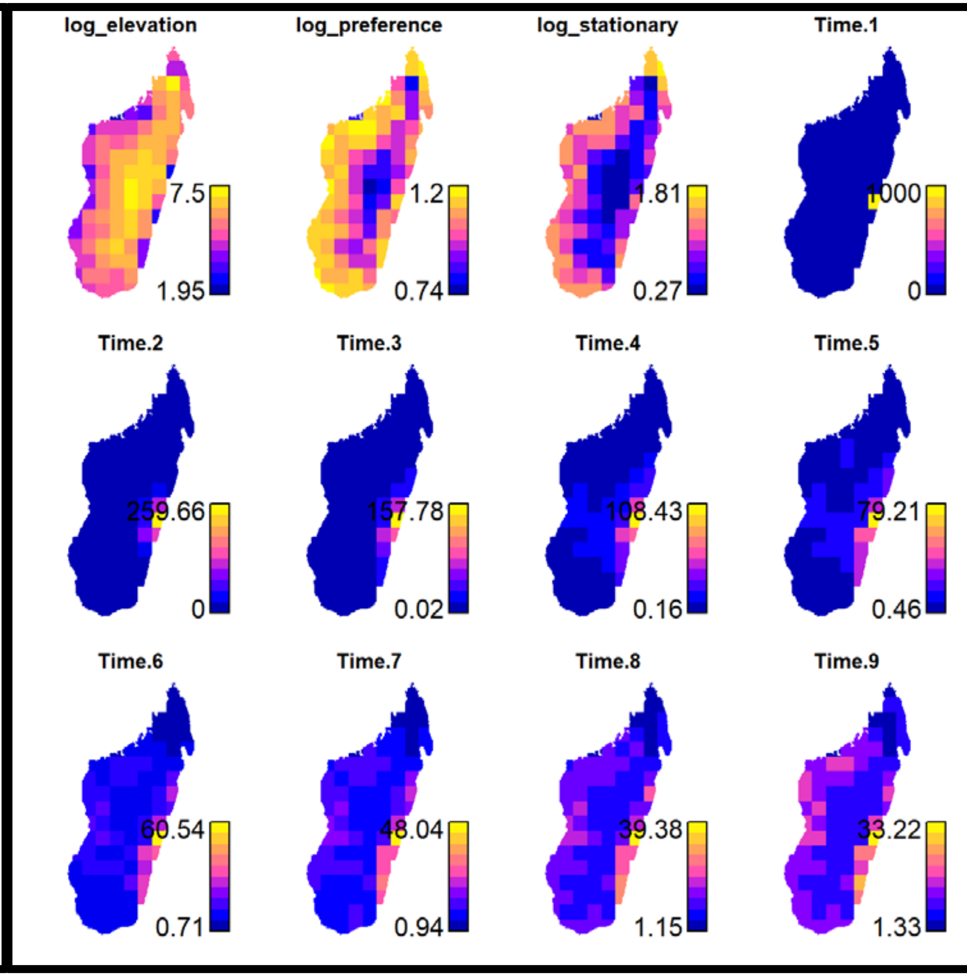
Discretize space for density (Eulerian) models

- Can use scale-invariant parameters, so results don't depend on scale

High resolution simulation of species invasion



Low resolution simulation of species invasion



Hidden Markov model

Goals:

- Estimate likelihood of parameters $\mathcal{L}(\theta; p_{g,t})$
  - Where $p_{g,t}$ is the data likelihood for archival tags
  - θ includes habitat preference parameters
- Estimate probability of states $\pi_{g,t}$ in each time t
- Predict habitat utilization from preference parameters

Approach:

- Apply forward algorithm to get
$$\Pr(Current\ state|prior\ and\ current\ data)$$
- Apply backward algorithm to get:
$$\Pr(Future\ data|state)$$
- Optimal state estimate is their product

Hidden Markov model

Apply forward algorithm to get

$$Pr(Current\ state|prior\ and\ current\ data)$$

Algorithm

$$\mathbf{f}_t = \begin{cases} \mathbf{p}_t & \text{if } t = 1 \\ \mathbf{p}_t \odot \mathbf{f}_{t-1}\mathbf{M} & \text{if } t > 1 \end{cases}$$

Where:

$$\mathcal{L}(\theta; p_{g,t}) = \sum_{g=1}^{G} f_{g,T}$$

And:

- T is the maximum time
- $\mathbf{f}_t$ is the forward algorithm for the probability $f_{g,t}$ of each state g in time t
- $\mathbf{M}$ is the integrated movement matrix

Hidden Markov model

Apply backwards algorithm to get

$$\Pr(Future\ data|state)$$

Algorithm

$$\mathbf{b}_t = \begin{cases} \mathbf{1} & \text{if } t = T \\ \mathbf{M}(\mathbf{p}_{t+1} \odot \mathbf{b}_{t+1}) & \text{if } t > 1 \end{cases}$$

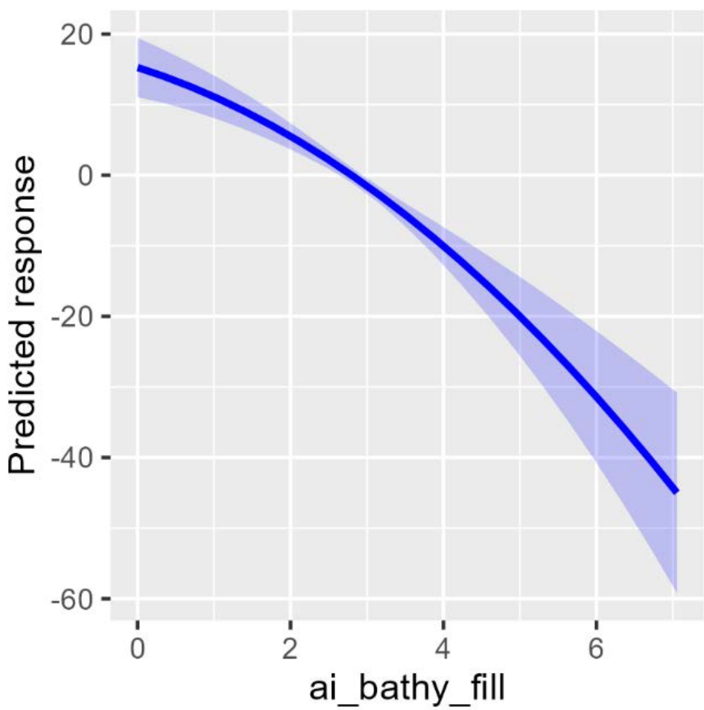
And then get empirical Bayes probability:

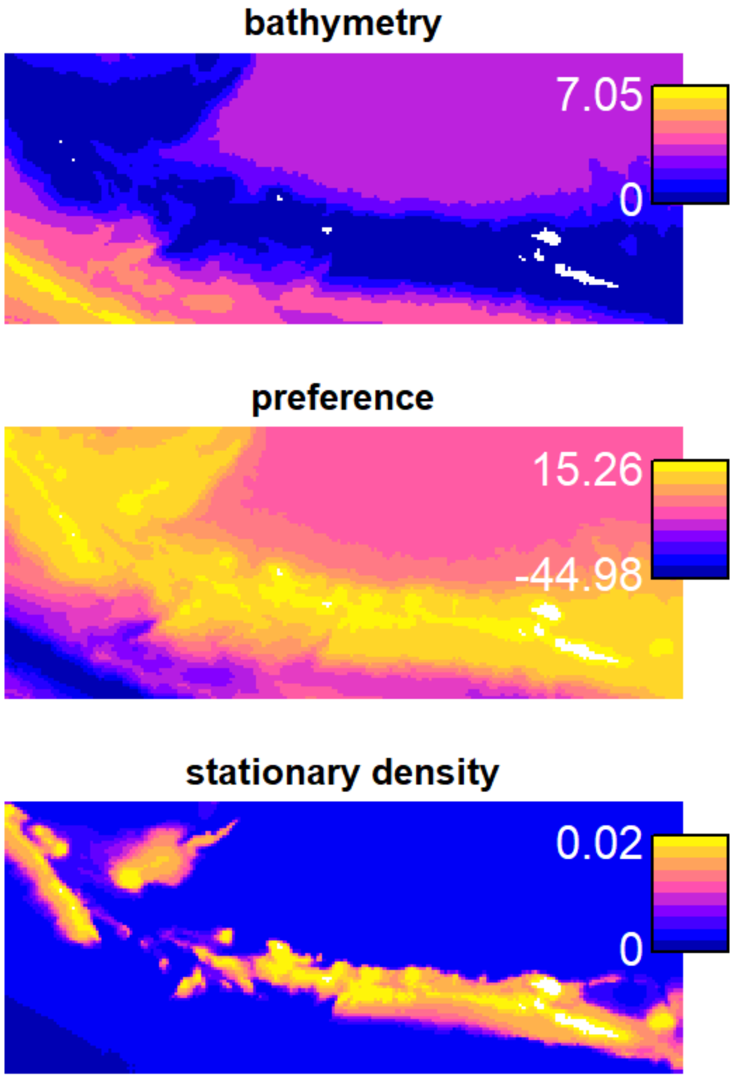
$$\boldsymbol{\pi}_t \propto \mathbf{f}_t \mathbf{b}_t$$

Discretize space for density (Eulerian) models

- Can fit to archival tags
- Showing a Pacific cod tagged

Predicted response to bathymetric depth

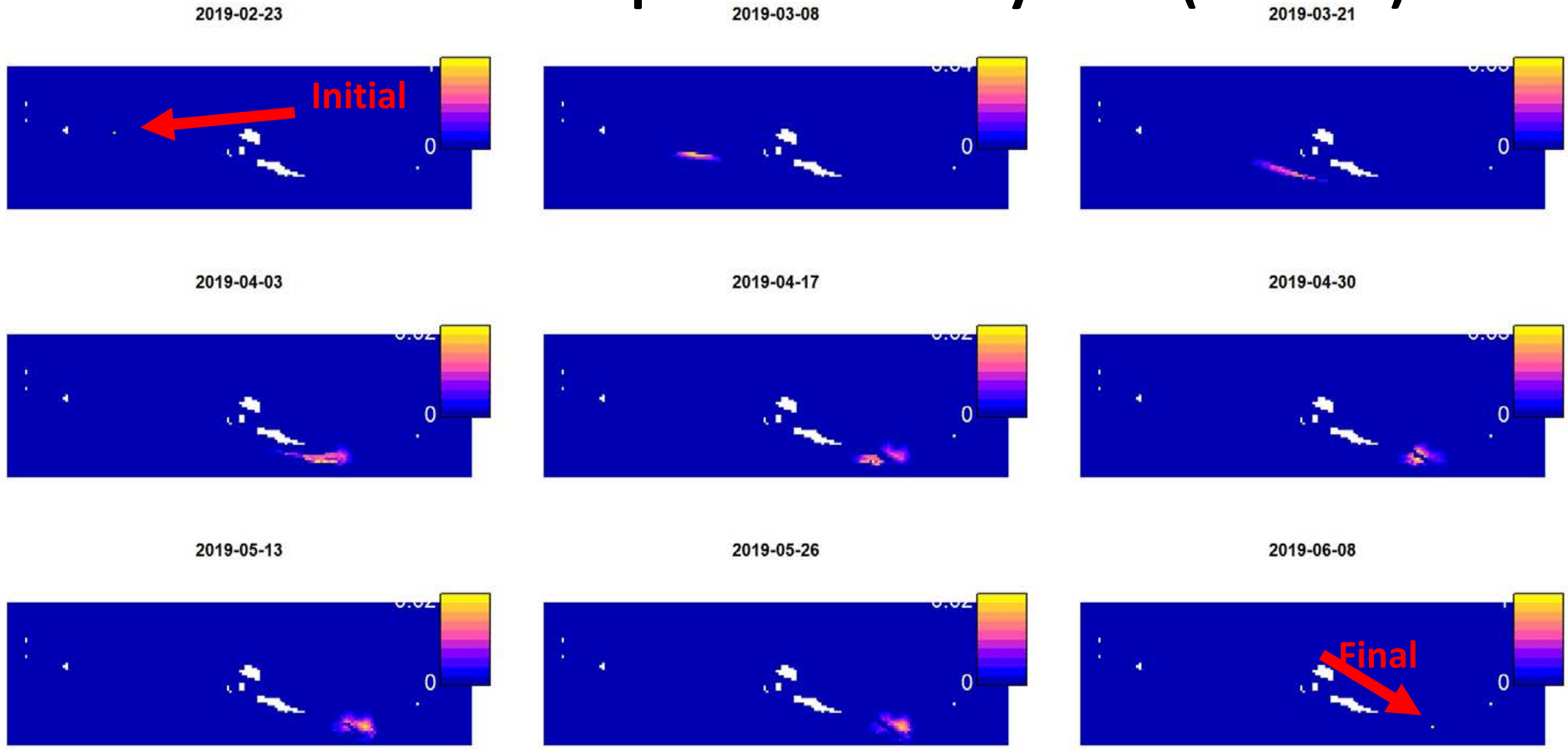




Discretize space for density (Eulerian) models

- Can fit to archival tags
- Apply filter-smoother algorithm to reconstruct tracks for 91 day tag deployment

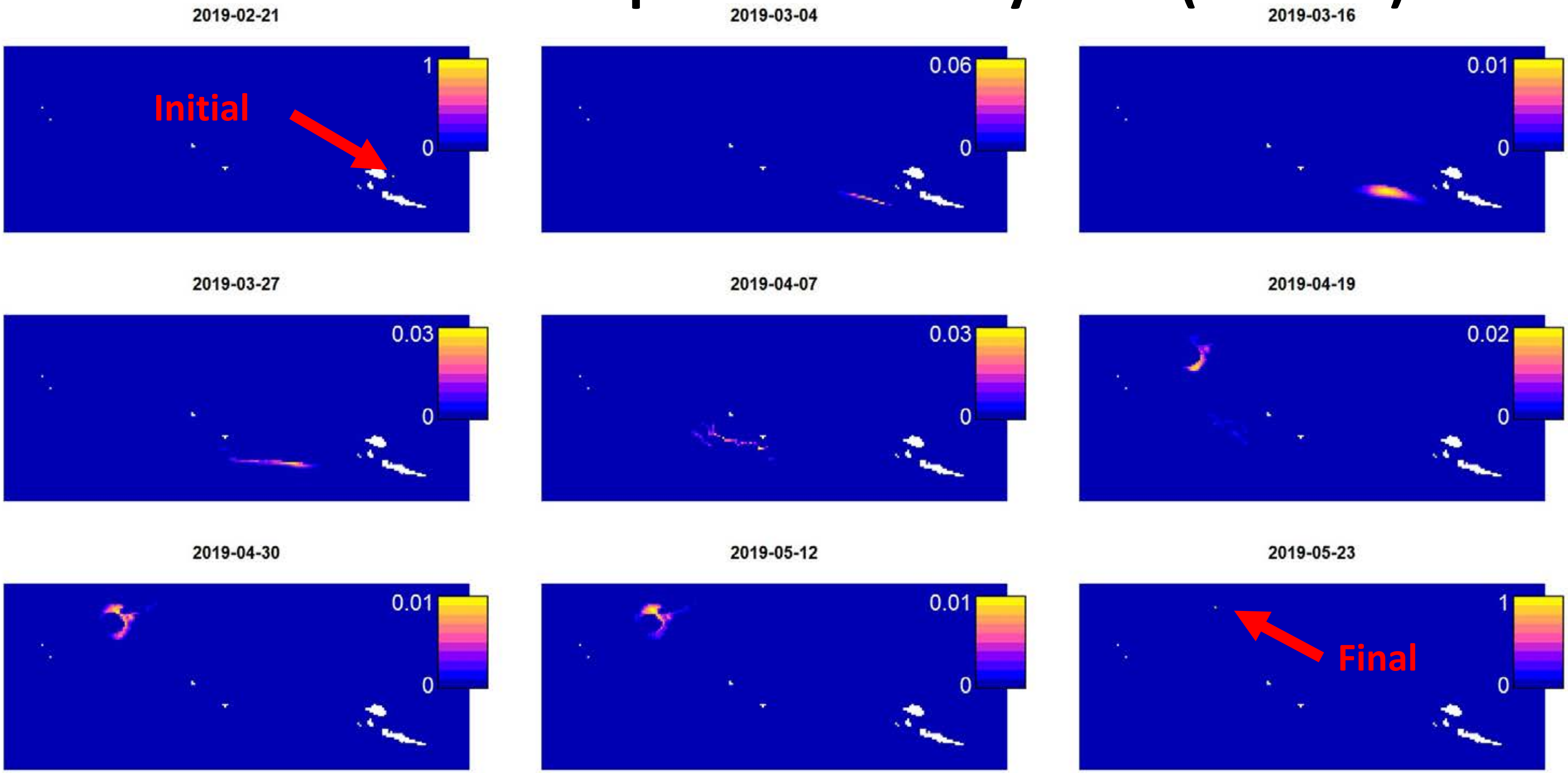
Predicted location probabilities by date (178709)



Discretize space for density (Eulerian) models

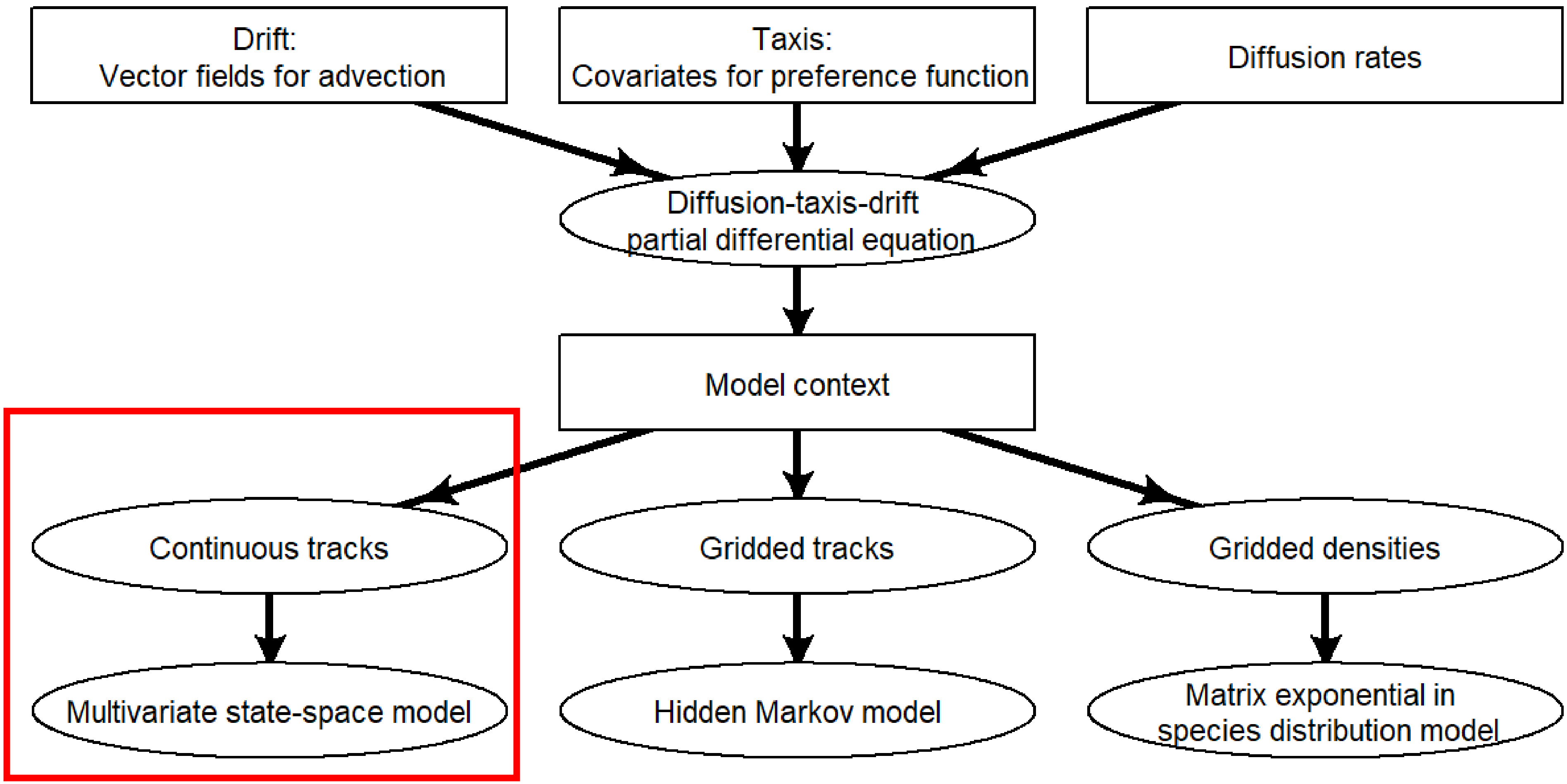
- Can fit to archival tags
- Apply filter-smoother algorithm to reconstruct tracks for 91 day tag deployment

Predicted location probabilities by date (178690)



$$\frac{\partial}{\partial t} d(s,t) = \underbrace{D\nabla^2 d(s,t)}_{\text{diffusion}} - \underbrace{\mathbf{v}(s)\cdot\nabla d(s,t)}_{\text{drift}} - \underbrace{\nabla h(s)\cdot\nabla d(s,t)}_{\text{taxis}}$$

Decision tree for movement models

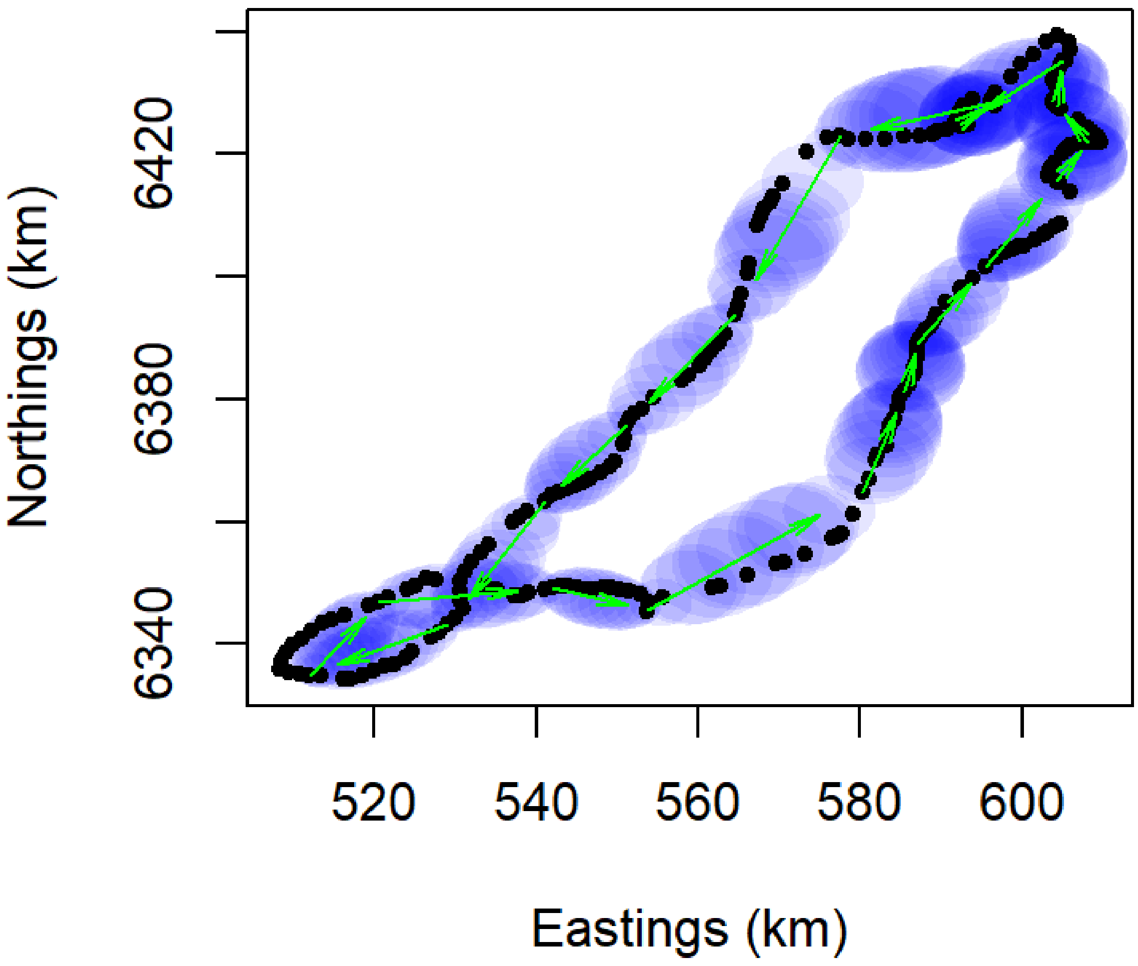


Continuous space for individual (Lagrangian) models

Estimate movement

(northern fur seal , tagged by AFSC Polar Ecosystem Program at St. Paul in 2016)

- Black dots – location from telemetry
- Thinning to 20 locations for fitting
- Blue circles – confidence elipse for predictions
- Green arrows – estimated path



Acknowledgements

Throughout:

- Kasper Kristensen

Co-authors:

- Julie Nielsen and Kevin Siwicke

Questions/Requests for Plan Team

Endorsement:

- Any concerns or additional research needed before using diffusion-taxis movement for archival tags in council process?

Prioritization:

- Is it worth pursuing funds to identify covariates that explain habitat preference for cod using archival tags?

Distribution:

- Should we aim to have an R-package that can do this, to facilitate access to method, or are simple scripts sufficient?