AGENDA D-1(b)
APRIL 1996

MEMORANDUM

TO: Council, SSC and AP Members

FROM: Clarence G. Pautzke
Executive Director

DATE: April 11, 1996

SUBJECT: Overfishing Definition Amendment

ESTIMATED TIME
4 HOURS
(for all D-1 items)

ACTION REQUIRED

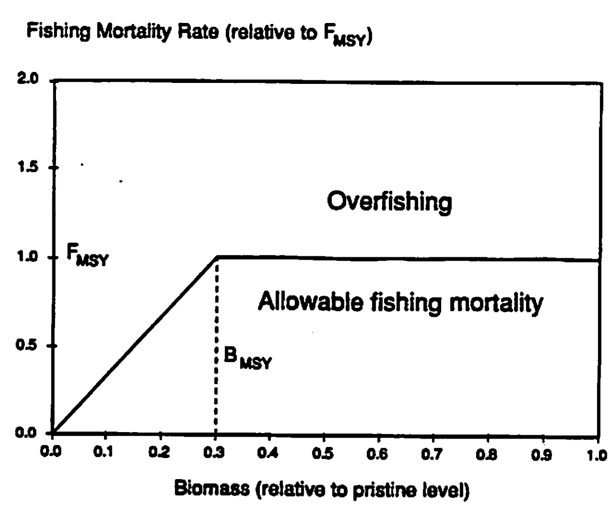
Initial review of plan amendment to revise the overfishing definition for BSAI and GOA groundfish.

BACKGROUND

In 1990, the *602 Guidelines* mandated that overfishing be defined in FMPs as follows:

> "Overfishing is a level or rate of fishing mortality that jeopardizes the long-term capacity of a stock or stock complex to produce maximum sustainable yield on a continuing basis", and that "Each FMP must specify, to the maximum extent possible, an objective and measurable definition of overfishing for each stock or stock complex covered by that FMP, and provide an analysis of how the definition was determined and how it relates to reproductive potential."

The Council added overfishing definitions to the GOA (Amendment 21) and BSAI (Amendment 16) fishery management plans in 1990, defining a maximum fishing mortality rate that declines at low stock sizes. Specifically, for any stock or stock complex under management, the maximum allowable mortality rate is set at the level corresponding to maximum sustainable yield (F_{msy}) for all biomass levels in excess of the level corresponding to maximum sustainable yield (B_{msy}). For lower biomass levels, the maximum allowable fishing mortality rate varies linearly with biomass, starting from a value of zero at the origin and increasing to a value of F_{msy} at B_{msy}, consistent with other applicable laws. These relationships are shown- in the figure below.



If data are insufficient to calculate F_{msy} or B_{msy}, the maximum allowable fishing mortality rate will be set equal to the following (in order of preference):

(1) the value that results in the biomass-per-recruit ratio (measured in terms of spawning biomass) falling to 30% of its pristine value;

(2) the value that results in the biomass-per-recruit ratio (measured in terms of exploitable biomass) falling to 30% of its pristine value; or

(3) the natural mortality rate (M).

If data are insufficient to estimate any of the above, the TAC shall not exceed the average catch taken since 1977.

The current overfishing definitions do not necessarily provide a buffer between acceptable biological catch (ABC) and the overfishing level (OFL). The Plan Teams and SSC have expressed concern about harvesting stocks to the OFL level as an acceptable target. In January 1995, the Council adopted for analysis a Scientific and Statistical Committee proposal (Item D-1(b)(1)) to evaluate the OFL and amend the plans as necessary. Grant Thompson, NMFS-AFSC will be on hand to present his analysis.

GROUNDFISH FISHERY MANAGEMENT PLAN AMENDMENT PROPOSAL
North Pacific Fishery Management Council

Name of Proposer: Scientific and Statistical Committee **Date:** 12/7/94
Address:
Telephone:

Fishery Management Plan: GOA/BSAI Groundfish

Brief Statement of Proposal:

Reconsider overfishing definition to provide buffer between ABC and OFL and to respond to "Scientific Review of Definitions of Overfishing" prepared for NMFS.

Objectives of Proposal: (What is the problem?)

Problems have occurred in the groundfish specification process when ABC and OFL turn out to be the same. Conceptually, ABC should be a "target" and OFL should be a "threshold" level to be avoided, so that there should be a buffer between them.

Need and Justification for Council Action: (Why can't the problem be resolved through other channels?)

The OFL process is specified in the plans. The Teams sometimes adjust the ABC downward to provide a buffer. The SSC does not agree with this approach and the desirability of the downward adjustment has not been evaluated. The "Scientific Review" claims that the NPFMC overfishing definition is somewhat ambiguous and may not be conservative in some cases. It recommends an evaluation mechanism based on recruitment falling to 1/2 the pristine level that may not be appropriate. The Council should be proactive in addressing overfishing.

Foreseeable Impacts of Proposal: (Who wins, who loses?)

Evaluation of the OFL process is needed to provide credibility for the desired conservatism of the NPFMC TAC's, ABC's, and OFL's. Overfishing is one of the most important issues in fisheries management at the current time, and the Council needs the assurance that its management avoids overfishing.

Are There Alternative Solutions? If so, what are they and why do you consider your proposal the best way of solving the problem?

NO

Supportive Data & Other Information: What data are available and where can they be found?

"Scientific Review of Definitions of Overfishing in U.S. Fishery Management Plans" by A. Rosenberg, et al. (1994).

Signature:

Terrance J. Quinn II, Chair, SSC

DRAFT FOR COUNCIL REVIEW

ENVIRONMENTAL ASSESSMENT/REGULATORY IMPACT REVIEW/ INITIAL REGULATORY FLEXIBILITY ANALYSIS

FOR

AN AMENDMENT TO THE BERING SEA/ALEUTIAN ISLANDS
AND GULF OF ALASKA
GROUNDFISH FISHERY MANAGEMENT PLANS

TO

REDEFINE ACCEPTABLE BIOLOGICAL CATCH AND OVERFISHING

Prepared by

Staff
National Marine Fisheries Service
Alaska Fisheries Science Center

April 16, 1996

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Executive Summary

Reviews by NMFS' Overfishing Definitions Review Panel (ODRP) and the Council's Scientific and Statistical Committee (SSC) have indicated that the definitions of "acceptable biological catch" and "overfishing" contained in the fishery management plans for groundfish of the Bering Sea/Aleutian Islands and Gulf of Alaska could and should be improved. Suggestions for improvement include the following: A) as parameter estimates become more imprecise, fishing mortality rates should become more conservative; B) for a stock below its target abundance level, fishing mortality rates should vary directly with biomass and ultimately fall to zero should the stock become critically depleted; and C) a buffer should be maintained between acceptable biological catch and the overfishing level.

This plan amendment proposal contains two alternatives: Alternative 1 (No Action) maintains the current definitions, and Alternative 2 (Proposed Revision) modifies the current definitions in response to the suggestions made by the ODRP and SSC. The differences between the two alternatives can perhaps best be illustrated by considering the case in which a point estimate of the fishing mortality rate at maximum sustainable yield (F_{MSY}) is available together with a reliable description of the amount of uncertainty surrounding that estimate. Under the current definitions, the target fishing mortality rate (F_{ABC}) and the maximum allowable fishing mortality rate (F_{OFL}, the rate above which overfishing is defined to occur) are both set equal to the point estimate of F_{MSY}, regardless of the level of uncertainty associated with that estimate. Under the proposed revision, the ratio between F_{ABC} and F_{OFL} varies inversely with the level of uncertainty (i.e., the greater the uncertainty in the estimate of F_{MSY}, the lower F_{ABC} is in relation to F_{OFL}).

Even in cases where reliable descriptions of the level of uncertainty associated with a point estimate of F_{MSY} are not available, the proposed revision maintains an appropriate buffer between F_{ABC} and F_{OFL}. Also, whenever a target abundance level can be reasonably identified, the proposed revision reduces fishing mortality rates as stock size falls below that target level. The current definitions do neither of these.

Because the proposed revision institutes new safeguards against overly aggressive harvest rates, particularly under conditions of high uncertainty or low stock size, the revision is expected to result in positive environmental impacts. Because the proposed revision is based explicitly on harvest policies designed to optimize long-term fishery performance, the revision is also expected to result in positive long-term economic impacts. However, it is possible that negative economic impacts could be generated in the short term for a few fisheries, particularly rockfish fisheries targeting on species other than Pacific ocean perch, where TAC might be reduced by 15-25%.

1.0 INTRODUCTION

The groundfish fisheries in the Exclusive Economic Zone (EEZ) (3 to 200 miles offshore) off Alaska are managed under the Fishery Management Plan for the Groundfish Fisheries of the Gulf of Alaska and the Fishery Management Plan for the Groundfish Fisheries of the Bering Sea and Aleutian Islands Area. Both fishery management plans (FMP) were developed by the North Pacific Fishery Management Council (Council) under the Magnuson Fishery Conservation and Management Act (Magnuson Act). The Gulf of Alaska (GOA) FMP was approved by the Secretary of Commerce and become effective in 1978 and the Bering Sea and Aleutian Islands Area (BSAI) FMP become effective in 1982.

Actions taken to amend FMPs or implement other regulations governing the groundfish fisheries must meet the requirements of Federal laws and regulations. In addition to the Magnuson Act, the most important of these are the National Environmental Policy Act (NEPA), the Endangered Species Act (ESA), the Marine Mammal Protection Act (MMPA), Executive Order (E.O.) 12866, and the Regulatory Flexibility Act (RFA).

NEPA, E.O. 12866 and the RFA require a description of the purpose and need for the proposed action as well as a description of alternative actions which may address the problem. This information is included in Section 1 of this document. Section 2 contains information on the biological and environmental impacts of the alternatives as required by NEPA. Impacts on endangered species and marine mammals are also addressed in this section. Section 3 contains a Regulatory Impact Review (RIR) which addresses the requirements of both E.O. 12866 and the RFA that economic impacts of the alternatives be considered. Section 4 contains the Initial Regulatory Flexibility Analysis (IRFA) required by the RFA which specifically addresses the impacts of the proposed action on small businesses.

This Environmental Assessment/Regulatory Impact Review/Initial Regulatory Flexibility Analysis (EA/RIR/IRFA) addresses a pair of plan amendments (one each for the BSAI and GOA Groundfish FMPs) to redefine "acceptable biological catch" (ABC) and "overfishing."

1.1 Purpose of and Need for the Action

The Magnuson Fishery Conservation and Management Act (MFCMA) contains a set of "national standards" with which all fishery management plans and implementing regulations must be consistent. The first national standard states,

> "Conservation and management measures shall prevent overfishing while achieving, on a continuing basis, the optimum yield from each fishery for the United States fishing industry."

Thus, the MFCMA places a high priority on the prevention of overfishing. However, nowhere in the MFCMA is overfishing defined. In 50 CFR Part 602, the National Oceanic and Atmospheric Administration (NOAA) presented its Guidelines for Fishery Management Plans (the "602

Guidelines"), which contain the following general definition:

> "Overfishing is a level or rate of fishing mortality that jeopardizes the long-term capacity of a stock or stock complex to produce maximum sustainable yield (MSY) on a continuing basis."

Because of the generality of this definition, NOAA felt that it would be difficult to apply unambiguously. Therefore, the 602 Guidelines also contain the following directive:

> "Each FMP must specify, to the maximum extent possible, an objective and measurable definition of overfishing for each stock or stock complex covered by that FMP, and provide an analysis of how the definition was determined and how it relates to reproductive potential."

In response to that directive, the BSAI and GOA Groundfish FMPs were amended to include an objective and measurable definition of overfishing, effective in 1991.

The 602 Guidelines also make allowance for the use of ABC as a step in the TAC specification process. The definition of ABC contained in the BSAI and GOA Groundfish FMPs was last amended in 1987.

During the years since the Council's current ABC and overfishing definitions were first implemented, it has been possible to examine how well these definitions are serving their intended purpose. In addition, there has been opportunity for the development of increased understanding within the fishery science community as to desirable properties of reference fishing mortality rates such as those used to define ABC and overfishing. As a result, several concerns regarding the definitions used in the BSAI and GOA Groundfish FMPs have been raised, particularly by NMFS' Overfishing Definitions Review Panel (ODRP, Rosenberg et al. 1994) and the Council's Scientific and Statistical Committee (SSC). These concerns are paraphrased below, where the following notation is used: OFL is the overfishing level (i.e., the catch during the coming year that would correspond to the overfishing definition), MSY is maximum sustainable yield, B represents projected biomass at the start of the coming harvest year, B_{MSY} is the biomass corresponding to MSY, B_{thr} is a "threshold" biomass level greater than zero, B_{pre} is a "precautionary" biomass level greater than B_{thr}, F represents fishing mortality rate, F_{ABC} is the F used to set ABC for the coming harvest year, F_{OFL} is the F corresponding to the overfishing definition, and F_{MSY} is the F corresponding to MSY.

ODRP concerns (paraphrased):

1) F_{OFL} should vary directly with biomass when the latter is between B_{thr} and B_{pre}. Currently, B_{thr} is implicitly set equal to zero, and B_{pre} is defined only for those few cases in which a reliable estimate of B_{MSY} is available.

2) For healthy stocks, F_{OFL} should exceed F_{MSY}. Currently, F_{OFL} is set equal to F_{MSY} whenever biomass exceeds B_{MSY} (provided that reliable estimates for F_{MSY} and B_{MSY} exist).

3) The authority for determining reliability of information should be specified. Currently, the definition of overfishing is cast in terms of the "sufficiency" of the available data to estimate various quantities, but no single authority (e.g., Council, SSC, Plan Team) is given specific responsibility for determining what constitutes "sufficient."

4) Ambiguity should be eliminated in any text relating SPR to exploitable biomass. Currently, language describing the measurement of spawning per recruit could potentially be misconstrued as referring to absolute biomass.

SSC concerns (paraphrased):

5) F_{ABC} should be reduced when $B<B_{MSY}$. Currently, F_{ABC} is not tied to biomass except in the (hypothetical) case where a "threshold" has been identified for a particular stock.

6) More caution should be required when less information is available. Currently, the level of uncertainty surrounding an estimate (e.g., an estimate of F_{MSY}) has no explicit relationship to the value of F_{ABC}, so long as the problematic "sufficiency" criterion referenced in Concern #3 (above) is satisfied.

7) F_{OFL} should exceed F_{ABC}. Currently, there is no requirement for a buffer between F_{ABC} and F_{OFL}.

8) OFL should remain constant over time when catch history is the only information available. Currently, in cases where catch history is the only information available, OFL is set equal to the average catch since 1977, meaning that OFL should tend to decrease over time (assuming that catch never exceeds OFL).

It is in response to the above concerns that the present amendment proposal is presented.

1.2 Alternatives Considered

1.2.1 Alternative 1: No Action. Under this alternative, the following (current) definitions of acceptable biological catch and overfishing would remain in place:

Acceptable biological catch is a seasonally determined catch or range of catches that may differ from MSY for biological reasons. It may be lower or higher than MSY in some years for species with fluctuating recruitments. Given suitable biological justification by the Plan Team and/or Scientific and Statistical Committee, the ABC may be set anywhere between zero and the current biomass less the threshold value. The ABC may be modfied to incorporate safety factors and risk assessment due to uncertainty. Lacking other biological justification, the ABC is defined as the maximum sustainable yield exploitation rate multiplied by the size of the biomass for the relevant time period. The ABC is defined as zero when the stock is at or below its threshold.

Threshold is the minimum size of a stock that allows sufficient recruitment so that the stock can eventually reach a level that produces MSY. Implicit in this definition are rebuilding schedules. They have not been specified since the selection of a schedule is a part of the OY determination process. Interest instead is on the identification of a stock level below which the ability to rebuild is uncertain. The estimate given should reflect use of the best scientific information available. Whenever possible, upper and lower bounds should be given for the estimate.

Overfishing is defined as a maximum allowable fishing mortality rate. For any stock or stock complex under management, the maximum allowable mortality rate will be set at the level corresponding to maximum sustainable yield (F_{MSY}) for all biomass levels in excess of the level corresponding to maximum sustainble yield (B_{MSY}). For lower biomass levels, the maximum allowable fishing mortality rate will vary linearly with biomass, starting from a value of zero at the origin and increasing to a value of F_{MSY} at B_{MSY}, consistent with other applicable laws. If data are insufficient to calcluate F_{MSY} or B_{MSY}, the maximum allowable fishing mortality rate will be set equal to the following (in order of preference):

1) the value that results in the biomass-per-recruit ratio (measured in terms of spawning biomass) falling to 30% of its pristine value;

2) the value that results in the biomass-per-recruit ratio (measured in terms of exploitable biomass) falling to 30% of its pristine value; or

3) the natural mortality rate (M).

If data are insufficient to estimate any of the above, the TAC shall not exceed the average catch taken since 1977.

1.2.2 Alternative 2: Proposed Revision. The revision proposed is to strike the existing FMP language defining "threshold" and replace the existing FMP language defining ABC and overfishing with the following (the proposed ABC definition--except for the last sentence--is taken directly from the 602 Guidelines):

Acceptable Biological Catch is a preliminary description of the acceptable harvest (or range of harvests) for a given stock or stock complex. Its derivation focuses on the status and dynamics of the stock, environmental conditions, other ecological factors, and prevailing technological characteristics of the fishery. The fishing mortality rate used to calculate ABC is capped as described under "overfishing" below.

Overfishing is defined as any amount of fishing in excess of a prescribed maximum allowable rate. This maximum allowable rate is prescribed through a set of six tiers which are listed below in descending order of preference, corresponding to descending order of information availability. The SSC has final authority for determining whether a given item of information is "reliable" for the purpose of this definition, and may use either objective or subjective criteria in making such determinations. For tier (1), a "pdf" refers to a probability density function (Appendix A). For tiers (1-3), the coefficient α is set at a default value of 0.05, with the understanding that the SSC may establish a different value for a specific stock or stock complex as merited by the best available scientific information. Figure 1 provides a hypothetical illustration of the behavior of tiers (1-3). For tiers (2-4), a designation of the form "$F_{X\%}$" refers to the F associated with an equilibrium level of spawning per recruit (SPR) equal to $X\%$ of the equilibrium level of spawning per recruit in the absence of any fishing. If reliable information sufficient to characterize the entire maturity schedule of a species is not available, the SSC may choose to view SPR calculations based on a knife-edge maturity assumption as reliable. For tier (3), the term $B_{40\%}$ refers to the long-term average biomass that would be expected under average recruitment and $F=F_{40\%}$.

1) *Information available: Reliable point estimates of B and B_{MSY} and reliable pdf of F_{MSY}.*
   - 1a) *Stock status: $B/B_{MSY} > 1$*
     - $F_{OFL} = m_A$, the arithmetic mean of the pdf (Appendix A)
     - $F_{ABC} \leq m_H$, the harmonic mean of the pdf (Appendix A)
   - 1b) *Stock status:* $a < B/B_{MSY} \leq 1$
     - $F_{OFL} = m_A \times (B/B_{MSY} - a)/(1 - a)$
     - $F_{ABC} \leq m_H \times (B/B_{MSY} - a)/(1 - a)$
   - 1c) *Stock status: $B/B_{MSY} \leq a$*
     - $F_{OFL} = 0$
     - $F_{ABC} = 0$
2) *Information available: Reliable point estimates of B, B_{MSY}, F_{MSY}, $F_{30\%}$,* and $F_{40\%}$.
   - 2a) *Stock status: $B/B_{MSY} > 1$*
     - $F_{OFL} = F_{MSY} \times (F_{30\%}/F_{40\%})$
     - $F_{ABC} \leq F_{MSY}$
   - 2b) *Stock status:* $a < B/B_{MSY} \leq 1$
     - $F_{OFL} = F_{MSY} \times (F_{30\%}/F_{40\%}) \times (B/B_{MSY} - a)/(1 - a)$
     - $F_{ABC} \leq F_{MSY} \times (B/B_{MSY} - a)/(1 - a)$
   - 2c) *Stock status: $B/B_{MSY} \leq a$*
     - $F_{OFL} = 0$
     - $F_{ABC} = 0$
3) *Information available: Reliable point estimates of B, $B_{40\%}$, $F_{30\%}$,* and $F_{40\%}$.
   - 3a) *Stock status: $B/B_{40\%} > 1$*
     - $F_{OFL} = F_{30\%}$
     - $F_{ABC} \leq F_{40\%}$
   - 3b) *Stock status: $a < B/B_{40\%} \leq 1$*
     - $F_{OFL} = F_{30\%} \times (B/B_{40\%} - a)/(1 - a)$
     - $F_{ABC} \leq F_{40\%} \times (B/B_{40\%} - a)/(1 - a)$
   - 3c) *Stock status: $B/B_{40\%} \leq a$*
     - $F_{OFL} = 0$
     - $F_{ABC} = 0$
4) *Information available: Reliable point estimates of B, $F_{30\%}$,* and $F_{40\%}$.
   - $F_{OFL} = F_{30\%}$
   - $F_{ABC} \leq F_{40\%}$
5) *Information available: Reliable point estimates of B and natural mortality rate M.*
   - $F_{OFL} = M$

$F_{ABC} \leq 0.75 \times M$

6) *Information available: Reliable catch history from 1978 through 1995.*

OFL = the average catch from 1978 through 1995

ABC ≤ 75% of the average catch from 1978 through 1995

2.0 NEPA REQUIREMENTS: ENVIRONMENTAL IMPACTS OF THE ALTERNATIVES

An environmental assessment (EA) is required by the National Environmental Policy Act of 1969 (NEPA) to determine whether the action considered will result in significant impact on the human environment. The environmental analysis in the EA provides the basis for this determination and must analyze the intensity or severity of the impact of an action and the significance of an action with respect to society as a whole, the affected region and interests, and the locality. If the action is determined not to be significant based on an analysis of relevant considerations, the EA and resulting finding of no significant impact (FONSI) would be the final environmental documents required by NEPA. An environmental impact study (EIS) must be prepared for major Federal actions significantly affecting the human environment.

An EA must include a brief discussion of the need for the proposal, the alternatives considered, the environmental impacts of the proposed action and the alternatives, and a list of document preparers. The purpose and alternatives were discussed in Sections 1.1 and 1.2, and the list of preparers is in Section 8. This section contains the discussion of the environmental impacts of the alternatives including impacts on threatened and endangered species and marine mammals.

2.1 Environmental Impacts of the Alternatives

The environmental impacts generally associated with fishery management actions are effects resulting from 1) harvest of fish stocks which may result in changes in food availability to predators, changes in the abundance and population structure of target fish stocks, and changes in community structure; 2) changes in the physical and biological structure of the benthic environment as a result of fishing practices, e.g., effects of gear use and fish processing discards; and 3) entanglement/entrapment of non-target organisms in active or inactive fishing gear. A summary of the effects of the 1996 groundfish total allowable catch amounts on the biological environment and associated impacts on marine mammals, seabirds, and other threatened or endangered species are discussed in the final environmental assessment for the 1996 groundfish total allowable catch specifications.

2.1.1 Alternative 1: No Action. Because this alternative simply prerserves the status quo, no significant environmental impacts are anticipated.

2.1.2 Alternative 2: Proposed Revision.

In terms of ABC, the definition contained in Alternative 2 can be viewed as a restricted version of the status quo. That is, nothing in the proposed redefinition of ABC is disallowed under the current definition. Therefore, in the sense that the Council currently has the ability to follow the restrictions on ABC contained in Alternative 2, the environmental impacts of adopting this alternative may be minimal (i.e., the Council might choose, even under Alternative 1, to impose *voluntarily* the same restrictions on ABC that would be *required* under Alternative 2). However, because the current definition of ABC is essentially open-ended except in cases where an estimate of F_{MSY} is available, there are insufficient built-in safeguards against imprudent harvest rates. By instituting such safeguards, Alternative 2 is expected to generate *positive* environmental impacts relative to the status quo by reducing the risk of setting allowable catches too high. This is done primarily by placing an upper limit on the fishing mortality rate used to calculate ABC. When the amount of uncertainty associated with an estimate of F_{MSY} can be determined, Alternative 2 prescribes a cap on F_{ABC} based on the risk-averse optimization presented in Appendix B. When information is more limited, Alternative 2 caps F_{ABC} at the $F_{40\%}$ level, following the recommendation of Clark (1993) and Mace (1994). When information is extremely limited, Alternative 2 caps F_{ABC} at a level somewhat (viz., 25%) below the natural mortality rate, following the recommendation of Deriso (1982) and Thompson (1993).

In terms of overfishing, the definition contained in Alternative 2 is also expected to generate positive environmental impacts relative to the status quo by imposing additional safeguards under those conditions where they are most needed. Although Alternative 2 relaxes the current overfishing definition slightly for healthy stocks, it is more restrictive than the current definition for stocks that have fallen significantly below their target levels of abundance (Figure 1).

In terms of the need for action outlined in Section 1.1, Alternative 2 addresses the specific ODRP and SSC concerns as follows:

1) *F_{OFL} should vary directly with biomass when the latter is between B_{thr} and B_{pre}.* The proposed definition satisfies this concern in tiers (1-3) by establishing a linear scale for F_{OFL} when biomass is between B_{thr} and B_{pre} and by setting B_{pre} equal to either B_{MSY} (tiers [1-2]) or $B_{40\%}$ (tier [3]). This concern is not satisfied in tiers (4-6) because it is impossible to identify an appropriate precautionary biomass level when basic biological information is largely or entirely lacking.

2) *For healthy stocks, F_{OFL} should exceed F_{MSY}.* The proposed definition satisfies this concern in tiers (1-2) by setting a buffer based either on the ratio between the arithmetic and harmonic means of the pdf (tier [1]) or on the ratio between $F_{30\%}$ and $F_{40\%}$ (tier [2]). This concern is not satisfied in tiers (3-6) because it is impossible to ensure that any particular F is greater than F_{MSY} if F_{MSY} cannot be estimated.

3) *The authority for determining reliability of information should be specified.* The proposed definition satisfies this concern by vesting within the SSC final authority for determining reliability of information.

4) *Ambiguity should be eliminated in any text relating SPR to exploitable biomass.* The proposed definition satisfies this concern by eliminating the previous definition's text relating SPR to exploitable biomass.

5) *F_{ABC} should be reduced when $B<B_{MSY}$.* The proposed definition satisfies this concern in tiers (1-2) by establishing a linear scale for F_{ABC} when biomass is between B_{thr} and B_{MSY}. This concern is not satisfied in tiers (3-6) because it is impossible to measure biomass relative to B_{MSY} when B_{MSY} cannot be estimated.

6) *More caution should be required when less information is available.* The proposed definition satisfies this concern in tier (1) by setting the F_{ABC}/F_{OFL} ratio equal to the ratio between the harmonic and arithmetic means, a quantity which tends to decrease as the coefficient of variation (a measure of uncertainty or lack of information) increases. This concern is not satisfied in tiers (2-6) taken individually (e.g., comparing F_{OFL}s for two different stocks under tier [4]), because these tiers are designed to group stocks together on the basis of similarity of available information, making it difficult to distinguish between levels of uncertainty for stocks managed within any one of these tiers. Neither is this concern satisfied in tiers (2-6) taken sequentially (e.g., comparing F_{OFL}s calculated for the same stock under tiers [5] and [6]), because it is impossible to ensure that an F computed under any given tier is lower than the F that would have been computed under a more information-intensive tier if the requisite information is lacking (which it necessarily is).

7) *F_{OFL} should exceed F_{ABC}.* The proposed definition satisfies this concern by providing an explicit buffer between F_{OFL} and F_{ABC}.

8) *OFL should remain constant over time when catch history is the only information available.* The proposed definition satisfies this concern in tier (6) by terminating the catch time series in 1995 (i.e., the endpoint of the catch time series would be fixed at 1995, not set at the current year as in the status quo). This concern is not relevant to tiers (1-5).

2.2 Impacts on Endangered, Threatened, or Candidate Species

Listed and candidate species under the Endangered Species Act (ESA) that may be present in the GOA and BSAI include:

Endangered

Northern right whale	*Balaena glacialis*
Sei whale	*Balaenoptera borealis*
Blue whale	*Balaenoptera musculus*
Fin whale	*Baleanoptera physalus*
Humpback whale	*Megaptera novaeangliae*
Sperm whale	*Physeter macrocephalus*
Snake River sockeye salmon	*Oncorhynchus nerka*
Snake R. fall chinook salmon	*Oncorhynchus tshawytscha*
Short-tailed albatross	*Diomedea albatrus*

Threatened

Steller sea lion	*Eumetopias jubatus*

Snake River spring and summer chinook salmon	*Oncorhynchus tshawytscha*
Spectacled eider	*Somateria fischeri*

The impact of BSAI and GOA groundfish fisheries on Steller sea lions was addressed in a formal consultation on April 19, 1991 and in various informal consulations since then. NMFS has determined that the groundfish fisheries are not likely to affect Steller sea lions in a way or to an extent not already considered in these consultations.

An informal consultation conducted on effects of the GOA and BSAI groundfish fisheries concluded that the continued operation of these fisheries would not adversely affect listed species of salmon as long as current observer coverage levels continued and salmon bycatch was monitored on a weekly basis. Consultation must be reinitiated if chinook salmon bycatch exceeds 40,000 fish in either the BSAI or GOA or sockeye salmon bycatch exceeds 200 fish in the BSAI or 100 fish in the GOA.

Endangered, threatened, and candidate species of seabirds that may be found within the regions of the GOA and BSAI where the groundfish fisheries operate, and potential impacts of the groundfish fisheries on these species are discussed in the EA prepared for the TAC specifications. The U.S. Fish and Wildlife Service (USFWS), in the informal consultation on the 1996 specifications, concluded that groundfish operations are likely to result in an unquantified level of mortality to short-tailed albatrosses, a listed species, but will not jeopardize the continued existence of the population. The take level was not expected to exceed that authorized in the USFWS consultation conducted on the implementation of the Marine Mammal Exemption Program (1988).

Neither Alternative 1 (No Action) nor Alternative 2 (Proposed Revision) is anticipated to impact threatened, endangered, or candidate species in a way or to an extent not already considered in the above-mentioned consultations.

2.3 Impacts on Marine Mammals

Marine mammals not listed under the Endangered Species Act that may be present in the GOA and BSAI include cetaceans [minke whale (*Balaenoptera acutorostrata*), killer whale (*Orcinus orca*), Dall's porpoise (*Phocoenoides dalli*), harbor porpoise (*Phocoena phocoena*), Pacific white-sided dolphin (*Lagenorhynchus obliquidens*), and the beaked whales (e.g., *Berardius bairdii* and *Mesoplodon* spp.)] as well as pinnipeds [northern fur seals (*Callorhinus ursinus*) and Pacific harbor seals (*Phoca vitulina*)] and the sea otter (*Enhydra lutris*).

Relative to the status quo, neither Alternative 1 (No Action) nor Alternative 2 (Proposed Revision) is anticipated to have an adverse impact on any marine mammal species.

2.4 Coastal Zone Management Act

Implementation of either alternative would be conducted in a manner consistent, to the maximum extent practicable, with the Alaska Coastal Management Program within the meaning of Section 30(c)(1) of the Coastal Zone Management Act of 1972 and its implementing regulations.

2.5 Conclusions or Finding of No Significant Impact

Neither of the alternatives is likely to significantly affect the quality of the human environment, and the preparation of an environmental impact statement for the proposed action is not required by Section 102(2)(C) of the National Environmental Policy Act or its implementing regulations.

3.0 REGULATORY IMPACT REVIEW

3.1 Economic and Socioeconomic Impacts of the Alternatives

This section provides information about the economic and socioeconomic impacts of the alternatives including identification of the individuals or groups that may be affected by the action, the nature of these impacts, quantification of the economic impacts if possible, and discussion of the trade offs between qualitative and quantitative benefits and costs.

The requirements for all regulatory actions specified in E.O. 12866 are summarized in the following statement from the order:

> In deciding whether and how to regulate, agencies should assess all costs and benefits of available regulatory alternatives, including the alternative of not regulating. Costs and benefits shall be understood to include both quantifiable measures (to the fullest extent that these can be usefully estimated) and qualitative measures of costs and benefits that are difficult to quantify, but nevertheless essential to consider. Further, in choosing among alternative regulatory approaches, agencies should select those approaches that maximize net benefits (including potential economic, environment, public health and safety, and other advantages; distributive impacts; and equity), unless a statute requires another regulatory approach.

This section also addresses the requirements of both E.O. 12866 and the Regulatory Flexibility Act to provide adequate information to determine whether an action is "significant" under E.O. 12866 or will result in "significant" impacts on small entities under the RFA.

E. O. 12866 requires that the Office of Management and Budget review proposed regulatory programs that are considered to be "significant". A "significant regulatory action" is one that is likely to:

> (1) Have an annual effect on the economy of $100 million or more or adversely affect in a material way the economy, a sector of the economy, productivity, competition, jobs, the environment, public health or safety, or State, local, or tribal governments or communities;

(2) Create a serious inconsistency or otherwise interfere with an action taken or planned by another agency;

(3) Materially alter the budgetary impact of entitlements, grants, user fees, or loan programs or the rights and obligations of recipients thereof; or

(4) Raise novel legal or policy issues arising out of legal mandates, the President's priorities, or the principles set forth in this Executive Order.

A regulatory program is "economically significant" if it is likely to result in the effects described above. The RIR is designed to provide information to determine whether the proposed regulation is likely to be "economically significant."

3.1.1 Alternative 1: No Action. Because this alternative simply prerserves the status quo, no significant economic or socioeconomic impacts are anticipated.

3.1.2 Alternative 2: Proposed Revision. As noted in Section 2.4 above, the definition of ABC contained in Alternative 2 can be viewed as a restricted version of the status quo. That is, nothing in the proposed redefinition of ABC is disallowed under the current definition. Therefore, in the sense that the Council currently has the ability to follow the restrictions on ABC contained in Alternative 2, the economic and socioeconomic impacts of adopting this alternative may be minimal (i.e., the Council might choose, even under Alternative 1, to impose *voluntarily* the same restrictions on ABC that would be *required* under Alternative 2).

Nevertheless, had Alternative 2 been in place when the 1996 groundfish specfications were put into place, it appears that some short-term economic impacts would have been felt. From Table 1, for example, it appears that 1996 ABCs for most flatfish stocks would have decreased on the order of 15-20% and that 1996 ABCs for most rockfish stocks other than Pacific ocean perch would have decreased on the order of 25%. However, changes in TAC would in most cases have been less extreme, since TAC was already well below ABC for many species. Table 2 shows the relative amount by which 1996 TACs differed with respect to the ABCs recommended by the Plan Teams. Note that the proportionate reductions (TAC relative to ABC) were already *greater* than the amounts that would have been required under Alternative 2 for BSAI Pacific cod, all BSAI flatfish, BSAI squid, BSAI "other species," GOA "other"slope rockfish, and GOA Atka mackerel. Although the magnitudes of the necessary reductions in 1996 ABCs for the various GOA flatfish categories are unknown, it seems fairly certain that they would have been smaller than the reductions which were actually made at the TAC stage. This leaves only AI pollock (with a reduction of perhaps 15-20% relative to 1996 TAC); BSAI rockfish other than Pacfic ocean perch (with reductions of about 15% relative to 1996 TACs); GOA sablefish (with a reduction of about 5-10% relative to 1996 TAC); and GOA shortraker/rougheye, northern, pelagic shelf, and demersal shelf rockfish (with reductions of about 25% relative to 1996 TACs) as requiring modification in the final TAC had

Alternative 2 been in place during the 1996 specification process.

While some short-term negative economic impacts may result from adoption of Alternative 2, it should be remembered the measures incorporated into this alternative were developed with long-term optimization explicitly in mind, meaning that increases in long-term benefits are expected to outweigh any short-term losses.

3.2 Administrative, Enforcement, and Information Costs

No additional administrative, enforcement, or information costs are expected under either alternative. Moreover, because Alternative 2 would require the maintenance of a reasonable buffer between ABC and OFL, its adoption is expected to make administration of the fishery management system easier and to reduce the average amount of unharvested TAC, the rationale being that it is easier to achieve a target harvest amount if the goal is to come as close to the target as possible than if the goal is to come as close as possible without going over.

4.0 INITIAL REGULATORY FLEXIBILITY ANALYSIS

The objective of the Regulatory Flexibility Act is to require consideration of the capacity of those affected by regulations to bear the direct and indirect costs of regulation. If an action will have a significant impact on a substantial number of small entities an Initial Regulatory Flexibility Analysis (IRFA) must be prepared to identify the need for the action, alternatives, potential costs and benefits of the action, the distribution of these impacts, and a determination of net benefits.

NMFS has defined all fish-harvesting or hatchery businesses that are independently owned and operated, not dominant in their field of operation, with annual receipts not in excess of $2,000,000 as small businesses. In addition, seafood processors with 500 employees or fewer, wholesale industry members with 100 employees or fewer, not-for-profit enterprises, and government jurisdictions with a population of 50,000 or less are considered small entities. A "substantial number" of small entities would generally be 20% of the total universe of small entities affected by the regulation. A regulation would have a "significant impact" on these small entities if it reduced annual gross revenues by more than 5 percent, increased total costs of production by more than 5 percent, or resulted in compliance costs for small entities that are at least 10 percent higher than compliance costs as a percent of sales for large entities.

If an action is determined to affect a substantial number of small entities, the analysis must include:

(1) a description and estimate of the number of small entities and total number of entities in a particular affected sector, and total number of small entities affected; and

(2) analysis of economic impact on small entities, including direct and indirect compliance costs, burden of completing paperwork or recordkeeping requirements, effect on the competitive position of small entities, effect on the small entity's cashflow and liquidity,

and ability of small entities to remain in the market.

4.1 Economic Impact on Small Entities

According to Table 2, the only 1996 TACs that would have needed modification had Alternative 2 been in place were AI pollock (a reduction of perhaps 15-20%); BSAI rockfish other than Pacfic ocean perch (reductions of about 15%); GOA sablefish (a reduction of about 5-10%); and GOA shortraker/rougheye, northern, pelagic shelf, and demersal shelf rockfish (reductions of about 25%). Given these results, it is likely that less than 20% of the groundfish fleet would suffer losses amounting to more than 5% of gross revenues as a result of implementing Alternative 2. For example, the largest reductions (in percentage terms) would have come in the GOA rockfish fisheries (excluding Pacific ocean perch, other slope rockfish, and thornyheads). However, in 1994 only about 200 vessels targeted GOA rockfish of *any* type, compared with a total of about 1,900 vessels which participated in the overall GOA groundfish fishery in 1994 (Angie Greig, NMFS/AFSC, pers. commun.). In order for a vessel to experience a 5% drop in revenue as the result of a 25% drop in its rockfish catches, rockfish catches would need to have accounted for at least 20% of the vessel's revenue prior to the drop (assuming that the vessel would not make up the difference in some other fishery). Or, in the case of GOA sablefish, catches of this species would need to account for at least 50% of a vessel's revenue in order for a 10% drop in sablefish catches to result in a 5% drop in overall revenue. On the basis of considerations such as these, then, it seems reasonable to conclude that implentation of Alternative 2 would not be expected to result in significant impacts on a substantial number of small entities, even in the short run. In the long run, since Alternative 2 is designed with optimality considerations explicitly in mind, it is assumed that implentation would tend to be beneficial to resource users *in general*, small entities included.

5.0 SUMMARY AND CONCLUSIONS

Reviews by the ODRP and SSC have indicated that the definitions of ABC and overfishing contained in the BSAI and GOA Groundfish FMPs could and should be improved. Suggestions for improvement include the following: A) as parameter estimates become more imprecise, fishing mortality rates should become more conservative; B) for a stock below its target abundance level, fishing mortality rates should vary directly with biomass and ultimately fall to zero should the stock become critically depleted; and C) a buffer should be maintained between acceptable biological catch and the overfishing level.

This plan amendment proposal contains two alternatives: Alternative 1 (No Action) maintains the current definitions, and Alternative 2 (Proposed Revision) modifies the current definitions in response to the suggestions made by the ODRP and SSC. The differences between the two alternatives can perhaps best be illustrated by examining the case in which a point estimate of F_{MSY} is available together with a reliable description of the amount of uncertainty surrounding that estimate. Under the current definitions, F_{ABC} and F_{OFL} are both set equal to the point estimate of F_{MSY}, regardless of the level of uncertainty associated with that estimate. Under the proposed definitions, the ratio between F_{ABC} and F_{OFL} varies inversely with the level of uncertainty (i.e., the greater the uncertainty in the estimate of F_{MSY}, the lower F_{ABC} is in relation to F_{OFL}).

Even in cases where reliable descriptions of the level of uncertainty associated with a point estimate are not available, the proposed revision maintains an appropriate buffer between ABC and OFL. Also, whenever a target abundance level can be reasonably identified, the proposed revision reduces fishing mortality rates as stock size falls below that target level. The current definitions do neither of these.

Because the proposed revision institutes new safeguards against overly aggressive harvest rates, particularly under conditions of high uncertainty or low stock size, the revision is expected to result in positive environmental impacts. Because the proposed revision is based explicitly on harvest policies designed to optimize long-term fishery performance, the revision is also expected to result in positive long-term economic impacts. However, it is possible that negative economic impacts could be generated in the short term for a few fisheries, particularly rockfish fisheries targeting on species other than Pacific ocean perch, where TAC might be reduced by 15-25%.

Neither of the alternatives is expected to result in a "significant regulatory action" as defined in E.O. 12866.

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7.0 AGENCIES AND INDIVIDUALS CONSULTED

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Table 1: Summary of Impacts of the Proposed Definition on Current ABC and OFL fishing mortality rates.

Gulf of Alaska

	F(ABC)			F(OFL)		
Species	1996 (Plan Team)	Proposed	% Change	1996 (Plan Team)	Proposed	% Change
Walleye pollock	FABC=0.30	same	0	F30%=0.50	same	0
Pacific cod	F40%=0.40	same	0	F30%=0.57	same	0
Deepwater flatfish	F35%=0.125	F40%=?	-?	F30%=0.146	same	0
Rex sole	F35%=0.125	F40%=?	-?	F30%=0.146	same	0
Flathead sole	F35%=0.145	F40%=?	-?	F30%=0.159	same	0
Shallow water flatfish (yellowfin sole)	F35%=0.149	F40%=?	-?	F30%=0.175	same	0
Arrowtooth flounder	F35%=0.125	F40%=?	-?	F30%=0.146	same	0
Sablefish	F35%(adj.)=0.112	F40%=0.103	-8	F30%=0.153	same	0
Pacific ocean perch	F44%(adj.)=0.052	same	0	FMSY(adj.)=0.065	FMSY(adj.)=0.082	+26
Shortraker	F=M=0.03	M x 0.75 = 0.023	-25	F=M=0.03	same	0
Rougheye	F=M=0.025	M x 0.75 = 0.019	-25	F30%=0.046	same	0
Other slope rockfish (sharpchin)	F=M=0.05	M x 0.75 = 0.038	-25	F30%=0.08	same	0
Northern rockfish	F=M=0.06	M x 0.75 = 0.045	-25	F30%=0.113	same	0
Dusky rockfish	F=M=0.09	M x 0.75 = 0.068	-25	F30%=0.151	same	0
Other pelagic shelf rockfish	Fave x 0.81 = ?	Fave x 0.75 = ?	-7	Fave	same	0
Demersal shelf rockfish	F=M=0.02	M x 0.75 = 0.015	-25	F30%=0.04	same	0
Thornyhead rockfish	F40%=0.06	same	0	F30%=0.09	same	0
Atka mackerel	F=M=0.30	M x 0.75 = 0.225	-25	F30%=0.45	same	0

Bering Sea and Aleutian Islands

	F(ABC)			F(OFL)		
Species	1996 (Plan Team)	Proposed	% Change	1996 (Plan Team)	Proposed	% Change
EBS Walleye pollock	F40%=0.30	same	0	FMSY=0.38	FMSY(adj.)=0.46	+21
AI Walleye pollock	F35%=0.42	F40%=?	-?	F30%=0.45	same	0
Bogoslof Walleye pollock	F35%=0.33	F40%=?	-?	F30%=0.40	same	0
Pacific cod	F35%=0.36	F40%=0.30	-17	F30%=0.43	same	0
Yellowfin sole	F35%=0.13	F40%=0.11	-15	F30%=0.16	same	0
Greenland turbot	F40%=0.24	same	0	F30%=0.37	same	0
Arrowtooth flounder	F35%=0.27	F40%=0.22	-19	F30%=0.34	same	0
Rock sole	F35%=0.18	F40%=0.15	-17	F30%=0.22	same	0
Flathead sole	F35%=0.19	F40%=0.16	-16	F30%=0.23	same	0
Other flatfish (Alaska plaice)	F35%=0.17	F40%=0.14	-18	F30%=0.20	same	0
Sablefish	F40%=0.10	same	0	F30%=0.15	same	0
EBS True POP	F44%=0.06	same	0	F30%=0.096	same	0
EBS Other red rockfish	F=M=0.05	M x 0.75 = 0.038	-25	F=M=0.05	same	0
AI True POP	F44%=0.06	same	0	F30%=0.096	same	0
AI Sharpchin/northern	F=M=0.06	M x 0.75 = 0.045	-25	F=M=0.06	same	0
AI Shortraker/rougheye	F=M=0.03	M x 0.75 = 0.023	-25	F=M=0.03	same	0
EBS Other rockfish	F=M=0.07	M x 0.75 = 0.053	-25	F=M=0.07	same	0
AI Other rockfish	F=M=0.07	M x 0.75 = 0.053	-25	F=M=0.07	same	0
Atka mackerel	F40%=0.49	same	0	F30%=0.75	same	0
Squid	Fave=?	Fave x 0.75 = ?	-25	Fave=?	same	0
Other species	Fave=?	Fave x 0.75 = ?	-25	Fave=?	same	0

Note: Listings under "proposed" above do not include any adjustments that might occur as a result of biomass falling below B40%.
Also, a listing of "same" under "proposed" means that the proposed definition would not have required any change in the Plan Team's 1996 ABC.

Table 2: Relationship of 1996 TAC to 1996 Plan Team ABC.

Gulf of Alaska

Species	ABC	TAC	% Change
Walleye pollock	54810	54810	0
Pacific cod	65000	65000	0
Deepwater flatfish	14590	11080	-24
Rex sole	11210	9690	-14
Shallow water flatfish	52270	9740	-81
Flathead sole	28790	18630	-35
Arrowtooth flounder	198130	35000	-82
Sablefish	17090	17080	0
Pacific ocean perch	8060	6959	-14
Shortraker/rougheye	1910	1910	0
Other slope rockfish	7110	2020	-72
Northern rockfish	5270	5270	0
Pelagic shelf rockfish	5430	5190	-4
Demersal shelf rockfish	950	950	0
Thornyhead	1560	1248	-20
Atka mackerel	6480	3240	-50

Bering Sea and Aleutian Islands

Species	ABC	TAC	% Change
EBS Walleye pollock	1290000	1190000	-8
AI Walleye pollock	26200	35600	36
Bogoslof Walleye pollock	286000	1000	-100
Pacific cod	357000	270000	-24
Yellowfin sole	278000	200000	-28
Greenland turbot	17000	7000	-59
Arrowtooth flounder	129000	9000	-93
Rock sole	361000	70000	-81
Flathead sole	116000	30000	-74
Other flatfish	102000	35000	-66
EBS Sablefish	1100	1100	0
AI Sablefish	1200	1200	0
EBS True POP	1800	1800	0
EBS Other red rockfish	1400	1260	-10
AI True POP	12100	12100	0
AI Sharpchin/northern	5810	5229	-10
AI Shortraker/rougheye	1250	1125	-10
EBS Other rockfish	497	447	-10
AI Other rockfish	952	857	-10
Atka mackerel	116000	106157	-8
Squid	3000	1000	-67
Other species	27600	20125	-27

Note: Council ABC for AI walleye pollock was set at the value recommended by the SSC, so that TAC=ABC(SSC).

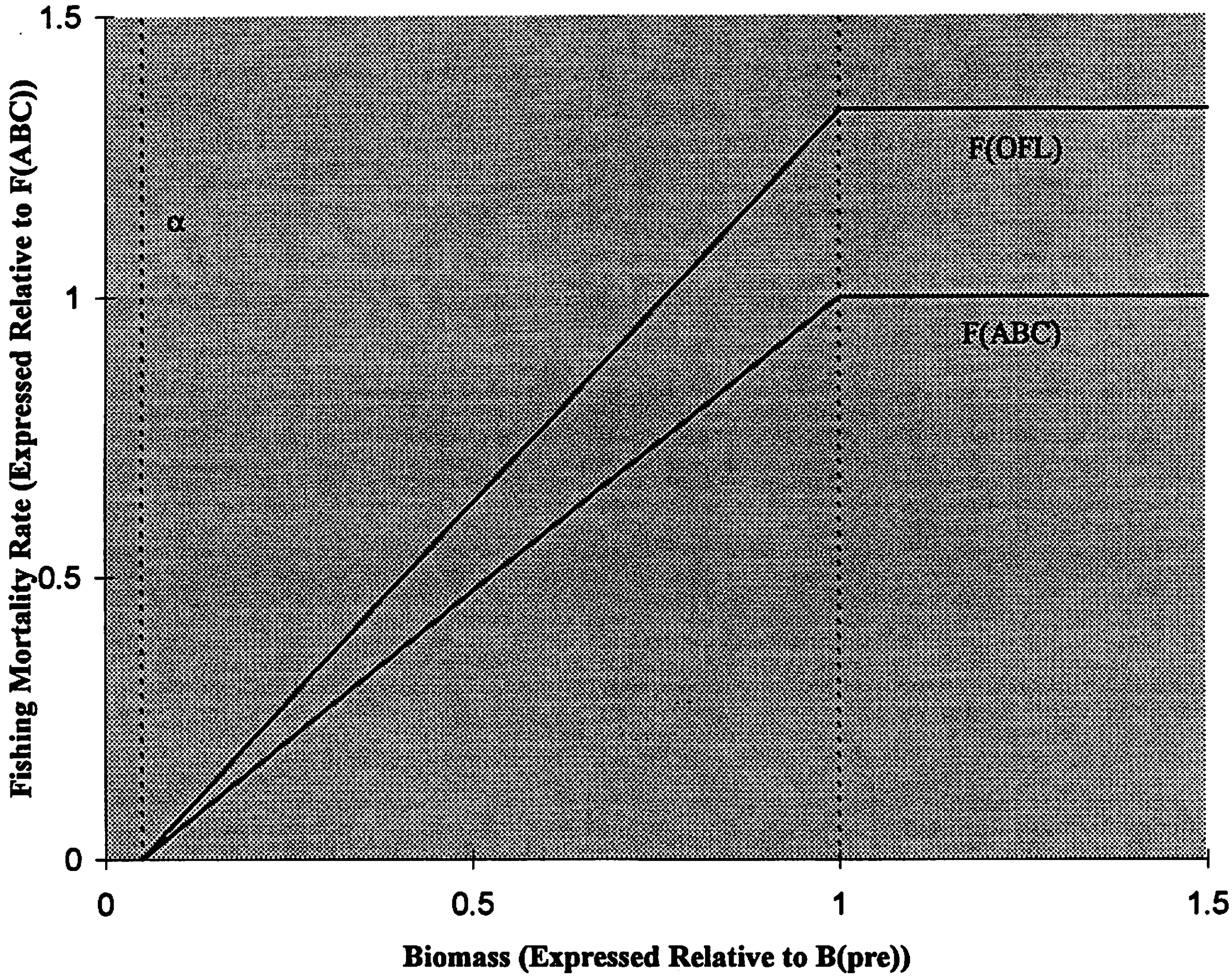

Figure 1. Hypothetical example illustrating relationship of F(OFL) to F(ABC) as a function of biomass.

Appendix A: Nontechnical Definitions of Statistical Terms

Probability density function (pdf): A description of the probability associated with different values of a variable. For example, in a coin flip the probability of tossing "heads" is 50% and the probability of tossing "tails" is 50%. As another example, in tossing a six-sided die, the probability of tossing a "1" is 16.667% and the probability of tossing something other than a "1" is 83.333%. The probabilities in a pdf must always sum to 100%.

Arithmetic mean: For a random variable *X*, the arithmetic mean is the sum of the possible values of *X* weighted by the respective probabilities of those values. For example, consider a game of chance based on a coin flip, where the random variable *X* denotes the prize associated with the game. The player gets $72 if he or she tosses "heads" and $24 if he or she tosses "tails." The arithmetic mean prize for this game is

$$(50\% \times \$72) + (50\% \times \$24) = \$48.$$

As another example, consider a game of chance based on the toss of a six-sided die, where again the random variable *X* denotes the prize associated with the game. The player gets $72 if he or she tosses a "1" and $24 if he or she tosses anything else. The arithmetic mean prize associated with this game is

$$(16.667\% \times \$72) + (83.333\% \times \$24) = \$32.$$

Harmonic mean: Unfortunately, when written out in words, the definition of harmonic mean is a little complicated, but here goes (hopefully, the examples which follow will make things clearer): For a random variable *X*, the harmonic mean is the reciprocal of the sum of the reciprocals of the possible values of *X* weighted by the respective probabilities of those values. For example, consider the game of chance based on a coin flip described under "arithmetic mean" above. The harmonic mean prize associated with this game is

$$\frac{1}{\dfrac{50\%}{\$72} + \dfrac{50\%}{\$24}} = \$36.$$

As another example, consider the game of chance based on the toss of a six-sided die described under "arithmetic mean" above. The harmonic mean prize associated with this game is

$$\frac{1}{\frac{16.667\%}{\$72} + \frac{83.333\%}{\$24}} = \$27.$$

Note that the harmonic mean is less than the arithmetic mean in both of these examples (\$36 versus \$48 for the coin flip and \$27 versus \$32 for the die toss). For all practical purposes, this relationship always holds (i.e., the harmonic mean is always less than the arithmetic mean). Thus, if the random variable *X* represents a fishing mortality rate, the harmonic mean is a more conservative (i.e., lower) rate than the arithmetic mean.

Appendix B:
Risk-Averse Optimal Harvesting in a Biomass Dynamics Model

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Abstract

Considerable interest has been expressed in the fishery science literature toward finding an objectively risk averse long-term management strategy that takes account of both measurement error and process error, factors which affect estimates of present stock size and projections of future stock sizes under alternative harvest strategies. The present paper will take for its underlying model of stock dynamics a stochastic differential equation deriving from the deterministic Gompertz growth model. This model exhibits, among others, the following two convenient properties: 1) the harvest rate that maximizes the expected value of the logarithm of stationary yield (a formally risk averse harvest objective) is simply the harmonic mean of the distribution of the estimate of the Gompertz growth parameter, and 2) process error is formally lognormal. This second property, combined with an assumption of lognormal measurement error, renders the (log transformed) model amenable to estimation via the Kalman filter, which can be interpreted as a Bayesian method of updating stock size estimates. The Kalman filter defines a likelihood function which, given prior distributions on certain model parameters, can then be used to obtain Bayesian estimates of those parameters from their respective posterior distributions.

Introduction

Considerable interest has been expressed in the fishery science literature toward finding an objectively risk averse long-term management strategy that takes account of both measurement error and process error (e.g., various articles in the volumes edited by Smith et al. (1993) and Kruse et al. (1994) and the review article by Rosenberg and Restrepo (1994)), factors which affect estimates of present stock size and projections of future stock sizes under alternative harvest strategies. The present paper will take for its underlying model of stock dynamics a stochastic differential equation owing to Capocelli and Ricciardi (1974) which in turn derives from the deterministic Gompertz (1825) growth model. Parameter estimation and derivation of the risk-averse optimal fishing mortality rate will be based on a Bayesian methodology (e.g., Berger 1985, Lee 1989), applying the principles of decision theory to posterior distributions of model parameters. The likelihood function will be generated by a Kalman filter approach (e.g., Harvey 1990, Pella 1994, Schnute 1994), which itself can be interpreted as a Bayesian methodology (e.g., Meinhold and Singpurwalla 1983).

As an example, the model will be applied to the eastern Bering Sea stock of flathead sole (*Hippoglossoides elassodon*), a lightly exploited stock which has been assessed by a standardized trawl survey annually since 1982 (Walters and Wilderbuer 1995).

The outline of the paper is as follows:

Introduction
Statistical Terminology and Notation
Model Development
- *Deterministic Dynamics*
- *Stochastic Dynamics (Process Error)*
- *Measurement Error*
- *The Kalman Filter*
- *Likelihood Function*

A Theory of Relative Risk Aversion
Parameter Estimation
- *Overview*
- *Optimal Fishing Mortality Rate*
  - *Case I: Parameter Values Certain*
  - *Case II: Parameter Values Uncertain*
- *Growth Rate and Process Error Scale*
  - *Case I: Parameter Values Certain*
  - *Case II: Parameter Values Uncertain*
- *Catchability and Range*
  - *Case I: Parameter Values Certain*
  - *Case II: Parameter Values Uncertain*

Conclusions

Statistical Terminology and Notation

Some notational conventions will be helpful to note early on: 1) Single capital Roman letters will refer to logarithms of their lower-case counterparts, except when used as an acronym (e.g., *Y* will refer to the logarithm of yield *y* except when it appears in an acronym such as *MSY*, the abbreviation for "maximum sustainable yield"). 2) The symbols μ and σ will refer to the mean and standard deviation of a normal distribution. 3) A "prime" symbol will designate a parameter of a prior distribution, while the absence of a "prime" symbol will designate a parameter of a posterior distribution (e.g., μ'_X would represent the prior mean of *X*, while μ_X would represent the posterior mean of *X*). 4) For coefficients that are functions of time (t), the limit as t goes to infinity will be indicated by the absence of a time argument (e.g., μ_X will denote the limit of $\mu_X(t)$ as t approaches infinity). 5) The symbol $g_X(X)$ will be used to designate the probability density function (pdf) of *X*.

Because the normal and lognormal distributions play such an important role in the remainder of the paper, a brief review of their functional forms is in order. If the variable *X* is normally distributed, that is, if it has a probability density function (pdf) of the form

$$g_X(X) = \sqrt{\frac{1}{2\pi}}\left(\frac{1}{\sigma_X}\right)\exp\left(-\left(\frac{1}{2}\right)\left(\frac{X-\mu_X}{\sigma_X}\right)^2\right), \tag{1}$$

where μ_X and σ_X represent the mean and standard deviation of *X*, respectively, then the variable $x=e^X$ is distributed lognormally with pdf

$$g_x(x) = \sqrt{\frac{1}{2\pi}}\left(\frac{1}{\sigma_X x}\right)\exp\left(-\left(\frac{1}{2}\right)\left(\frac{\ln(x)-\mu_X}{\sigma_X}\right)^2\right). \tag{2}$$

If $g_x(x)$ represents a lognormal pdf of the variable x, the jth moment about zero is given by

$$\int_0^\infty x^j g_x(x)dx = \exp\left(j\mu_X + \frac{j^2\sigma_X^2}{2}\right). \tag{3}$$

Note that j need not be restricted to integer values.

The jth root of the jth moment of a pdf is known as the "mean of order j" (Mitrinovic et al. 1993), and will be denoted here by

$$m_x(j) = \left(\int_0^{\infty} x^j g_x(x)dx\right)^{1/j}. \quad (4)$$

If the coefficients defining $g_x(x)$ are time variant, the jth-order mean may be written $m_x(t,j)$.

A well-known characteristic of the function $m_x(j)$ is that it is monotone increasing with respect to j, regardless of the form of $g_x(x)$, provided that $g_x(x)=0$ for all $x \leq 0$ (Mitrinovic et al. 1993). Important special cases correspond to $j=1$ (the arithmetic mean), the limit as j approaches 0 (the geometric mean), and $j=-1$ (the harmonic mean).

For the lognormal distribution, the jth-order mean is given by

$$m_x(j) = \exp\left(\mu_X + \frac{j\sigma_X^2}{2}\right). \quad (5)$$

Model Development

Deterministic Dynamics

Define some basic model parameters as follows: a growth rate a, a fishing mortality rate f, a carrying capacity k, and an initial stock size x_0. Their respective logarithms will be denoted A, F, K, and X_0. The time derivative of stock size x in the Gompertz growth model can then be written

$$\frac{dx}{dt} = ax(K - \ln(x)) - fx. \quad (6)$$

For a given value of f, equilibrium stock size b is given by

$$b = ke^{-f/a}, \quad (7)$$

which simplifies the time derivative to

$$\frac{dx}{dt} = ax(B - \ln(x)), \quad (8)$$

where $B=\ln(b)$.

The time derivative of log stock size X is simply the linear relationship

$$\frac{dX}{dt} = a(B - X). \tag{9}$$

The parameter a thus represents: 1) the per-capita growth rate of x at $x=b/e$, or 2) the per-capita growth rate of X at $X=B/2$.

Given an initial stock size x_0, the population trajectory is given by

$$x(t) = b\left(\frac{x_0}{b}\right)^{e^{-at}}. \tag{10}$$

For initial log stock size X_0, the trajectory of log stock size is given by

$$X(t) = e^{-at}X_0 + (1 - e^{-at})B. \tag{11}$$

Yield y is given by the simple relationship $y=fx$. As shown by Fox (1970, using a slightly different parametrization), *MSY* is obtained by fishing at a rate equal to a, which results in an equilibrium stock size of k/e or an equilibrium log stock size of K-1.

Stochastic Dynamics (Process Error)

To introduce a stochastic component into the deterministic model presented above, it is convenient to begin with the well-known Ornstein-Uhlenbeck process of mathematical physics (Uhlenbeck and Ornstein 1930, Ricciardi 1977), which can be written, for arbitrary parameters a and B and arbitrary variable X as

$$\frac{dX}{dt} = a(B - X) + s\Lambda(t), \tag{12}$$

where Λ is a standard white noise process and s is a scale parameter describing the intensity of the noise. Note that Equation (12) is identical to Equation (9) except for the specific interpretation of a, B, and X in Equation (9) and the fact that Equation (12) includes the term $s\Lambda(t)$ on the RHS.

The transition pdf of the Ornstein-Uhlenbeck process is normal with parameters

$$\mu_X(t) = e^{-at}X_0 + (1 - e^{-at})B \tag{13}$$

and

$$\sigma_{PX}(t) = s\sqrt{\frac{1 - e^{-2at}}{2a}}, \tag{14}$$

where X_0 is the (known) value of X at time t=0. The subscript "PX" (rather than just "X") is used in Equation (14) to indicate that this is the variability due to *process* error only.

Using the Stratonovich interpretation of stochastic differential equations (e.g., Ricciardi 1977), Equation (12) can be transformed into a stochastic version of Equation (8) by the chain rule of ordinary calculus, giving

$$\frac{dx}{dt} = ax(B - \ln(x)) + sx\Lambda(t). \tag{15}$$

The transition pdf of x is then lognormal (Capocelli and Ricciardi 1974) with parameters given by Equations (13) and (14).

Suppose instead that the value of X_0 is not known, but is assumed to follow a lognormal distribution with parameters μ_0 and σ_0. In this case, the coefficients of the transition pdf are

$$\mu'_X(t) = e^{-at}\mu_0 + (1 - e^{-at})B \tag{16}$$

and

$$\sigma'_X(t) = \sqrt{\sigma_{PX}(t)^2 + e^{-2at}\sigma_0^2}. \tag{17}$$

In the limit as t goes to infinity, the above equations (either (13-14) or (16-17)) reduce to the coefficients of the stationary distribution, namely

$$\mu'_X = B \tag{18}$$

and

$$\sigma'_X = \frac{s}{\sqrt{2a}}. \tag{19}$$

Assume that the conditional transition distribution of log yield Y is normal with parameters F+X and σ_{PY}. Note that this is not the same as substituting K-f /a for B in Equation (17) and then substituting y for fx in the resulting expression, which would lead to a two-dimensional (and thus considerably less tractable) stochastic differential equation. Instead, the simpler assumption is made that error in the harvest process y=fx affects y but not x.

Given this assumption, the marginal transition distribution of yield y is lognormal (i.e., Y is normal) with coefficients

$$\mu'_Y(t) = F + \mu'_X(t) \tag{20}$$

and

$$\sigma'_Y(t) = \sqrt{\sigma'_X(t)^2 + \sigma^2_{PY}}. \tag{21}$$

Measurement Error

Suppose the following: A stock of size (biomass) x is distributed over a range of area r. The stock's size has been estimated n+1 times by a survey, specifically at times t_i, i=0,1,2,...,n. Each survey consists of a large number of observations, each of which in turn measures, on a per-unit-area basis, the segment of the population contained in some sampling site or quadrat (e.g., the portion of the seabed swept by a single haul in a trawl survey). Survey observations may be biased (either upward or downward) by a "catchability coefficient" q.

For example, a standardized trawl survey has been used to assess groundfish stocks of the eastern Bering Sea annually since 1982 (Walters and Wilderbuer 1995). The survey includes sampling stations distributed throughout an area of approximately 46,338 ha, a figure which is typically used as a proxy for the area inhabited by the flathead sole stock. The survey is typically viewed as unbiased for this stock. Thus, for the flathead sole example, one might set r=46,338 ha, n=13 (through 1995), and q=1.

It will be assumed here that the observations generated by a given survey represent a random draw from some pdf with mean z, where Z=ln(z) is a consistent estimator of log stock size plus a constant. Given an assumption about the form of the pdf, Z can be estimated by the method of maximum likelihood (a straightforward extension of the method suggested by Kappenman 1994). Then, if the survey sample size is large enough, the asymptotic normality of maximum likelihood estimates can be invoked, meaning that Z_i is normal with expected value

$X(t_i)+Q-R$, where $Q=\ln(q)$ adjusts the survey estimate for bias and $R=\ln(r)$ adjusts the survey estimate to reflect the stock's abundance over its entire range rather than just its density per unit area basis. The standard deviation of Z_i, σ_{MX_i}, represents the variability in the survey estimate due to measurement error (the subscript label "*MX*" stands for "measurement error in *X*").

The following estimates were obtained by this method for the flathead sole survey time series (Figure 1):

i:	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Z:	1.568	1.695	1.992	1.759	2.074	2.128	2.555	2.422	2.610	2.454	2.710	2.532	2.712	2.551
σ_{MX}:	0.212	0.077	0.117	0.085	0.146	0.116	0.150	0.108	0.130	0.093	0.148	0.066	0.072	0.122

In similar fashion, true yield y_i is not known, but rather is measured by an estimate w_i. The logarithm of this estimate, W_i, may be viewed as normal with parameters $Y(t_i)$ and σ_{MY_i}. Unfortunately, an empirical estimate of σ_{MY} is not available for the flathead sole fishery, but the following point estimate of W can be identified:

i:	0	1	2	3	4	5	6	7	8	9	10	11	12	13
W:	8.414	8.564	8.403	8.637	8.558	8.284	8.822	8.190	9.916	9.655	9.564	9.523	9.700	9.196

The Kalman Filter

The model described above is ideally suited to estimation via the Kalman filter. In the present context, the Kalman filter consists of iteratively reweighting the coefficients of the transition pdf of X. Let the time difference between each successive pair of surveys be given by $\tau_i=t_i-t_{i-1}$, $i=1,2,\ldots,n$. For the flathead sole example, $\tau_i=1$ for all i. Then, the prior and posterior estimates of σ_X are given recursively for observations $i=1,2,\ldots,n$ by

$$\sigma'_{X_i} = \sqrt{e^{-2\alpha\tau_i}\sigma_{X_{i-1}}^{\;2} + \sigma_{PX_i}^{\;2}} \tag{22}$$

and

$$\sigma_{X_i} = \sqrt{\frac{1}{\dfrac{1}{\sigma'^{\,2}_{X_i}} + \dfrac{1}{\sigma_{MX_i}^{\;2}} + \dfrac{1}{\sigma_{PY_i}^{\;2} + \sigma_{MY_i}^{\;2}}}}, \tag{23}$$

where

$$\sigma_{X_0} = \sqrt{\frac{1}{\frac{1}{\sigma_{MX_0}^2} + \frac{1}{\sigma_{PY_0}^2 + \sigma_{MY_0}^2}}}, \tag{24}$$

that is, where it is assumed that the estimate of the standard deviation of X prior to the first observation (i=0) is infinite.

The prior estimates of the standard deviations of Z and W are given by

$$\sigma'_{Z_i} = \sqrt{\sigma'^2_{X_i} + \sigma_{MX_i}^2} \tag{25}$$

and

$$\sigma'_{W_i} = \sqrt{\sigma'^2_{X_i} + \sigma_{PY_i}^2 + \sigma_{MY_i}^2}, \tag{26}$$

with correlation coefficient

$$\rho_{ZW_i} = \frac{\sigma'^2_{X_i}}{\sigma'_{Z_i}\sigma'_{W_i}}. \tag{27}$$

The prior and posterior estimates of μ_X are given recursively for observations i=1,2,...,n by

$$\mu'_{X_i} = e^{-a\tau_i}\mu_{X_{i-1}} + (1 - e^{-a\tau_i})B \tag{28}$$

and

$$\mu_{X_i} = \sigma_{X_i}^2\left(\frac{\mu'_{X_i}}{\sigma'^2_{X_i}} + \frac{Z_i - Q + R}{\sigma_{MX_i}^2} + \frac{W_i - F}{\sigma_{PY_i}^2 + \sigma_{MY_i}^2}\right), \tag{29}$$

where

$$\mu_{X_0} = \sigma_{X_0}^{\;2}\left(\frac{Z_0 - Q + R}{\sigma_{MX_0}^{\;2}} + \frac{W_0 - F}{\sigma_{PY_0}^{\;2} + \sigma_{MY_0}^{\;2}}\right). \tag{30}$$

The prior means of Z and W are given, respectively, by

$$\mu'_{Z_j} = \mu'_{X_j} + Q - R \tag{31}$$

and

$$\mu'_{W_j} = \mu'_{X_j} + F. \tag{32}$$

Likelihood Function

Given Equations (23-25) and (31-32), the log likelihood is

$$L = -n\ln(2\pi) - \left(\sum_{i=1}^{n} \ln(\sigma'_{Z_i}) + \ln(\sigma'_{W_i}) + \left(\frac{1}{2}\right)\ln(1 - \rho_{ZW_i}^{\;2}) + \frac{\left(\frac{Z_i - \mu'_{Z_i}}{\sigma'_{Z_i}}\right)^2 - 2\rho_{ZW_i}\left(\frac{Z_i - \mu'_{Z_i}}{\sigma'_{Z_i}}\right)\left(\frac{W_i - \mu'_{W_i}}{\sigma'_{W_i}}\right) + \left(\frac{W_i - \mu'_{W_i}}{\sigma'_{W_i}}\right)^2}{2(1 - \rho_{ZW_i}^{\;2})}\right). \tag{33}$$

In Equation (33), the vectors σ'_Z, σ'_W, and ρ are all functions of parameters a and s and vectors σ_{MX}, σ_{PY}, and σ_{MY}. The vectors μ'_Z and μ'_W are all functions of the same, plus parameters f, k, q, and r.

The topic of parameter estimation will be considered in the following sections. For now, suffice it to say that at least some parameters, for example a and s, could potentially be estimated by the method of maximum likelihood, that is, by maximizing Equation (33). However, an interior maximum to the likelihood surface does not always exist in this model. That is, there is always a maximum to the likelihood surface at a=0 with s positive and another at s=0 with a positive, but there is not always a maximum with both a and s positive. In addition to this practical difficulty, the maximum likelihood estimates (MLEs) suffer from a lack of any clear relationship to alternative levels of risk aversion, and they ignore prior knowledge about the relative believability of alternative parameter values. To address these shortcomings, a Bayesian estimation methodology is set forth below.

A Theory of Relative Risk Aversion

For some quantity which can be thought of as a proxy for nominal income, such as yield y in the case of a fishery, Pratt (1964) defined relative risk aversion RRA as

$$RRA = -y\left(\frac{\frac{d^2L}{dy^2}}{\frac{dL}{dy}}\right), \tag{34}$$

where L is the "loss" function which, when multiplied by a negative constant, describes the level of well-being or "utility" associated with a given level of y. A loss function may pertain to an individual, a group, or a society. The scale of L is arbitrary.

A convenient choice for L is the following:

$$L(y) = \frac{1 - y^j}{j}. \tag{35}$$

When j=1 Equation (35) gives a linear loss function, and in the limit as j approaches zero Equation (35) gives a logarithmic loss function. Using the definition of relative risk aversion given in Equation (34), Equation (35) implies a constant (i.e., y-independent) level of relative risk aversion, namely RRA=1-j. Thus, j=1 corresponds to the "risk neutral" approach where RRA=0, and j=0 corresponds to a distinctly risk averse approach where RRA=1.

In Bayesian decision theory, the objective is to minimize risk, where risk is defined as expected loss. For example, let L be given by Equation (35) and write y as a function of target fishing mortality ϕ and some uncertain parameter θ with pdf $g_\theta(\theta)$. Then, the objective is to choose ϕ so as to minimize

$$EL(\phi) = \int_{-\infty}^{\infty} L(y(\phi,\theta))g_\theta(\theta)d\theta = \frac{1 - m_y(j)^j}{j}. \tag{36}$$

Differentiating Equation (36) with respect to ϕ and setting the result equal to zero gives

$$\frac{dEL}{d\phi} = -m_y(j)^{j-1}\left(\frac{dm_y(j)}{d\phi}\right) = 0. \tag{37}$$

That is, the value of ϕ which *minimizes* the expected loss (the Bayes optimum) is simply the value of ϕ which *maximizes* the jth-order mean of y, where j is equal to 1 minus the chosen level of relative risk aversion. For example, if RRA=0 (i.e., j=1) the Bayes optimum is the value of ϕ which maximizes the *arithmetic* mean of y, while if RRA=1 (i.e., j=0) the Bayes optimum is the value of ϕ which maximizes the *geometric* mean of y.

Note that ϕ is a special type of parameter in that its value can be *chosen*, that is, ϕ is a decision variable. Other parameters, however, may best be thought of as "states of nature," not readily subject to manipulation. Parameters such as a, f, k, q, r, and s in the present model would be examples. In general, such parameters cannot be estimated within the framework outlined above. For instance, future yield y could be written as a function of carrying capacity k as well as target fishing mortality ϕ. However, the derivative of $m_y(j)$ with respect to k is a positive constant, so no solution to Equation (37) would exist.

However, it is possible to modify the framework slightly so as to enable states of nature to be estimated in a manner very analogous to decision variables. To begin with, let stock size x be written as a function of some uncertain state of nature θ and let L be redefined as follows:

$$L(\theta,\hat{\theta}) = \left(\frac{x(\theta)^j - x(\hat{\theta})^j}{j}\right)^2, \tag{38}$$

where $\hat{\theta}$ is an estimate of θ. Since RRA was shown to be equal to 1-j when RRA and L were defined by Equations (34) and (35), respectively, assume for the present that RRA can again be equated with 1-j when L is defined by Equation (38), even though the definition given by Equation (34) is not used in the "state of nature" case.

The expected loss is given by

$$EL(\hat{\theta}) = \int_{-\infty}^{\infty} L(\theta,\hat{\theta})g_\theta(\theta)d\theta = \frac{m_x(2j)^{2j} - 2m_x(j)^j x(\hat{\theta})^j + x(\hat{\theta})^{2j}}{j^2}. \tag{39}$$

Differentiating Equation (39) with respect to $\hat{\theta}$ and setting the result equal to zero gives

$$\frac{dEL}{d\hat{\theta}} = -2x(\hat{\theta})^{j-1}\left(\frac{dx}{d\hat{\theta}}\right)\left(\frac{m_x(j)^j - x(\hat{\theta})^j}{j}\right) = 0. \tag{40}$$

That is, the value of $\hat{\theta}$ which minimizes the expected loss is simply the value which sets $x=m_x(j)$. (If the derivative of x with respect to $\hat{\theta}$ can be set equal to zero, this will also be a minimum or maximum.)

In summary, the approaches to choosing a value for a decision variable and for a state of nature are as follow: For a decision variable, choose the value that maximizes $m_y(j)$. For a state of nature, choose the value that sets $x=m_x(j)$. Even though the definition of relative risk aversion given by Equation (34) is not meaningful for the loss function defined by Equation (38), the fact that the solutions to Equations (37) and (40) are so similar suggests that the relationship $RRA=1-j$ derived from the combination of Equations (34) and (35) is also a reasonable measure of relative risk aversion when L is defined by Equation (38), at least for the case where dy/dx is always positive (as it is here).

For the remainder of this paper, results will focus primarily on a risk averse approach corresponding to an *RRA* value of unity.

Parameter Estimation

Overview

The parameters in this model are the vectors σ_{MX}, σ_{PY}, and σ_{MY} and the scalars ϕ, a, f, k, q, r, and s. Note that the *historical* fishing mortality rate f may in general be different than the *optimal* rate ϕ. Estimation of ϕ will follow the method for decision variables described in the preceding section.

It will be assumed that the vector σ_{MX} is known, which in practice means viewing an independent estimate of σ_{MX} (such as the one presented in the "Measurement Error" section for the flathead sole example) as certain. Ideally, an independent estimate of the vector σ_{MY} would be available as well, but in practice it often is not, as is the case with the flathead sole example. If σ_{MY} is unknown, it is impossible to estimate the vector σ_{PY}, since the two terms never appear separately. For the flathead sole example, then, an *ad hoc* value of

$$\sqrt{\sigma_{PY_i}^{\ 2} + \sigma_{MY_i}^{\ 2}} = 0.5 \qquad (41)$$

will be assumed for all i. This value is high enough that it has the effect of downweighting the importance of the catch time series, thereby letting the model focus on tracking the survey abundance time series. Such an approach seems fairly reasonable for the flathead sole example, since the complexities of the eastern Bering Sea groundfish fishery and its management are such that anything close to a time-invariant proportionality between catch and biomass for this stock is probably not a very appropriate assumption.

To facilitate estimation of the remaining parameters, define two new parameters which, given A, Q, and R, prescribe a one-to-one mapping into F and K:

$$C = F - Q + R \tag{42}$$

and

$$D = K - e^{F-A} + Q - R. \tag{43}$$

Note that B=D-Q+R. The important thing abound C and D (or, alternatively, c and d) is that their MLEs can be computed in closed form (Attachment 1), and that these MLEs are independent of both Q and R. Thus, it is convenient to reduce the dimensionality of the model by setting parameters c and d at their MLEs, conditional on the other parameters (i.e., c and d become explicit functions of the other parameters). Walters and Ludwig (1994) call this an "approximate Bayesian" procedure.

This leaves parameters a, q, r, and s to be estimated. These four parameters are of two distinct types in terms of their estimability. Parameters a and s appear separately in the terms making up the likelihood (i.e., they are not formally confounded, though they may be correlated) and their values directly influence the likelihood even when c and d are set at their respective MLEs. Parameters q and r, on the other hand, *are* formally confounded (specifically, they always appear in the form r/q), and their values have no influence on the likelihood when c and d are set at their respective MLEs. Thus, it is natural to consider estimation of a and s separately from estimation of q and r.

Parameters a and s will be estimated by applying the method for states of nature described in the preceding section, where the computation of $m_x(j)$ will involve integrating across the joint posterior distribution of a and s. The joint posterior distribution, in turn, is obtained by assuming a joint prior distribution for a and s, then multiplying by the likelihood (Equation (33)). A convenient form for a joint prior distribution is the bivariate lognormal:

$$g'_{a,s}(a,s) = \left(\frac{1}{2\pi\sigma_A\sigma_S a s}\right)\sqrt{\frac{1}{1-\rho_{AS}^2}}\exp\left(-\frac{\left(\frac{\ln(a)-\mu_A}{\sigma_A}\right)^2+\left(\frac{\ln(s)-\mu_S}{\sigma_S}\right)^2}{2(1-\rho_{AS}^2)}+\frac{\rho_{AS}\left(\frac{\ln(a)-\mu_A}{\sigma_A}\right)\left(\frac{\ln(s)-\mu_S}{\sigma_S}\right)}{1-\rho_{AS}^2}\right), \tag{44}$$

where μ_A, μ_S, σ_A, and σ_S represent the means and standard deviations of the marginal distributions of A and S, respectively, and ρ_{AS} is the correlation between A and S. For the flathead sole example, the parameter values μ_A=-1.96, μ_S=-0.99, σ_A=σ_S=0.833, and ρ_{AS}=0 were chosen. These give $m_a(1)$=0.2 (the point estimate of the natural mortality rate for flathead sole, Walters and Wilderbuer 1995) and a coefficient of variation (CV) equal to unity for the marginal prior pdfs of a and s as well as for the stationary distribution of x when Equation (19) is evaluated at the means of the respective marginal priors. (It should be noted that although the bivariate lognormal will be used in the flathead sole example, the estimation methodology presented below does not depend on the prior following this functional form.)

Estimation of parameters q and r will follow basically the same scheme, except that the joint posterior pdf is identical to the joint prior pdf because the value of the likelihood is invariant with respect to these two parameters. A bivariate lognormal pdf was assumed, with parameters μ_Q=-0.01961, μ_R=10.714, σ_Q=0.19804, σ_R=0.24290, and ρ_{QR}=0. These set $m_q(1)$ and $m_r(1)$ equal to the point estimates of 1.0 and 46338 given earlier, a CV of 0.20 for the marginal pdf of q, and a CV of just under 0.25 for the marginal pdf of r.

The following three subsections treat, in turn, estimation of the optimal fishing mortality rate ϕ, the parameters a and s, and the parameters q and r. In each subsection, the estimation process is divided into two cases: the first is based on the relevant jth-order mean when parameter values are known with certainty (i.e., process error only), and the second is based on the relevant jth-order mean when parameter uncertainty is incorporated.

Optimal Fishing Mortality ϕ

Case I: Parameter Values Certain

Using the loss function defined by Equation (35), the expected loss is given by Equation (38), where $m_y(t,j)$ is defined by Equations (20-21) with target fishing morality rate ϕ substituted for historic fishing mortality rate f:

$$m_y(t,j) = \phi \exp\left(\mu'_X(t) + \frac{j\sigma'_Y(t)^2}{2}\right)$$
$$= \phi \exp\left(e^{-at}\mu_0 + (1-e^{-at})\left(\ln(k) - \frac{\phi}{a}\right) + \frac{j\sigma'_Y(t)^2}{2}\right). \quad (45)$$

In the limit as t approaches infinity, Equation (45) becomes

$$m_y(j) = \phi \exp\left(\ln(k) - \frac{\phi}{a} + \left(\frac{j}{2}\right)\left(\frac{s^2}{2a} + \sigma_{PY}^{\ 2}\right)\right). \quad (46)$$

Differentiating $m_y(t,j)$ with respect to ϕ and setting the resulting expression equal to zero gives the solution for the fishing mortality rate at maximum expected utility (MEU):

$$\phi_{MEU}(t) = \frac{a}{1 - e^{-at}}. \tag{47}$$

Stated another way, the degree of relative risk aversion does not impact the choice of exploitation rates when model parameters are known with certainty *regardless of the level of process error*, and as t approaches infinity, the solution collapses to $\phi_{MEU}=\phi_{MSY}=a$.

Case II: Parameter Values Uncertain

When the parameters $a, f, k, q, r, s, \sigma_{PY}$ are uncertain, Equation (46) changes as shown below:

$$m_y(j) = \phi\left(\int_0^\infty \cdots \int_0^\infty \exp\left(j\left(\ln(k) - \frac{\phi}{a}\right) + \frac{j^2 {\sigma'_Y}^2}{2}\right) g_{\theta_1,\ldots,\theta_7}(\theta_1,\ldots,\theta_7)\, d\theta_1 \ldots d\theta_7\right)^{1/j}, \tag{48}$$

where each of the θ_i corresponds to one of the uncertain parameters (σ_{PY} is treated here as a scalar for notational convenience, though in general it could be viewed as a vector) and $g_{\theta_1,\ldots,\theta_7}(\theta_1,\ldots,\theta_7)$ represents the joint pdf of those parameters .

For the special case where j approaches 0, Equation (48) simplifies to

$$\begin{aligned} m_y(0) &= \phi \exp\left(\int_0^\infty \int_0^\infty \left(\ln(k) - \frac{\phi}{a}\right) g_{a,k}(a,k)\, da dk\right) \\ &= \phi\, m_k(0) \exp\left(-\frac{\phi}{m_a(-1)}\right), \end{aligned} \tag{49}$$

where $g_{a,k}(a,k)$ is the joint marginal pdf of a and k.

Differentiating Equation (49) with respect to ϕ and solving for zero gives the harvest rate that maximizes expected log stationary yield (*MELSY*), a harvest strategy suggested by Thompson (1992):

$$\phi_{MELSY} = m_a(-1). \tag{50}$$

That is, the risk-averse (*RRA*=1) long-term optimal harvest rate is simply the harmonic mean of the marginal posterior pdf of a. For the flathead sole example, $m_a(-1)=0.108$. By way of comparison, $m_a(1)=0.150$. The marginal prior and posterior pdfs of a for the flathead sole example are compared in Figure 2. Although the two distributions appear roughly similar, the CV of the posterior is actually 43% smaller than that of the prior.

Growth Rate *a* and Process Error Scale *s*

Case I: Parameter Values Certain

From the transition pdf of x, the jth-order mean of x can be written

$$m_x(t,j) = \exp\left(e^{-at}\mu_{X_0} + (1-e^{-at})\left(\ln(k) - \frac{\phi}{a}\right) + \left(\frac{j}{2}\right)\left(e^{-2at}\sigma_{X_0}^{\ 2} + (1-e^{-2at})\left(\frac{s^2}{2a}\right)\right)\right). \quad (51)$$

In the limit as t approaches infinity, Equation (51) reduces to

$$m_x(j) = k\exp\left(-\frac{\phi}{a} + \frac{js^2}{4a}\right). \quad (52)$$

Case II: Parameter Values Uncertain

When the parameters a and s are uncertain, Equation (52) changes to

$$m_x(j) = \left(\int_0^\infty \int_0^\infty \exp\left(j\left(\ln(k) - \frac{\phi}{a}\right) + \frac{j^2 s^2}{2a}\right) g_{a,s}(a,s)\,da\,ds\right)^{1/j}, \quad (53)$$

which, in the special case where j approaches 0, becomes

$$\begin{aligned} m_x(0) &= k\exp\left(\int_0^\infty \left(-\frac{\phi}{a}\right) g_a(a)\,da\right) \\ &= k\exp\left(-\frac{\phi}{m_a(-1)}\right). \end{aligned} \quad (54)$$

Thus, the optimal estimator of a (for this limiting case) is $m_a(-1)$, which, as noted earlier, has a value of 0.108 in the flathead sole example. Because $m_a(-1)$ turns out to be both the optimal estimator of a and the risk-averse optimal fishing mortality rate, the deterministic result equating a

with the optimal fishing mortality rate is preserved in the stochastic case.

An optimal estimator of s is not so obvious, since s does not appear in Equation (54). However, a reasonable choice can be initiated by noting that the φth-order mean of $m_x(j)$ can be written

$$m_{m_x(j)}(\varphi) = k\left(\int_0^\infty \int_0^\infty \exp\left(-\frac{\varphi\phi}{a} + \frac{\varphi j s^2}{2a}\right) g_{a,s}(a,s)\,da\,ds\right)^{1/\varphi}, \tag{55}$$

which, in the limit as φ approaches zero, becomes

$$\begin{aligned} m_{m_x(j)}(0) &= k\exp\left(\int_0^\infty \int_0^\infty \left(-\frac{\phi}{a} + \frac{js^2}{2a}\right) g_{a,s}(a,s)\,da\,ds\right) \\ &= k\exp\left(-\frac{\phi}{m_a(-1)} + \frac{j m_s(2)^2}{2m_a(-1)}\right). \end{aligned} \tag{56}$$

Thus, since the geometric mean of $m_x(j)$ is given by setting $s=m_s(2)$ for arbitrary j, it makes sense to set $s=m_s(2)$ for the special case where j=0. In the flathead sole example, $m_s(2)$=0.128.

Catchability q and Range r

Case I: Parameter Values Certain

Noting that

$$\mu_{X_0} = \mu_{Z_0} - \ln(q) + \ln(r) \tag{57}$$

and

$$k = d\left(\frac{r}{q}\right)\exp\left(\frac{cq}{ar}\right), \tag{58}$$

and defining

$$h(t,j) = \exp\left(e^{-at}\mu_{Z_0} - (1-e^{-at})\left(\frac{\phi}{a}\right) + \left(\frac{j}{2}\right)\left(e^{-2at}\sigma_{X_0}^{\ 2} + (1-e^{-2at})\left(\frac{s^2}{2a}\right)\right)\right), \tag{59}$$

it is possible to rewrite Equation (51) as

$$m_x(t,j) = \left(\frac{r}{q}\right)\exp\left((1-e^{-at})\left(\ln(d) + \frac{cq}{ar}\right)\right)h(t,j). \tag{60}$$

Importantly, nothing in $h(t,j)$ depends on either q or r. The fact that neither q nor r enters into the calculation of σ_X can be confirmed from Equations (14) and (22-24), and the independence of μ_Z from q and r may be established by substituting the expression for μ_X shown in Attachment 1 into Equation (57).

In the limit as t approaches infinity, Equation (60) becomes

$$m_x(j) = d\left(\frac{r}{q}\right)\exp\left(\frac{cq}{ar} - \frac{\phi}{a} + \frac{js^2}{4a}\right). \tag{61}$$

Case II: Parameter Values Uncertain

When q and r are uncertain, Equation (61) changes as shown below:

$$m_x(j) = d\left(\int_0^\infty\int_0^\infty\left(\frac{r}{q}\right)^j\exp\left(\frac{jcq}{ar} - \frac{j\phi}{a} + \frac{j^2s^2}{4a}\right)g_{q,r}(q,r)\,dq\,dr\right)^{1/j}, \tag{62}$$

which, in the special case where j approaches 0, is simply

$$m_x(0) = d\left(\frac{m_r(0)}{m_q(0)}\right)\exp\left(\frac{cm_q(1)}{am_r(-1)} - \frac{\phi}{a}\right). \tag{63}$$

Note that Equation (63) can also be obtained by evaluating Equation (58) at the point $(q,r)=(m_q(1),m_r(-1))$ and then multiplying by the terms $m_q(1)/m_q(0)$, $m_r(0)/m_r(-1)$, and $e^{-\phi/a}$. In general, the value of k implied by Equation (63) may be less than or greater than the value of k given by evaluating Equation (58) at the point $(q,r)=(m_q(1),m_r(1))$. In the flathead sole example, for instance, the two quantities are the same. That is, the risk-averse optimal estimates of q and r are simply the arithmetic means of the respective marginal pdfs (by coincidence). The value of k under the risk-averse optimal parameter estimates is approximately 1,110,000 t. The corresponding estimate of $m_x(0,0)$, that is, the geometric mean estimate of biomass at the time of the most recent survey, is approximately 640,000 t.

Conclusions

In summary, the following conclusions may be drawn from the above:

1) In the field of fishery stock assessment, it is possible to model both process and measurement error simultaneously in a formal, rigorous fashion.

2) The existence of lognormal process error in population dynamics, often assumed on an ad hoc basis, can actually be derived in the case of the model presented here, though the relationship between error magnitude and stock size is more complex than generally assumed.

3) The Kalman filter provides a straightforward means of addressing the time-series nature of at least some of the estimation problems typically encountered in fishery stock assessment.

4) The jth-order means of the pdfs of stationary yield and stock size provide a straightforward and heuristic mapping into alternative levels of relative risk aversion: For example, a risk-averse optimal fishing mortality rate can be defined as that which maximizes the geometric mean of stationary yield, equivalent to the MELSY (maximum expected log stationary yield) strategy.

5) In the model presented here, the harmonic mean of the posterior pdf of the Gompertz growth parameter is the MELSY solution.

6) Because the Gompertz growth parameter is identical to the MSY fishing mortality rate in the deterministic case, the preceding result suggests the hypothesis that the harmonic mean of the posterior pdf of the MSY fishing mortality rate may be a good proxy for the risk-averse optimum *in general* (i.e., not just for the model presented here).

7) Even when information regarding the age structure of the stock is not available or is ignored, a time series of trawl survey stock size estimates may provide sufficient information to achieve a substantial reduction in the CV of the MSY fishing mortality rate (i.e., comparing the CV of the posterior pdf to that of the prior pdf).

8) It should be possible, at least in some models, to estimate the level of process error internally.

9) In terms of computational overhead, the cost of viewing the catchability coefficient and similar quantities as uncertain parameters rather than as known constants may be minimal.

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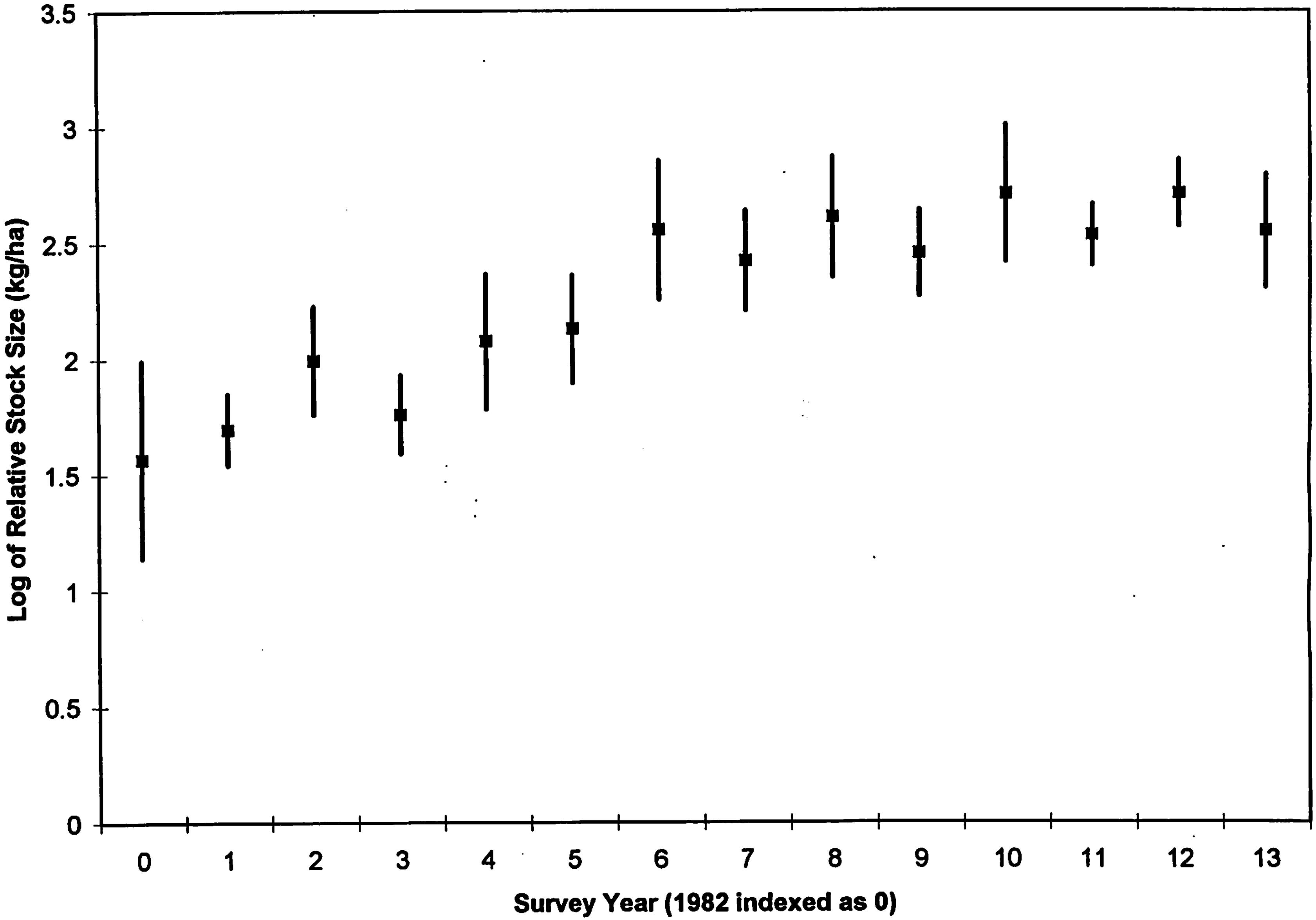

Figure 1. Trawl survey estimates of relative flathead sole abundance on a log scale, plus or minus two standard deviations.

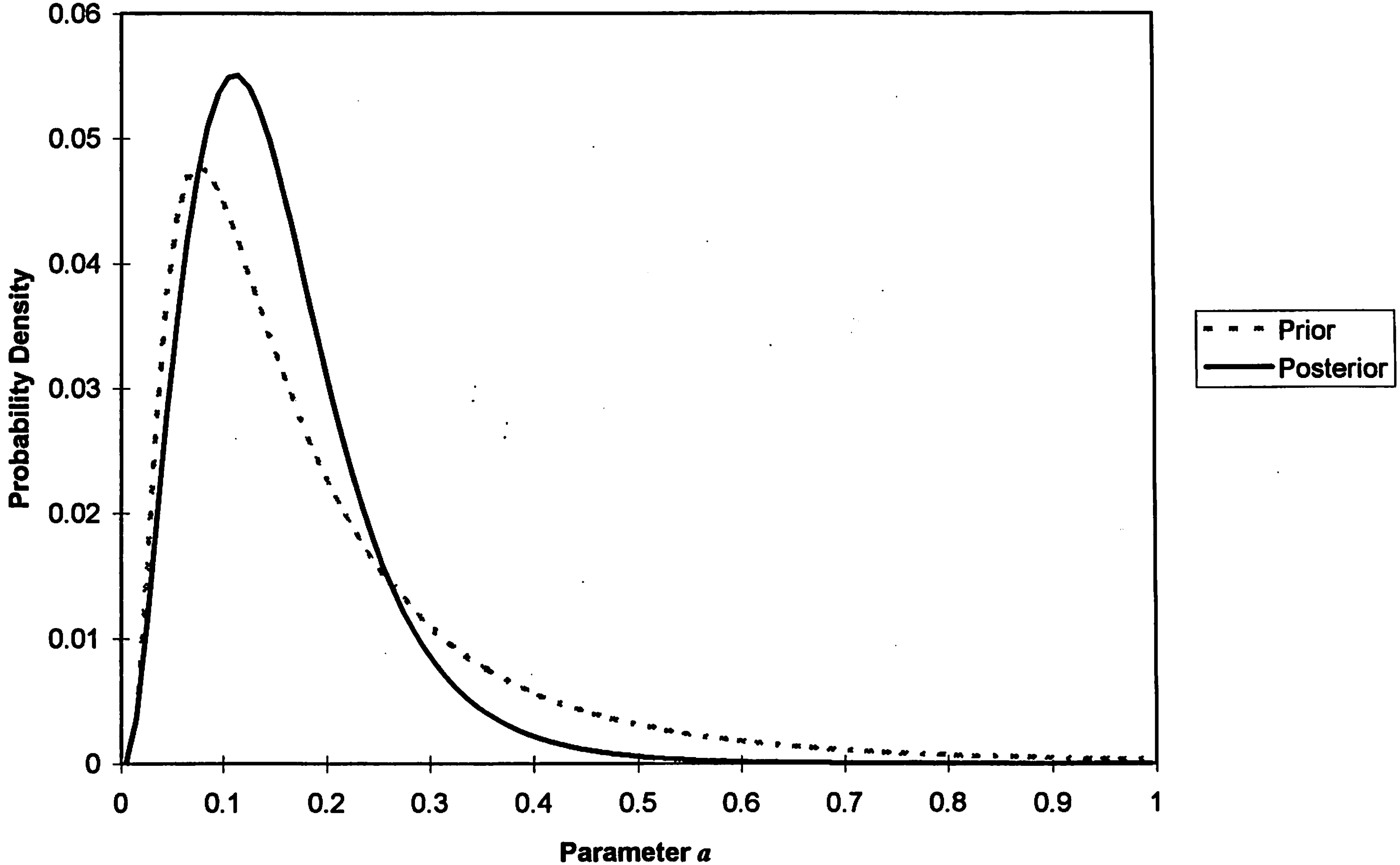

Figure 2. Prior and posterior probability density functions for parameter a in the flathead sole example.

Attachment 1:
Maximum Likelihood Estimation of Parameters *C* and *D*

Define some coefficients:

$$\alpha_0 := \left(\frac{\sigma X_0}{\sigma MX_0}\right)^2 \qquad \alpha_i := e^{-a\cdot\tau_i}\cdot\left(\frac{\sigma X_i}{\sigma' X_i}\right)^2\cdot\alpha_{i-1} + \left(\frac{\sigma X_i}{\sigma MX_i}\right)^2$$

$$\beta_0 := \left(\frac{\sigma X_0}{\sigma MX_0}\right)^2\cdot Z_0 \qquad \beta_i := e^{-a\cdot\tau_i}\cdot\left(\frac{\sigma X_i}{\sigma' X_i}\right)^2\cdot\beta_{i-1} + \left(\frac{\sigma X_i}{\sigma MX_i}\right)^2\cdot Z_i$$

$$\chi_0 := \frac{\left(\sigma X_0\right)^2}{\left(\sigma PY_0\right)^2 + \left(\sigma MY_0\right)^2} \qquad \chi_i := e^{-a\cdot\tau_i}\cdot\left(\frac{\sigma X_i}{\sigma' X_i}\right)^2\cdot\chi_{i-1} + \frac{\left(\sigma X_i\right)^2}{\left(\sigma PY_i\right)^2 + \left(\sigma MY_i\right)^2}$$

$$\delta_0 := \left[\frac{\left(\sigma X_0\right)^2}{\left(\sigma PY_0\right)^2 + \left(\sigma MY_0\right)^2}\right]\cdot W_0 \qquad \delta_i := e^{-a\cdot\tau_i}\cdot\left(\frac{\sigma X_i}{\sigma' X_i}\right)^2\cdot\delta_{i-1} + \left[\frac{\left(\sigma X_i\right)^2}{\left(\sigma PY_i\right)^2 + \left(\sigma MY_i\right)^2}\right]\cdot W_i$$

Define some more coefficients which are linear combinations of the above:

$$\upsilon_i := e^{-a\cdot\tau_i}\cdot\left(\beta_{i-1} + \delta_{i-1}\right) \qquad \varpi_i := e^{-a\cdot\tau_i}\cdot\chi_{i-1} \qquad \omega_i := 1 - e^{-a\cdot\tau_i}\cdot\left(\alpha_{i-1} + \chi_{i-1}\right)$$

The above coefficients enable the prior means of *X* to be written as linear functions of *C* and *D*:

$$\mu' X_i = \upsilon_i - \varpi_i\cdot C + \omega_i\cdot D - Q + R$$

and the posterior means as linear functions of *C*, *D*, *Z*, and *W*:

$$\mu X_j = \left(\sigma X_j\right)^2\cdot\left[\frac{\upsilon_j - \varpi_j\cdot C + \omega_j\cdot D}{\left(\sigma' X_j\right)^2} + \frac{Z_j}{\left(\sigma MX_j\right)^2} + \frac{W_j - C}{\left(\sigma PY_j\right)^2 + \left(\sigma MY_j\right)^2}\right] - Q + R$$

Define partial derivatives of posterior means (of *X*) with respect to *C* and *D*:

$$\delta\mu\delta C_0 := -\frac{\left(\sigma X_0\right)^2}{\left(\sigma PY_0\right)^2 + \left(\sigma MY_0\right)^2} \qquad \delta\mu\delta C_i := e^{-a\cdot\tau_i}\cdot\left(\frac{\sigma X_i}{\sigma' X_i}\right)^2\cdot\delta\mu\delta C_{i-1} - \frac{\left(\sigma X_i\right)^2}{\left(\sigma PY_i\right)^2 + \left(\sigma MY_i\right)^2}$$

$$\delta\mu\delta D_0 := 0 \qquad \delta\mu\delta D_i := \left(\frac{\sigma X_i}{\sigma' X_i}\right)^2\cdot\left[e^{-a\cdot\tau_i}\cdot\left(\delta\mu\delta D_{i-1} - 1\right) + 1\right]$$

Define a pair of vectors:

$$\gamma := \begin{bmatrix} -\displaystyle\sum_{i=1}^{n} \frac{\left[\begin{array}{l}(o' W_i)^2 \cdot e^{-a\cdot\tau_i}\cdot\delta\mu\delta C_{i-1}\cdot\varpi_i + (o' Z_i)^2\cdot\left(e^{-a\cdot\tau_i}\cdot\delta\mu\delta C_{i-1}+1\right)\cdot(\varpi_i - 1) \ldots \\ + -(o' X_i)^2\cdot\left[\left(e^{-a\cdot\tau_i}\cdot\delta\mu\delta C_{i-1}+1\right)\cdot\varpi_i + e^{-a\cdot\tau_i}\cdot\delta\mu\delta C_{i-1}\cdot(\varpi_i - 1)\right]\end{array}\right]}{(o' Z_i)^2\cdot(o' W_i)^2 - (o' X_i)^4} \\ \displaystyle\sum_{i=1}^{n} \frac{\left[\begin{array}{l}(o' W_i)^2\cdot\left[e^{-a\cdot\tau_i}\cdot\delta\mu\delta C_{i-1}\cdot(Z_i - \upsilon_i)\right] + (o' Z_i)^2\cdot\left(e^{-a\cdot\tau_i}\cdot\delta\mu\delta C_{i-1}+1\right)\cdot(W_i - \upsilon_i) \ldots \\ + -(o' X_i)^2\cdot\left[\left(e^{-a\cdot\tau_i}\cdot\delta\mu\delta C_{i-1}+1\right)\cdot(Z_i - \upsilon_i) + e^{-a\cdot\tau_i}\cdot\delta\mu\delta C_{i-1}\cdot(W_i - \upsilon_i)\right]\end{array}\right]}{(o' Z_i)^2\cdot(o' W_i)^2 - (o' X_i)^4} \\ -\displaystyle\sum_{i=1}^{n} \frac{\left[(o' W_i)^2\cdot e^{-a\cdot\tau_i}\cdot\delta\mu\delta C_{i-1} - (o' X_i)^2\cdot\left(2\cdot e^{-a\cdot\tau_i}\cdot\delta\mu\delta C_{i-1}+1\right) + (o' Z_i)^2\cdot\left(e^{-a\cdot\tau_i}\cdot\delta\mu\delta C_{i-1}+1\right)\right]\cdot\varpi_i}{(o' Z_i)^2\cdot(o' W_i)^2 - (o' X_i)^4} \end{bmatrix}$$

$$\eta := \begin{bmatrix} \displaystyle\sum_{i=1}^{n} \frac{\left[e^{-a\cdot\tau_i}\cdot\left(\delta\mu\delta D_{i-1} - 1\right)+1\right]\cdot\left[(o' W_i)^2 - 2\cdot(o' X_i)^2 + (o' Z_i)^2\right]\cdot\varpi_i}{(o' Z_i)^2\cdot(o' W_i)^2 - (o' X_i)^4} \\ \displaystyle\sum_{i=1}^{n} \frac{\left[e^{-a\cdot\tau_i}\cdot\left(\delta\mu\delta D_{i-1} - 1\right)+1\right]\cdot\left[\left[(o' W_i)^2 - (o' X_i)^2\right]\cdot(Z_i - \upsilon_i) + \left[(o' Z_i)^2 - (o' X_i)^2\right]\cdot(W_i - \upsilon_i)\right]}{(o' Z_i)^2\cdot(o' W_i)^2 - (o' X_i)^4} \\ \displaystyle\sum_{i=1}^{n} \frac{\left[e^{-a\cdot\tau_i}\cdot\left(\delta\mu\delta D_{i-1} - 1\right)+1\right]\cdot\left[\left[(o' W_i)^2 - (o' X_i)^2\right]\cdot\varpi_i + \left[(o' Z_i)^2 - (o' X_i)^2\right]\cdot(\varpi_i - 1)\right]}{(o' Z_i)^2\cdot(o' W_i)^2 - (o' X_i)^4} \end{bmatrix}$$

Solve for the maximum likelihood estimates of *C* and *D* simultaneously:

$$C := \frac{\gamma_1\cdot\eta_0 + \gamma_2\cdot\eta_1}{\gamma_0\cdot\eta_0 - \gamma_2\cdot\eta_2} \qquad D := \frac{\gamma_0\cdot\eta_1 + \gamma_1\cdot\eta_2}{\gamma_0\cdot\eta_0 - \gamma_2\cdot\eta_2}$$

Note that the maximum likelihood estimates of *C* and *D* can also be written as linear functions of each other:

$$C := \frac{\gamma_1 + \gamma_2\cdot D}{\gamma_0} \qquad D := \frac{\eta_1 + \eta_2\cdot C}{\eta_0}$$

Solve for *F* and *K* as functions of *A*, *C*, *D*, and *Q*:

$$F := C + Q - R \qquad K := D + e^{C+Q-R-A} - Q + R$$