PIBKC: Random Effects Model Comparisons

William T. Stockhausen

5/6/23

Introduction

The Pribilof Islands blue king crab (PIBKC) stock assessment is currently conducted using a mixed Tier 4/Tier 5 approach (Stockhausen, 2021): stock status (**overfished/not overfished**) is determined using a Tier 4 model to estimate B_{MSY} and current mature male biomass (MMB) at the time of mating while the OFL is determined using a Tier 5 approach based on historical catch. For PIBKC, the variances associated with annual survey estimates of MMB are so large that, prior to estimating B_{MSY} and "current" MMB-at-mating, the survey MMB time series is first smoothed to reduce overall variability and batter capture. A random walk model is fit to design-based estimates of annual mature male biomass from the NMFS EBS shelf summer trawl survey using a state-space, random effects approach to better estimate the underlying (unobserved) time series. The code implementing this random walk model, adopted over an inverse variance-weighted three-year moving average (Stockhausen, 2015), was modified by the author from original AD Model Builder (ADMB; Fournier et al., 2012) code developed for certain groundfish stock assessments by J. Ianelli (AFSC). Recently, the state-space random effects code used by groundfish assessment authors has been converted to Template Model Builder (TMB; Kristensen, 2016), a model-building platform similar to ADMB but with better characteristics with respect to random effects models, and provided in the R package **"rema"** (R Core Team, 2023; Sullivan, 2022; Sullivan et al., 2022). In order to take advantage of a wider user base for analysis, testing and development, it has been suggested that the PIBKC model be converted to use **rema** for the random walk model estimation. Here, I present results from a comparison of running the ADMB and **rema** random walk models on the PIBKC mature male biomass time series from the NMFS EBS shelf trawl survey.

Random walk estimation

The state-space random effects model is a statistical approach which models annual log-scale changes in "true" survey MMB as a random walk process using

$$p(< ln(MMB_s) >_y \mid < ln(MMB_s) >_{y-1}) \sim N(0, \phi^2) \tag{1}$$

as the state equation, where $p(x|y)$ denotes the probability of x conditional on y, $N(\mu, v)$ indicates the normal distribution with mean μ and variance v, and

$$ln(MMB_{s_y}) = < ln(MMB_s) >_y + \eta_y, \text{ where } \eta_y \sim N(0, \sigma^2_{s_y}) \tag{2}$$

as the observation equation. $< ln(MMB_s) >_y$ in equations 1 and 2 is the estimated "true" log-scale survey MMB in year y, while ϕ^2 in Equation 1 represents the estimated (ln-scale) process error variance. MMB_{s_y} in equation 2 is the observed survey MMB in year y, η_y represents normally-distributed ln-scale observation error, and $\sigma^2_{s_y}$ is the ln-scale survey MMB variance in year y. The MMB_s's and σ_s's are observed quantities, while the $< ln(MMB_s) >$'s are estimated parameters regarded as random effects in the likelihood function. The process error variance ϕ^2 is parameterized on the ln-scale using $\phi^2 = exp(2 \cdot \lambda)$, where λ is an estimated fixed effect parameter.

Parameter estimates are obtained by minimizing the joint negative log-likelihood objective function

$$\Lambda = \sum_y \left[ln(2\pi\phi) + \left(\frac{< ln(MMB_s) >_y - < ln(MMB_s) >_{y-1}}{\phi} \right)^2 \right] + \sum_y \left(\frac{ln(MMB_{s_y}) - < ln(MMB_s) >_y}{\sigma_{s_y}} \right)^2 \tag{3}$$

and integrating out the random effects using the Laplace approximation.

Results

For the purposes of the PIBKC assessment, both the ADMB model and the **rema** TMB model implement the same random walk model and should provide essentially the same estimates for parameters and derived quantities. Both models were fit to the time series of design-based estimates for PIBKC mature male biomass from the NMFS EBS shelf trawl survey used in the 2021 PIBKC assessment (Table 1; Figures 1 and 2). Both models converged with small maximum gradients, although the TMB gradient was much smaller in absolute scale than the ADMB value (7.219e-15 vs. 4.549e-08). The **rema** model also achieved a slightly smaller objective function value (28.35 vs. 28.38), although the difference is negligible from a likelihood standpoint. The arithmetic-scale estimates of process error variance are identical to 3 significant figures (Table 2), while the estimates of mature male survey biomass agree to 5 digits (Table 1; Figures 1-4). Finally, the confidence intervals from the ADMB model are slightly narrower than those from the **rema** model.

Discussion

Results from the two models are quite similar, with the **rema** model exhibiting slightly better characteristics (smaller maximum gradient, smaller objective function). In addition, using **rema** has the added benefits of extended diagnostics (e.g., one step ahead residuals [not discussed here]), the ability to fit multiple time series simultaneously, an extended user community, and continued development. Consequently, the author recommends that the **rema** model be adopted in place of the ADMB model in the PIBKC assessment to reduce the variance and better capture the trends in the estimated time series of mature male survey biomass used in the process to determine the Tier 4 overfished status.

Acknowledgments

I would like to thank Jane Sullivan (AFSC) for a very helpful review of a first draft of this paper.

References

Daly, B., C. Armistead and R. Foy. The 2015 Eastern Bering Sea Continental Shelf Bottom Trawl Survey: Results for Commercial Crab Species. NOAA Technical Memorandum NMFS-AFSC-XX 172 p.

Fournier, D.A., H.J. Skaug, J. Ancheta, J. Ianelli, A. Magnusson, M.N. Maunder, A. Nielsen, and J. Sibert. 2012. AD Model Builder: using automatic differentiation for statistical inference of highly parameterized complex nonlinear models. Optim. Methods Softw. 27:233-249.

Kristensen, K., A. Nielsen, C.W. Berg, H. Skaug, B.M. Bell. 2016. TMB: Automatic Differentiation and Laplace Approximation. Journal of Statistical Software, 70(5), 1-21. doi:10.18637/jss.v070.i05

R Core Team (2023). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. https://www.R-project.org/.

Stockhausen, W. 2021. 2021 Stock Assessment and Fishery Evaluation Report for the Pribilof Islands Blue King Crab Fisheries of the Bering Sea and Aleutian Islands Regions. In: Stock Assessment and Fishery Evaluation Report for the King and Tanner Crab Fisheries of the Bering Sea and Aleutian Islands: 2021 Final Crab SAFE. North Pacific Fishery Management Council. Anchorage, AK.

Sullivan J. 2022. rema: A generalized framework to fit the random effects (RE) model, a state-space random walk model developed at the Alaska Fisheries Science Center (AFSC) for apportionment and biomass estimation of groundfish and crab stocks. https://github.com/JaneSullivan-NOAA/rema, https://afsc-assessments.github.io/rema/.

Sullivan, J., C. Monnahan, P. Hulson, J. Ianelli, J. Thorson, and A. Havron. 2022. REMA: a consensus version of the random effects model for ABC apportionment and Tier 4/5 assessments. Plan Team Report, Joint Groundfish Plan Teams, North Pacific Fishery Management Council. 605 W 4th Ave, Suite 306 Anchorage, AK 99501.

Tables

Table 1. Comparison of results from the rema and ADMB random effects models for the time series of mature male biomass for Pribilof Islands blue king crab from the EBS Shelf Trawl Survey.

		value			lci			uci	
year	observed	ADMB	**rema**	observed	ADMB	**rema**	observed	ADMB	**rema**
1975	38,054	26,785	26,783	20,760	17,035	16,789	69,754	42,116	42,728
1976	14,059	19,947	19,947	8,104	13,547	13,426	24,391	29,369	29,634
1977	42,618	21,190	21,189	17,814	13,764	13,593	101,958	32,620	33,029
1978	17,370	16,960	16,960	8,912	11,463	11,356	33,852	25,093	25,328
1979	10,959	13,352	13,352	7,386	9,817	9,774	16,262	18,159	18,241
1980	23,553	15,539	15,538	13,894	11,082	11,015	39,925	21,788	21,917
1981	11,628	11,412	11,412	9,321	9,362	9,351	14,507	13,911	13,928
1982	7,389	7,448	7,448	5,825	6,063	6,055	9,373	9,148	9,161
1983	5,409	5,075	5,075	4,316	4,157	4,152	6,778	6,194	6,202
1984	2,216	2,352	2,352	1,659	1,850	1,846	2,959	2,989	2,996
1985	1,055	1,357	1,357	754	1,030	1,027	1,476	1,787	1,793
1986	1,505	1,557	1,557	1,030	1,164	1,160	2,199	2,083	2,091
1987	2,923	1,923	1,923	1,761	1,360	1,351	4,853	2,718	2,736
1988	842	1,436	1,436	446	964	955	1,591	2,138	2,160
1989	827	1,610	1,610	392	1,051	1,039	1,749	2,465	2,496
1990	3,078	2,603	2,603	1,513	1,741	1,723	6,261	3,893	3,933
1991	4,690	3,800	3,800	2,910	2,691	2,674	7,556	5,367	5,400
1992	4,391	4,173	4,173	2,612	2,959	2,941	7,382	5,886	5,923
1993	4,556	4,324	4,324	3,100	3,214	3,201	6,694	5,819	5,842
1994	3,410	4,021	4,021	2,220	2,929	2,914	5,240	5,519	5,547
1995	8,360	4,922	4,921	4,091	3,363	3,334	17,086	7,204	7,265
1996	4,641	4,376	4,376	3,309	3,324	3,314	6,509	5,761	5,779
1997	3,233	3,322	3,322	2,284	2,534	2,527	4,575	4,354	4,368
1998	2,798	2,704	2,704	2,043	2,092	2,087	3,833	3,494	3,503
1999	1,729	1,978	1,979	1,136	1,461	1,455	2,631	2,678	2,690
2000	2,091	1,832	1,832	1,443	1,362	1,357	3,031	2,464	2,474
2001	1,599	1,262	1,262	689	840	831	3,710	1,896	1,916
2002	680	784	784	369	535	530	1,254	1,151	1,161
2003	702	548	548	428	385	383	1,150	781	786
2004	107	281	281	53	184	181	214	429	434
2005	344	267	267	152	172	170	780	414	420
2006	166	226	226	81	146	144	339	351	355
2007	306	231	231	125	145	143	753	368	374
2008	46	212	212	16	130	128	134	345	352
2009	497	294	294	219	189	186	1,130	458	465
2010	303	321	321	173	216	214	532	476	480
2011	461	371	371	180	235	232	1,180	583	592
2012	644	396	396	277	251	247	1,496	627	636
2013	250	344	344	102	218	214	615	542	550

(continued)

year	value observed	value ADMB	value **rema**	lci observed	lci ADMB	lci **rema**	uci observed	uci ADMB	uci **rema**
2014	233	336	336	104	219	216	524	516	522
2015	622	390	390	382	271	269	1,011	561	565
2016	129	247	247	62	164	163	265	371	375
2017	253	229	229	136	154	153	470	341	345
2018	154	197	197	78	129	127	303	302	306
2019	206	201	201	101	122	120	421	330	336
2020	NA	201	201	NA	99	94	NA	405	428
2021	NA	201	201	NA	87	78	NA	465	513

Table 2. Estimates for the arithmetic-scale process error variance from the ADMB and rema model fits to the time series of mature male biomass for Pribilof Islands blue king crab from the EBS Shelf Trawl Survey.

model	objective function	max gradient	estimate	std err
ADMB	28.3803	4.5486e-08	0.4332	0.0779
rema	28.3492	7.2188e-15	0.4331	0.0779

Figures

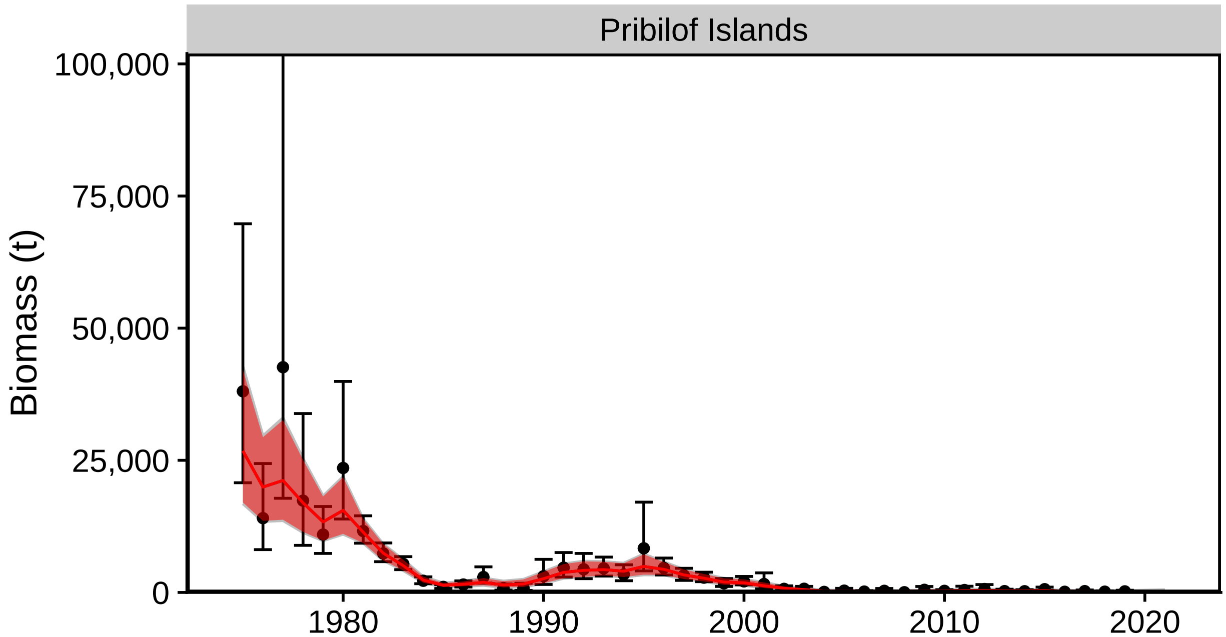

Figure 1. Arithmetic-scale comparison of RE smoothing results for the time series of mature male biomass for Pribilof Islands blue king crab from the EBS Shelf Trawl Survey. Red shading: **rema**. Grey shading: ADMB. Black circles and error bars: data.

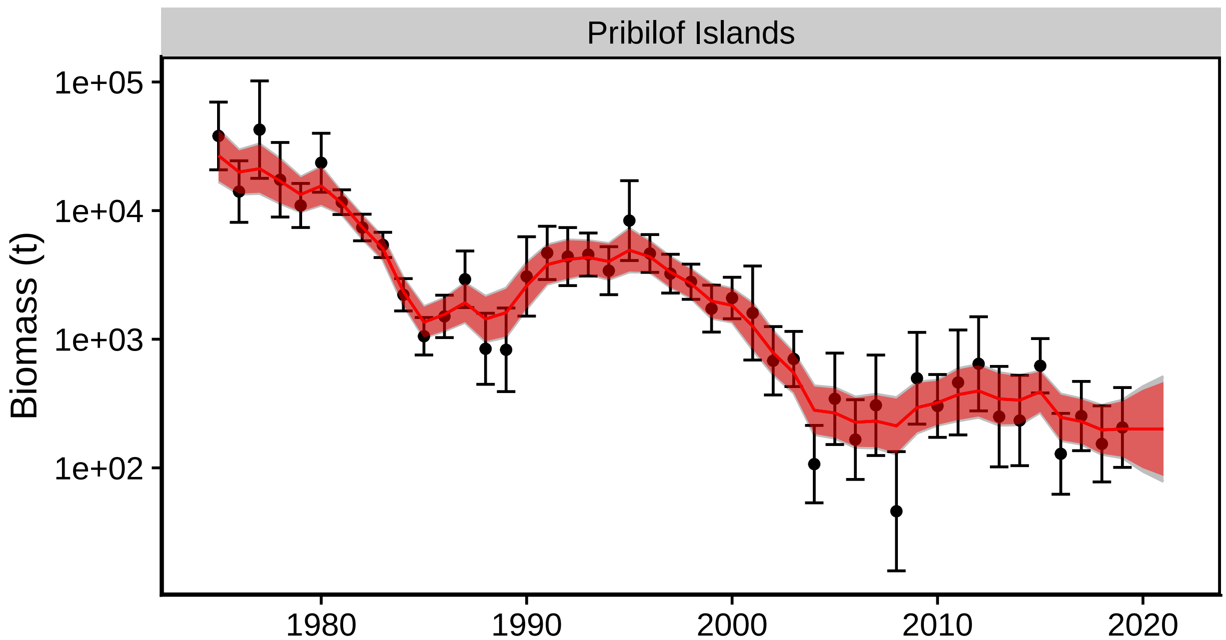

Figure 2. Log-scale comparison of RE smoothing results for the time series of mature male biomass for Pribilof Islands blue king crab from the EBS Shelf Trawl Survey. Red shading: **rema**. Grey shading: ADMB. Black circles and error bars: data.

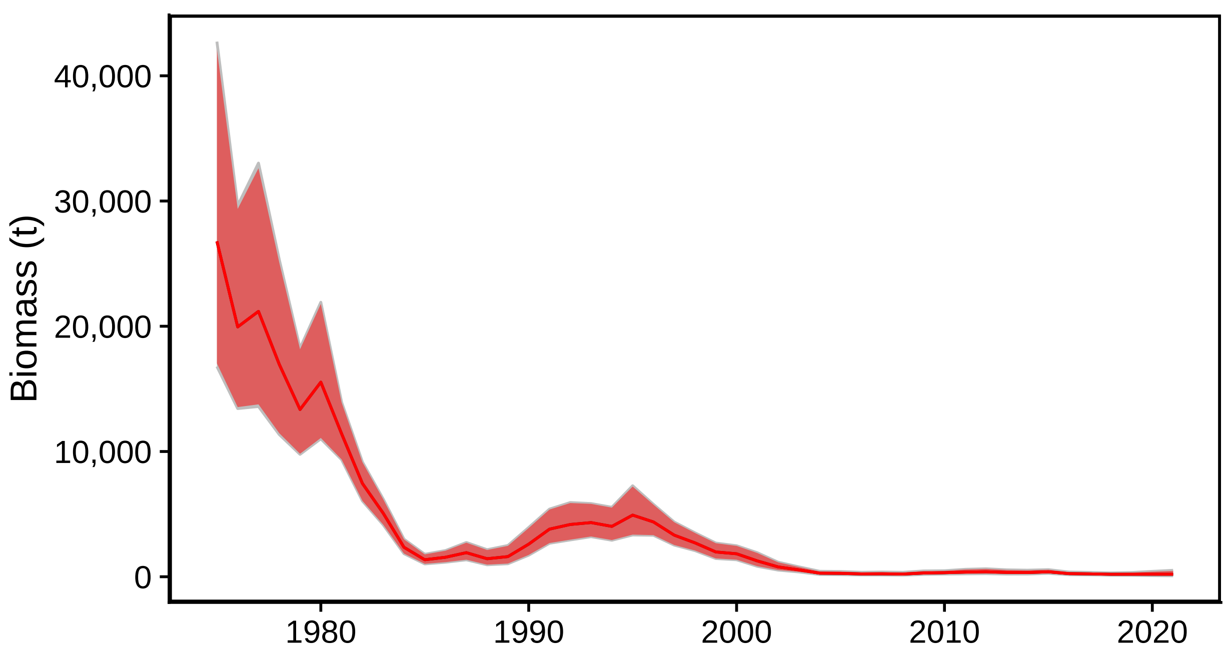

Figure 3. Arithmetic-scale comparison of RE smoothing results for the time series of mature male biomass for Pribilof Islands blue king crab from the EBS Shelf Trawl Survey. Red shading: **rema**. Grey shading: ADMB.

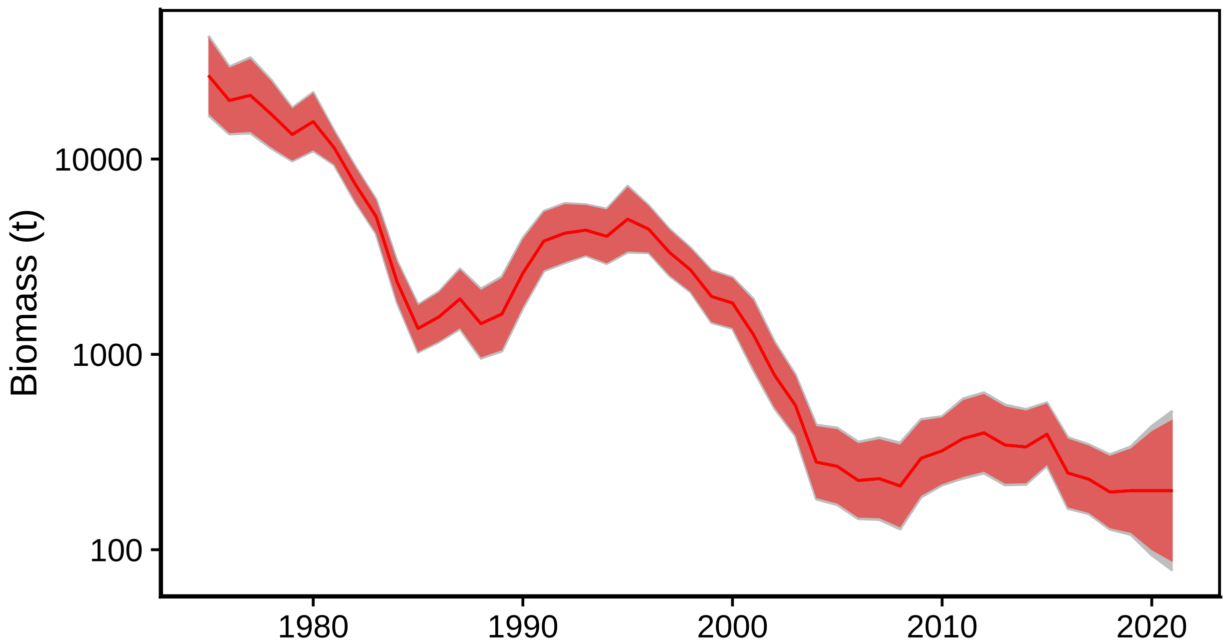

Figure 4. Log-scale comparison of RE smoothing results for the time series of mature male biomass for Pribilof Islands blue king crab from the EBS Shelf Trawl Survey. Red shading: **rema**. Grey shading: ADMB.