Appendix A. Description of GMACS with Bristol Bay Red King Crab Options (mainly from the GMACS document)

A. Model Description

a. Population model

The basic dynamics account for growth, mortality, maturity state and shell condition (although most of the equations below do not explicitly refer to maturity state and shell condition). For the case in which shell condition is not distinguished:

$$\underline{N}^g_{y,t} = ((\mathbf{I} - \mathbf{P}^g_{y,t-1}) + \mathbf{X}^g_{y,t-1}\mathbf{P}^g_{y,t-1})\mathbf{S}^g_{y,t-1}\,\underline{N}^g_{y,t-1} + \underline{\tilde{R}}^g_{y,t} \tag{A.1}$$

where $\underline{N}^g_{y,t}$ is the number of animals by size-class of gender g at the start of season t of year y, $\mathbf{P}^g_{y,t}$ is a matrix with diagonals given by vector of molting probabilities for animals of gender g at the start of season t of year y, $\mathbf{S}^g_{y,t}$ is a matrix with diagonals given by the vector of probabilities of surviving for animals of gender g during time-step t of year y (which may be of zero duration):

$$S^g_{y,t,l,l} = \exp\left(-Z^g_{y,t,l}\right) \tag{A.2}$$

$\mathbf{X}^g_{y,t}$ is the size-transition matrix (probability of growing from one size-class to each of the other size-classes or remains in the same size class) for animals of gender g during season t of year y, $\underline{\tilde{R}}^g_{y,t}$ is the recruitment (by size-class) to gear g during season t of year y (which will be zero except for one season – the recruitment season), and $Z^g_{y,t,l}$ is the total mortality for animals of gender g in size-class l during season t of year y. Note that mortality is continuous across a time-step.

The initial conditions for the model (i.e., the numbers-at-size at the start of the first year, y_1) is specified with an overall total recruitment multiplied by offsets for each size-class, i.e.:

$$N^g_{y_1,l} = R_{\text{Init}} e^{\delta^g_{y_1,l}} / \sum_{g'} \sum_{l'} e^{\delta^{g'}_{y_1,l'}} \tag{A.3}$$

The minimum carapace length for both males and females is set at 65 mm, and crab abundance is modeled with a length-class interval of 5 mm. The last length class includes all crab ≥160-mm CL for males and ≥140-mm CL for females. Thus, length classes/groups are 20 for males and 16 for females.

b. Recruitment

Recruitment occurs once during each year. Recruitment by sex and size-class is the product of total recruitment, the split of the total recruitment to sex and the assignment of sex-specific recruitment to size-classes, i.e.:

$$\tilde{R}^g_{y,t,l} = \bar{R}e^{\varepsilon_y} \begin{cases} (1+e^{\phi_y})^{-1}\, p_l^{\text{r,mal}} & \text{if } g\text{=males} \\ \phi_y (1+e^{\phi_y})^{-1}\, p_l^{\text{r,fem}} & \text{if } g\text{=females} \end{cases} \tag{A.4}$$

where $\bar{R}$ is median recruitment, ϕ_y determines the sex ratio of recruitment during year *y*, and $p_l^{r,g}$ is the proportion of the recruitment (by gender and year) that recruits to size-class *l*:

$$p_l^{r,g} = \int\limits_{L_l^{\text{low}}}^{L_l^{\text{hi}}} \tfrac{1}{\Gamma(\alpha^{r,g}/\beta^{r,g})} (l/\beta^{r,g})^{((\alpha^{r,g}/\beta^{r,g})-1)} e^{-l/\beta^{r,g}}\, dl \tag{A.5}$$

where $\alpha^{r,g}$ and $\beta^{r,g}$ are the parameters that define a gamma function for the distribution of recruits to size-class. Equation A.5 can be restricted to a subset of size-classes, in which case the results from Equation A.5 are normalized to sum to 1 over the selected size-classes.

c. Total mortality / probability of encountering the gear

Total mortality is the sum of fishing mortality and natural mortality, i.e.:

$$Z^g_{y,t,l} = \rho^{\text{M}}_{y,t} M^g_y \tilde{M}_l + \sum_f S^{f,g}_{y,t,l} (\lambda^{f,g}_{y,t,l} + \Omega^{f,g}_{y,t,l}(1-\lambda^{f,g}_{y,t,l})) F^{f,g}_{y,t} \tag{A.6}$$

where $\rho^{\text{M}}_{y,t}$ is the proportion of natural mortality that occurs during season *t* for year *y*, M^g_y is the rate of natural mortality for year *y* for animals of gender *g* (applies to animals for which $\tilde{M}_l = 1$), $\tilde{M}_l$ is the relative natural mortality for size-class *l* , $S^{f,g}_{y,t,l}$ is the (capture) selectivity for animals of gender *g* in size-class *l* by fleet *f* during season *t* of year *y*, $\lambda^{f,g}_{y,t,l}$ is the probability of retention for animals of gender *g* in size-class *l* by fleet *f* during season *t* of year *y*, $\Omega^{f,g}_{y,t,l}$ is the mortality rate for discards of gender *g* in size-class *l* by fleet *f* during season *t* of year *y*, and $F^{f,g}_{y,t}$ is the fully-selected fishing mortality for animals of gender *g* by fleet *f* during season *t* of year *y*.

The probability of encountering the gear (occurs instantaneously) is given by:

$$\tilde{Z}^g_{y,t,l} = \sum_f S^{f,g}_{y,t,l} F^{f,g}_{y,t} \tag{A.7}$$

Note that Equation A.7 is computed under the premise that fishing is instantaneous and hence that there is no natural mortality during season *t* of year *y*.

The logarithms of the fully-selected fishing mortalities by season are modelled as:

$$\ell\text{n}F^{f,\text{mal}}_{y,t} = \ell\text{n}F^{f,\text{mal}} + \xi^{f,\text{mal}}_{y,t} \tag{A.8}$$

$$\ell\text{n}F^{f,\text{fem}}_{y,t} = \ell\text{n}F^{f,\text{mal}}_{y,t} + \phi^f + \xi^{f,\text{fem}}_{y,t} \tag{A.9}$$

where $F^{f,\text{mal}}$ is the reference fully-selected fishing mortality rate for fleet *f*, ϕ^{f} is the offset between female and male fully-selected fishing mortality for fleet *f*, and $\xi_{y,t}^{f,g}$ are the annual deviation of fully-selected fishing mortality for fleet *f* (by gender).

Natural mortality can depend on time with blocked natural mortality (individual parameters). This option estimates natural mortality as parameters by block, i.e.:

$$M_{y}^{g} = e^{\psi_{y}^{g}} \tag{A.10}$$

where $M_{y_1}^{g}$ is the rate of natural mortality for gender *g* for the first year of the model, and ψ_{y}^{g} is the annual change in natural mortality and changes in blocks of years.

It is possible to 'mirror' the values for the ψ_{y}^{g} parameters (between genders and between blocks), which allows male and female natural mortality to be the same, and for natural mortality to be the same for discontinuous blocks (based on Equation A.10). It is also possible to estimate a ratio of natural mortality between genders. The deviations in natural mortality can also be penalized to avoid unrealistic changes in natural mortality to fit 'quirks' in the data.

d. Landings, discards, total catch

The model keeps track of (and can be fitted to) landings, discards, total catch by fleet in season with continuous mortality:

Landed catch

$$C_{y,t,l}^{\text{Land},f,g} = \frac{\lambda_{y,t,l}^{f,g}\, S_{y,t,l}^{f,g} F_{y,t}^{f,g}}{Z_{y,t,l}^{g}} N_{y,t,l}^{f,g}(1-e^{-Z_{y,t,l}^{g}}) \tag{A.11}$$

Discards

$$C_{y,t,l}^{\text{Disc},f,g} = \frac{(1-\lambda_{y,t,l}^{f,g})\, S_{y,t,l}^{f,g} F_{y,t}^{f,g}}{Z_{y,t,l}^{g}} N_{y,t,l}^{f,g}(1-e^{-Z_{y,t,l}^{g}}) \tag{A.12}$$

Total catch

$$C_{y,t,l}^{\text{Total}f,g} = \frac{S_{y,t,l}^{f,g} F_{y,t}^{f,g}}{Z_{y,t,l}^{g}} N_{y,t,l}^{f,g}(1-e^{-Z_{y,t,l}^{g}}) \tag{A.13}$$

Landings, discards, and total catches by fleet can be aggregated over gender (e.g., when fitting to removals reported as gender-combined). Equations A.11-13 are extended naturally for the case in which the population is represented by shell condition and/or maturity status (given the assumption that fishing mortality, retention and discard mortality depend on gender and time, but not on shell condition nor maturity status).

Landings, discards, and total catches by fleet can be reported in numbers (Equations A.11–13) or in terms of weight. For example, the landings, discards, and total catches by fleet, season, year, and gender for the total (over size-class) removals are computed as:

$$C_{y,t}^{\text{Land},g,f} = \sum_{l} C_{y,t,l}^{\text{Land},g,f}\, w_{y,l}^{g}\,;\; C_{y,t}^{\text{Disc},g,f} = \sum_{l} C_{y,t,l}^{\text{Disc},g,f}\, w_{y,l}^{g}\,;\; C_{y,t}^{\text{Total},g,f} = \sum_{l} C_{y,t,l}^{\text{Total},g,f}\, w_{y,l}^{g} \tag{A.14}$$

where $C_{y,t}^{\text{Land},g,f}$, $C_{y,t}^{\text{Disc},g,f}$, and $C_{y,t}^{\text{Total},g,f}$ are respectively the landings, discards, and total catches in weight by fleet, season, year, and gender for the total (over size-class) removals, and $w_{y,l}^{g}$ is the weight of an animal of gender *g* in size-class *l* during year *y*.

e. Selectivity / retention

Selectivity (the probability of encountering the gear) and retention (the probability of being landed given being captured) are logistic function:

$$S_l = 1 - \left(1 + \frac{exp((\bar{L}_l - S_{50})}{\sigma^S}\right))^{-1} \tag{A.15}$$

where S_{50} is the size corresponding to 50% selectivity, σ^S is the "standard deviation" of the selectivity curve, and $\bar{L}_l$ is the midpoint of size-class *l*.

It is possible to assume that selectivity for one fleet is the product of two of the selectivity patterns. This option is used to model cases in which one survey (NMFS trawl survey) is located within the footprint of another survey (BSFRF trawl survey).

The options to model retention are the same as those for selectivity, except that it is possible to estimate an asymptotic parameter, which allows discard of animals that would be "fully retained" according to the standard options for (capture) selectivity.

Selectivity and retention can be defined for blocks of contiguous years. Two blocks are used for NMFS survey selectivity (before 1982 and after 1981) due to gear modifications and two blocks are used for the directed pot fishery retention (before 2005 and after 2004) due to the fishery rationalization.

f. Growth

Growth is a key component of any size-structured model. It is modelled in terms of molt probability and the size-transition matrix (the probability of growing from each size-class to each of the other size-classes, constrained to be zero for sizes less than the current size). Note that the size-transition matrix has entries on its diagonal, which represent animals that molt but do not change size-classes.

(1) Molt probability

There are two options for modelling the probability of molting as a function of size, $P_{l,l}$:

- Constant probability (1 for females)
- Logistic probability (for males), i.e.:

$$P_{l,l} = 1 - (1 + \exp((\bar{L}_l - P_{50}) / \sigma^P))^{-1} \tag{A.16}$$

where P_{50} is the size at which the probability of molting is 0.5, and σ^S is the "standard deviation" of the molt probability function.

Molt probability is specified by gender and can change in blocks (one block before 1981 and one block after 1980 for males).

(2) Size-transition

The proportion of animals in size-class *j* that grow to be in size-class *i* ($X_{i,j}$) can be pre-specified as gamma-distributed size-increments:

$$X_{i,j} = \int_{L_j^{\text{low}}}^{L_j^{\text{hi}}} \frac{1}{\Gamma(I_i/\tilde{\beta})}((l-\bar{L}_i)/\tilde{\beta})^{(I_i/\tilde{\beta})-1} e^{-(l-\bar{L}_i)/\tilde{\beta}} dl \tag{A.17}$$

> where I_i is the 'expected' growth increment for an animal in size-class *i* (a linear function of the mid-point of size-class *i*), $\tilde{\beta}$ determines the variation in growth among individuals, and L_j^{low} and L_j^{hi} are respectively the lower and upper bounds of size-class *j*.

The size-transition matrix is specified by gender and can change in blocks (one block for males and three blocks for females (1975-1982, 1983-1993, and 1994-present based on changes in sizes at maturity).

B. Outputs, Projections and OFL Calculation

a. Core model outputs

The core model outputs are the N-matrix, the matrix of fully-selected fishing mortalities, the time-series of spawning stock biomass, mature male biomass (SSB), the values for the model parameters, and the predictions related to the observations. The spawning stock biomass (and hence mature male biomass) is defined according to:

$$SSB_y = \sum_g p^{\text{SSB},g} \sum_l N_{y,t^*,l}^g \tag{A.18}$$

where $p^{\text{SSB},g}$ is the relative contribution of gender *g* to spawning biomass ($p^{\text{SSB,mal}} = 1$; $p^{\text{SSB,fem}} = 0$ corresponds to spawning stock biomass equating to mature male biomass), and *t** is the season in which spawning takes place (spawning occurs at the start of the season).

Definition of model outputs:

1. Biomass: two population biomass measurements are used in this report: total survey biomass (crab >64 mm CL) and mature male biomass (males >119 mm CL). Mating time is assumed to Feb. 15.
2. Recruitment: new entry of number of males in the 1st seven length classes (65- 99 mm CL) and new entry of number of females in the 1st five length classes (65-89 mm CL).
3. Fishing mortality: full-selected instantaneous annual fishing mortality rate at the time of fishery.

b. Biological reference points

The key biological reference points are the proxy for F_{MSY}, the proxy for B_{MSY} and the Overfishing Level (OFL).

(1) The proxy for F_{MSY}

The specification for the proxy for F_{MSY} depends on the tier in which the stock is placed. BBRKC belongs to Tier 3, and the proxy for F_{MSY} is $F_{35\%}$, the value of a multiplier on the fully-selected fishing mortality rates for directed fisheries in the final year of the assessment such that spawning biomass-per-recruit is 35% of the unfished level. The fully-selected fishing mortality rates for non-directed fisheries are set to recent averages (recent 5 years for BBRKC). The unfished spawning biomass-per-recruit, $SSBPR(\underline{0})$, is calculated by projecting the population model forward where fishing mortality is zero for all fleets, and recruitment is constant (and ideally equal to 1). $F_{35\%}$ is then computed (using Newtons' method) such that:

$$SSBPR(\underline{\alpha}\underline{\bar{F}}) = 0.35\, SSBPR(\underline{0}) \tag{A.19}$$

where $\underline{\bar{F}}$ is the vector of recent average fully-selected fishing mortalities, and $\underline{\alpha}$ is a vector with 1 for the non-directed fisheries and a calculated constant for the directed fisheries.

(2) The proxy for B_{MSY}

The specification for the proxy for B_{MSY} depends on the tier in which the stock is placed. For stocks in Tier 4, the proxy for B_{MSY} is the average spawning stock biomass over a pre-specified number of years. For Tier 3, the proxy for B_{MSY} is $0.35\, SSBPR(\underline{0})$ multiplied by the mean recruitment over a pre-specified number of years. GMACS estimates annual recruitments by sex through estimating annual recruitment deviations and annual recruitment proportions by sex. Pre-specified numbers of years are needed in the control file for recruitment average and for mean recruitment sex ratio, respectively.

(3) Calculating the OFL

The OFL is the total catch (in weight) encountered by the gear that dies either due to being landed or due to being discarded when fully-selected fishing mortality is computed using the OFL control rule. The total catch

$$OFL = \sum_g \sum_t w^g_{y_2,l} \frac{S^{f,g}_{y_2,t,l}(\lambda^{f,g}_{y_2,t,l} + \Omega^{f,g}_{y_2,t,l}(1-\lambda^{f,g}_{y_2,t,l})\, S^{f,g}_{y_2,t,l})\alpha^{*,f}\bar{F}^{f,g}_t}{Z^g_{y_2+1,t,l}} N^{f,g}_{y_2+1,t,l}(1-e^{-Z^g_{y_2+1,t,l}}) \tag{A.20}$$

where y_2 is the final year of the assessment, $\alpha^{*,f}$ is the multiplier on average fully-selected fishing mortality for fleet f (1 for non-directed fisheries and a value computed from the OFL control rule for the directed fisheries), $\bar{F}^{f,g}_t$ is recent average fully-selected fishing mortality for fleet f and gender g during season t, and $Z^g_{y_2+1,t,l}$ is the total mortality on animals of gender g in size-class l during season t of year y_2+1:

$$Z^{g}_{y_2+1,t,l} = \rho^{\mathrm{M}}_{y_2,t} M^{g}_{y_2} \tilde{M}_l + \sum_f S^{f,g}_{y_2,t,l} (\lambda^{f,g}_{y_2,t,l} + \Omega^{f,g}_{y_2,t,l} (1-\lambda^{f,g}_{y_2,t,l})) \alpha^{*,f} \bar{F}^{f,g}_t \tag{A.21}$$

The values for entries of the vector α^* for the directed fisheries are determined using the OFL control rule:

- If the projected spawning stock biomass in year y$_2$+1 when $\underline{\alpha}^* = \underline{\alpha}$ exceeds the proxy for B_{MSY}, then $\alpha^{*,f} = \alpha^f$.
- If the projected spawning stock biomass in year y$_2$+1 when $\underline{\alpha}^* = \underline{\alpha}$ is less than 25% of the proxy for B_{MSY}, then $\alpha^{*,f} = 0$.
- If the projected spawning stock biomass in year y$_2$+1, $SSB^*_{y_2}$ when $\underline{\alpha}^* = \underline{\alpha}$ lies between less than 25% and 100% of the proxy for B_{MSY}, then $\alpha^{*,f}$ is tuned according to $\alpha^{*,f} = \frac{\alpha^f \left(\frac{SSB^*_{y_2}}{B_{MSY}} - 0.1\right)}{0.9}$ until convergence.

c. Projections

The specifications for the projections relate to:

- The duration of the projection.
- Whether the fully-selected fishing mortalities for the non-directed fisheries are set to zero or to recent averages by fleet.
- The way in which future recruitment is generated. The options available are:
  - Select a recruitment from a set of historical recruitments at random.
  - Generate a future recruitment from a Ricker stock-recruitment relationship, i.e.:

    $$R^g_y = SSB_{y-a^*} / SSB_0 e^{-1.25 \ell n h (SSB_{y-a^*}/SSB_0 - 1)} e^{\varepsilon_y - \sigma_R^2/2} \; ; \; \varepsilon_y \sim N(0;\sigma^2) \tag{A.22}$$

    where a^* is the time-lag between spawning and entering the first size-class in the model, SSB_0 is unfished spawning stock biomass, h is the steepness of the stock-recruitment relationship, σ_R is the variation in recruitment about the stock-recruitment relationship.
  - Generate a future recruitment from a Beverton-Holt stock-recruitment relationship, i.e.:

    $$R^g_y = \frac{4R_0 \, SSB_{y-a^*} / SSB_0}{(1-h) + (5h-1) SSB_{y-a^*} / SSB_0} e^{\varepsilon_y - \sigma_R^2/2} \quad \varepsilon_y \sim N(0;\sigma^2) \tag{A.23}$$

    where R_0 is unfished recruitment (i.e.. $SSB_0 / SSBPR(\underline{0})$).
- The control rule used to set fully-selected fishing mortality for the directed fisheries. The options are available
  - Pre-specified values for fully-selected fishing mortality for each fishery.
  - Pre-specified values subject to the dead catch not exceeding that corresponding to the OFL.

- Pre-specified values subject to the dead catch not exceeding that corresponding to the OFL and the landed catch not exceeding that corresponding to the State of Alaska harvest control rule.

The value for the steepness of the stock-recruitment relationship is computed such that the maximum sustainable yield occurs at $F_{35\%}$, i.e.:

$$\left.\frac{dC(\underline{F})}{dF}\right|_{\underline{F}=\underline{\alpha}^{*}\overline{F}} \tag{A.24}$$

where $C(\underline{F})$ is the equilibrium landed catch when the population model is projected forward deterministically under one of the two stock-recruitment relationships.

C. Parameter Estimation

a. *Estimating Bycatch Fishing Mortalities for Years without Observer Data*

Observer data are not available for the directed pot fishery before 1990 and the Tanner crab fishery before 1991. There are also extremely low observed bycatches in the Tanner crab fishery in 1994 and during 2006-2009. Bycatch fishing mortalities for male and females during 1975-1989 in the directed pot fishery were estimated as

$$F_t^{disc,s} = r^s\ F_t^{dir} \tag{A.25}$$

where r^s is the mean ratio of estimated bycatch discard fishing mortalities to the estimated directed pot fishing mortalities during 1990-2004 for sex s. Directed pot fishing practice has changed after 2004 due to fishery rationalization.

We used pot fishing effort (potlifts) east of 163° W in the Tanner crab fishery to estimate red king crab bycatch discard fishing mortalities in that fishery when observer data are not available (1975-1990, 1994, 2006-2009):

$$F_t^{Tanner,s} = a^s\ E_t \tag{A.26}$$

where a^s is the mean ratio of estimated Tanner crab fishery bycatch fishing mortalities to fishing efforts during 1991-1993 for sex s, and E_t is Tanner crab fishery fishing efforts east of 163° W in year t. Due to fishery rationalization after 2004, we used the data only during 1991-1993 to estimate the ratio.

b. Likelihood Components

A maximum likelihood approach was used to estimate parameters. For length compositions ($p_{l,t,s,sh}$), the likelihood functions are :

$$
\begin{gathered}
Rf = \prod_{l=1}^{L} \prod_{t=1}^{T} \coprod_{s=1}^{2} \prod_{sh=1}^{2} \frac{\left\{ exp\left[-\frac{\left(p_{l,t,s,sh} - \hat{p}_{l,t,s,sh}\right)^2}{2\sigma_{l,t,s,sh}^2} \right] + 0.01 \right\}}{\sqrt{2\pi\sigma_{l,t,s,sh}^2}} \\
\sigma_{l,t,s,sh}^2 = \frac{\left[p_{l,t,s,sh}\left(1 - p_{l,t,s,sh}\right) + \frac{0.1}{L}\right]}{n_t}
\end{gathered}
\tag{A.27}
$$

where L is the number of length groups, T the number of years, and n_t the effective sample size in year t, which was estimated for trawl survey, pot retained catch, total directed pot male catch, directed pot female discard, groundfish trawl discard, groundfish fixed gear discard, and Tanner crab fishery discard length composition data. $p_{l,t,s,sh}$ is the observed proportion of crab in length-class l, year t, sex s and shell condition sh, and $\hat{p}_{l,t,s,sh}$ is the model-estimate corresponding to $p_{l,t,s,sh}$.

The weighted negative log likelihood functions are:

$$
\begin{gathered}
Length\ compositions\text{:}\ -\sum \ln\left(Rf_i\right) \\
Catch\ and\ bycatch\ biomasses\text{:}\ \sum\left[ln\left(\frac{C_t}{\hat{C}_t}\right)^2 / \left(2\ln\left(cv_t^2 + 1\right)\right)\right] \\
NMFS\ survey\ biomass\text{:}\ \sum\left[ln\left(ln\left(CV_t^{\,2} + 1\right)\right)^{0.5} + \frac{ln\left(\frac{B_t}{\hat{B}_t}\right)^2}{\left(2ln\left(CV_t^{\,2}+1\right)\right)}\right] \\
BSFRF\ survey\ biomass\text{:}\ \sum\left[ln\left(ln\left(CV_t^{\,2} + AV^2 + 1\right)\right)^{0.5} + \frac{ln\left(\frac{B_t}{\hat{B}_t}\right)^2}{\left(2ln\left(CV_t^{\,2}+AV^2+1\right)\right)}\right] \\
R\ variation\text{:}\ \lambda_R \sum\left[ln\left(\frac{R_t}{\bar{R}}\right)^2\right] \\
R\ sex\ ratio\text{:}\ \lambda_s \sum\left[ln\left(\frac{\bar{R}_M}{\bar{R}_F}\right)^2\right] \\
Groundfish\ bycatch\ fishing\ mortalities\text{:}\ \lambda_t \sum\left[ln\left(\frac{F_{t,gf}}{\bar{F}_{gf}}\right)^2\right] \\
Pot\ female\ bycatch\ fishing\ mortalities\text{:}\ \lambda_p \sum\left[ln\left(\frac{F_{t,f}}{\bar{F}_f}\right)^2\right] \\
Trawl\ survey\ catchability\text{:}\ \frac{\left(Q - \hat{Q}\right)^2}{2\sigma^2}
\end{gathered}
\tag{A.28}
$$

where R_t is the recruitment in year t, $\bar{R}$ the mean recruitment, $\bar{R}_M$ the mean male recruitment, $\bar{R}_F$ the mean female recruitment, AV is additional CV and estimated in the model, $\bar{F}_{gf}$ the mean groundfish bycatch fishing mortality (this is separated into trawl and fixed gear fishery bycatch), $\bar{F}_f$ the mean pot female bycatch fishing mortality, Q summer trawl survey catchability, and σ the estimated standard deviation of Q (all models).

Weights λ_j are assumed to be 2 for recruitment variation, 10 for recruitment sex ratio, 0.2 for pot female bycatch fishing mortality, and 0.1 for trawl bycatch fishing mortality. These λ_j values correspond to CV values of 0.53, 0.23, 3.34, and 12.14, respectively.

c. Population State in Year 1.

The total abundance and proportions for the first year are estimated in the model.

d. Parameter estimation framework:

(1) Parameters estimated independently

Basic natural mortality, length-weight relationships, and mean growth increments per molt were estimated independently outside of the model. Mean length of recruits to the model depends on growth and was assumed to be 72.5 for both males and females. Handling mortality rates were set to 0.2 for the directed pot fishery, 0.25 for the Tanner crab fishery, 0.5 for the groundfish fixed gear fishery, and 0.8 for the groundfish trawl fishery.

i. Natural Mortality

Based on an assumed maximum age of 25 years and the 1% rule (Zheng 2005), a base M was estimated to be 0.18 for males. Model 23.0a using a strong prior (log-normal prior with a mean of 0.18 and a CV of 0.04) to estimate a base M of 0.23.

ii. Length-weight Relationship

Length-weight relationships for males and females were as follows:

$$\begin{aligned} &\text{Immature Females:} && W = 0.000408\, L^{3.127956} \\ &\text{Ovigerous Females:} && W = 0.003593\, L^{2.666076} \\ &\text{Males:} && W = 0.0004031\, L^{3.141334} \end{aligned} \qquad \text{(A.29)}$$

where W is weight in grams, and L CL in mm.

iii. Growth Increment per Molt

A variety of data are available to estimate male mean growth increment per molt for Bristol Bay RKC. Tagging studies were conducted during the 1950s, 1960s and 1990s, and mean growth increment per molt data from these tagging studies in the 1950s and 1960s were analyzed by Weber and Miyahara (1962) and Balsiger (1974). Modal analyses were conducted for the data during 1957-1961 and the 1990s (Weber 1967; Loher et al. 2001). Mean growth increment per molt may be a function of body size and shell condition and vary over time (Balsiger 1974; McCaughran and Powell 1977); however, for simplicity, mean growth increment per molt was assumed to be only a function of body size in the models. Tagging data were used to estimate mean growth increment per molt as a function of pre-molt length for males (Figure A2). The results from modal analyses of 1957-1961 and the 1990s were used to estimate mean growth increment per molt for immature females during 1975-1993 and 1994-2020, respectively, and the data presented in Gray (1963) were used to estimate those for mature females for model scenarios (Figure A2). To make a smooth transition of growth increment per molt from immature to mature females, weighted growth increment averages of 70% and 30% at 92.5 mm CL pre-molt length and 90% and 10% at 97.5 mm CL were used, respectively, for mature and immature females during 1983-1993. These percentages are roughly close to the composition of maturity.

During 1975-1982, females matured at a smaller size, so the growth increment per molt as a function of length was shifted to smaller increments. Likewise, during 1994-2021, females matured at a slightly higher size, so the growth increment per molt was shifted to high increments for immature crab (Figure A2). Once mature, the growth increment per molt for male crab decreases slightly and annual molting probability decreases, whereas the growth increment for female crab decreases dramatically but annual molting probability remains constant at 1.0 (Powell 1967).

iv. Sizes at Maturity for Females

The NMFS collected female reproductive condition data during the summer trawl surveys. Mature females are separated from immature females by a presence of egg clutches or egg cases. Proportions of mature females at 5-mm length intervals were summarized and a logistic curve was fitted to the data each year to estimate sizes at 50% maturity. Sizes at 50% maturity are illustrated in Figure A3 with mean values for three different periods (1975-82, 1983-93, and 1994-2021).

v. Sizes at Maturity for Males

Although size at sexual maturity for Bristol Bay red king crab males has been estimated (Paul et al. 1991), there are no data for estimating size of functional maturity collected in the natural environment. Sizes at functional maturity for Bristol Bay male RKC have been assumed to be 120 mm CL (Schmidt and Pengilly 1990). This is based on mating pair data collected off Kodiak Island (Figure A4). Sizes at maturity for Bristol Bay female RKC are about 90 mm CL, about 15 mm CL less than Kodiak female RKC (Pengilly et al. 2002). The size ratio of mature males to females is 1.3333 at sizes at maturity for Bristol Bay RKC, and since mature males grow at much larger increments than mature females, the mean size ratio of mature males to females is most likely larger than this ratio. Size ratios of the large majority of Kodiak mating pairs were less than 1.3333, and in some bays, only a small proportion of mating pairs had size ratios above 1.3333 (Figure A4).

In the laboratory, male RKC as small as 80 mm CL from Kodiak and Southeast Alaska can successfully mate with females (Paul and Paul 1990). But few males less than 100 mm CL were observed to mate with females in the wild. Based on the size ratios of males to females in the Kodiak mating pair data, setting 120 mm CL as a minimum size of functional maturity for Bristol Bay male RKC is proper in terms of managing the fishery.

vi. Potential Reasons for High Mortality during the Early 1980s

Bristol Bay red king crab abundance had declined sharply during the early 1980s. Many factors have been speculated for this decline: (i) completely wiped out by fishing: the directed pot fishery, the other directed pot fishery (Tanner crab fishery), and bottom trawling; and (ii) high fishing and natural mortality. With the survey abundance, harvest rates in 1980 and 1981 were among the highest, thus the directed fishing definitely had a big impact on the stock decline, especially legal and mature males. However, for the sharp decline during 1980-1984 for males, 3 out of 5 years had low mature harvest rates. During the 1981-1984 decline for females, 3 out of 4 years had low mature harvest rates. Also pot catchability for females and immature males are generally much lower than for legal males, so the directed pot fishing alone cannot explain the sharp decline for all segments of the stock during the early 1980s.

Red king crab bycatch in the eastern Bering Sea Tanner crab fishery is another potential factor (Griffin et al. 1983). The main overlap between Tanner crab and Bristol Bay red king crab is east of 163° W. No absolute red king crab bycatch estimates are available until 1991. So there are insufficient data to fully evaluate the impact. Tanner crab retained catch and potlifts from the eastern Bering Sea Tanner crab fishery are illustrated in Figure A5. The observed red king crab bycatch in the Tanner crab fishery during 1991-2015 and total potlifts east of 166° W during 1975 to 2015 were used to estimate the bycatch mortality in the current model. Because winter sea surface temperatures and air temperatures were warmer (which means a lower handling mortality rate) and there were fewer potlifts during the early 1980s than during the early 1990s, bycatch in the Tanner crab fishery is unlikely to have been a main factor for the sharp decline of Bristol Bay red king crab.

Several factors may have caused increases in natural mortality. Crab diseases in the early 1980s were documented by Sparks and Morado (1985), but inadequate data were collected to examine their effects on the stock. Stevens (1990) speculated that senescence may be a factor because many crab in the early 1980s were very old due to low temperatures in the 1960s and early 1970s. The biomass of the main crab predator, Pacific cod, increased about 10 times during the late 1970s and early 1980s. Yellowfin sole biomass also increased substantially during this period. Predation is primarily on juvenile and molting/softshell crab. But we lack stomach samples in shallow waters (juvenile habitat) and during the period when red king crab molt. Also cannibalism occurs during molting periods for red king crab. High crab abundance in the late 1970s and early 1980s may have increased the occurrence of cannibalism.

Overall, the likely causes for the sharp decline in the early 1980s are combinations of the above factors, such as pot fisheries on legal males, bycatch, and predation on females and juvenile and sublegal males, senescence for older crab, and disease for all crab. In our model, we estimated one mortality parameter for males and another for females during 1980-1984. We also estimated a mortality parameter for females during 1976-1979 and 1985-1993. These three mortality parameters are additional to the basic natural mortality of 0.18yr^{-1}, all directed fishing mortality, and non-directed fishing mortality. These three mortality parameters could be attributed to natural mortality as well as undocumented non-directed fishing mortality. The model fit the data much better with these three parameters than without them.

(2) Parameters estimated conditionally

The following model parameters were estimated for male and female crab: total recruits for each year (year class strength R_t for t = 1975 to 2020), total abundance in the first year (1975), growth parameter β, and recruitment parameter β_r for males and females separately. Molting probability parameters β and L_{50} were also estimated for male crab. Estimated parameters also include different sets of β and L_{50} for total selectivity and retained proportions, β and L_{50} for pot-discarded female selectivity, β and L_{50} for pot-discarded male and female selectivities from the eastern Bering Sea Tanner crab fishery, β

and L_{50} for groundfish trawl and fixed gear discarded selectivities, and different sets of β and L_{50} for NMFS trawl survey male and female selectivities separately. The NMFS survey catchabilities Q for some models were also estimated. Different sets of β and L_{50} for selectivity parameters were estimated for the survey data from the Bering Fisheries Research Foundation. Annual fishing mortalities were also estimated for the directed pot fishery for males (1975-2020), pot-discarded females from the directed fishery (1990-2020), pot-discarded males and females from the eastern Bering Sea Tanner crab fishery (1991-93, 2013-15), groundfish trawl discarded males and females (1976-2020), and groundfish fixed gear discarded males and females (1996-2020). One additional mortality parameter for years 1980-1984 for males and a constant to multiply male natural mortality for estimating female natural mortality were also estimated. Some estimated parameters were constrained in the model. For example, male and female recruitment estimates were forced to be close to each other for a given year.

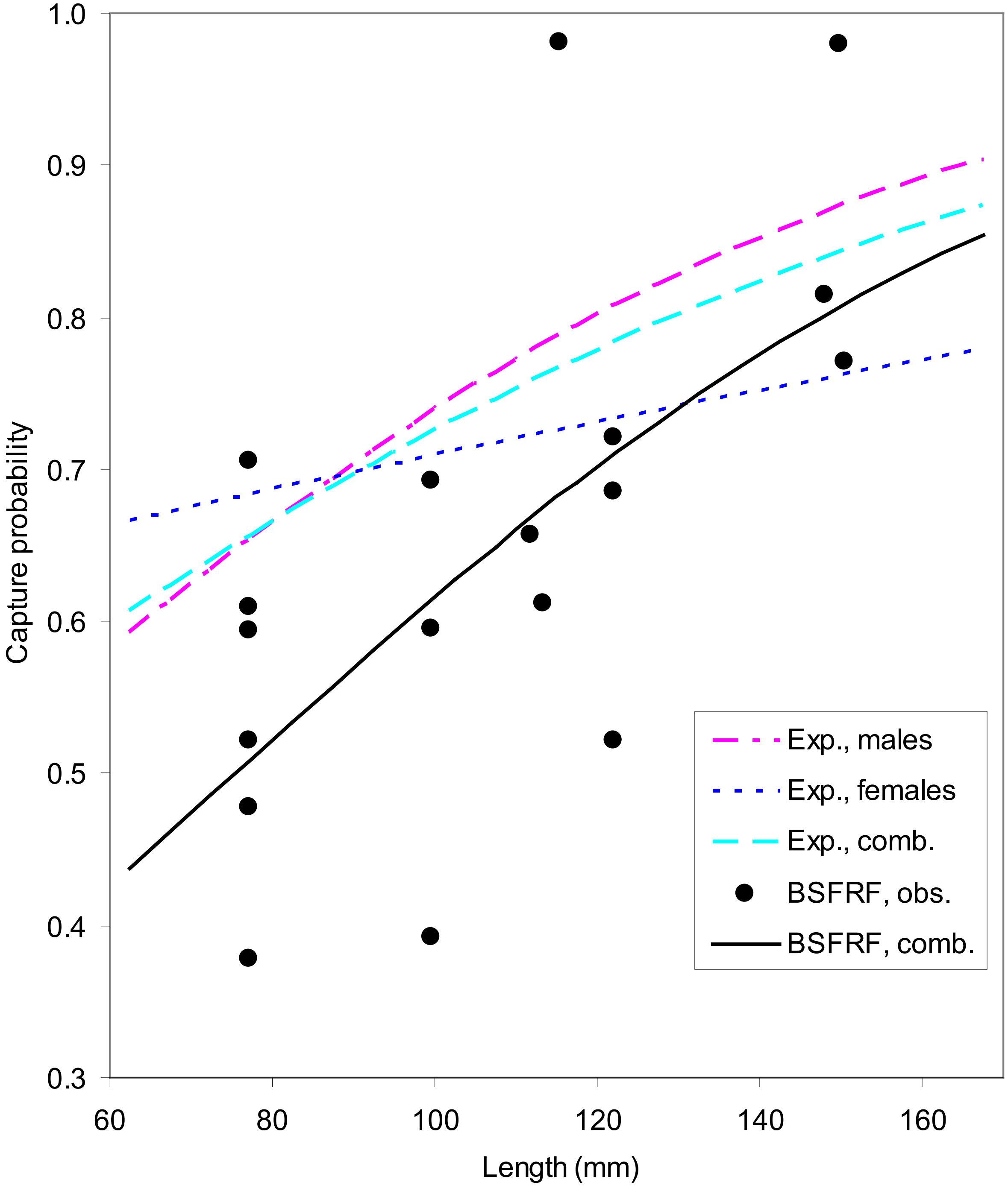

Figure A1. Estimated capture probabilities for NMFS Bristol Bay red king crab trawl surveys by Weinberg et al. (2004) and the Bering Sea Fisheries Research Foundation surveys.

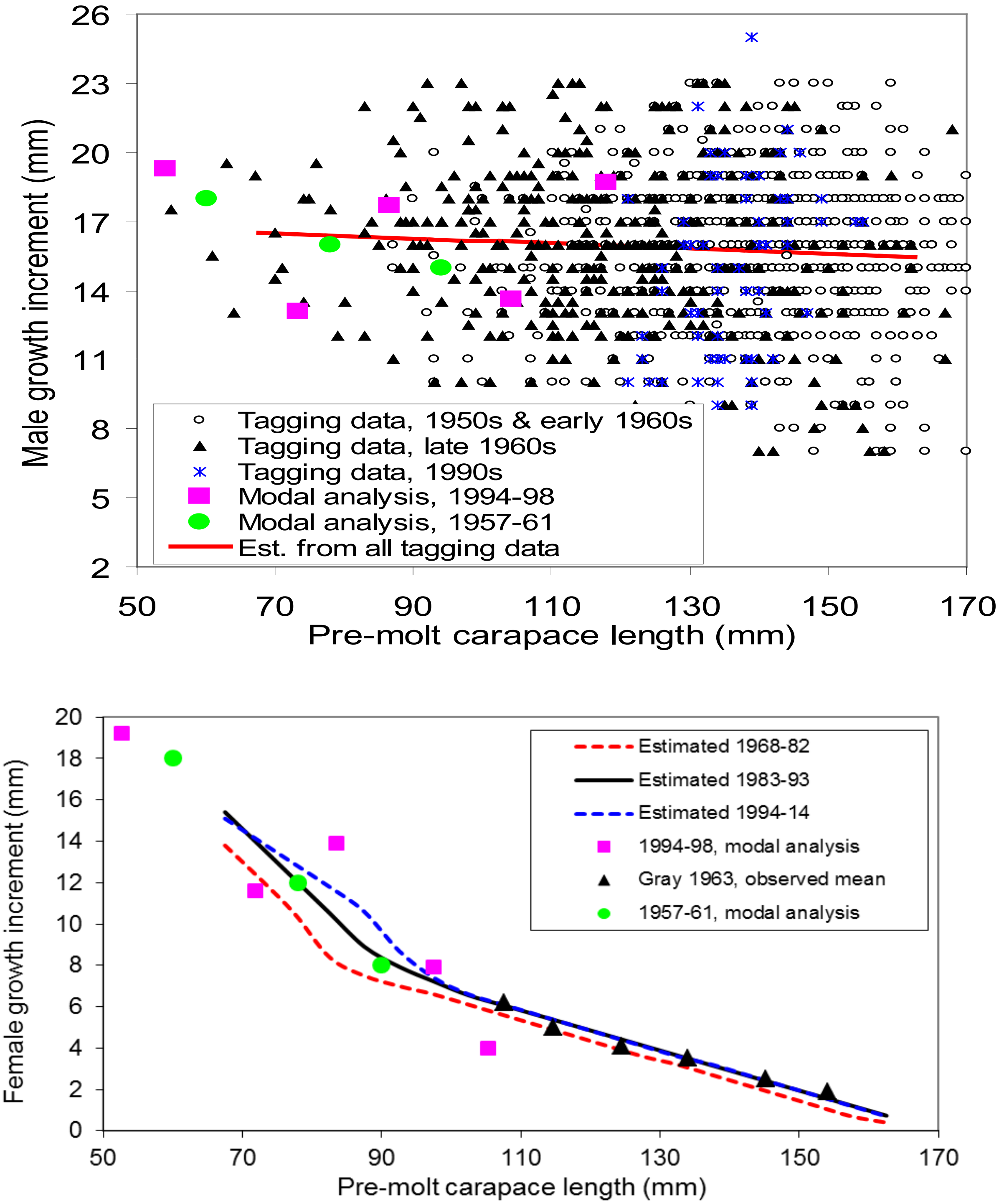

Figure A2. Mean growth increments per molt for Bristol Bay red king crab. Note: "tagging"---based on tagging data; "mode"---based on modal analysis. The female growth increments per molt are for different model scenarios.

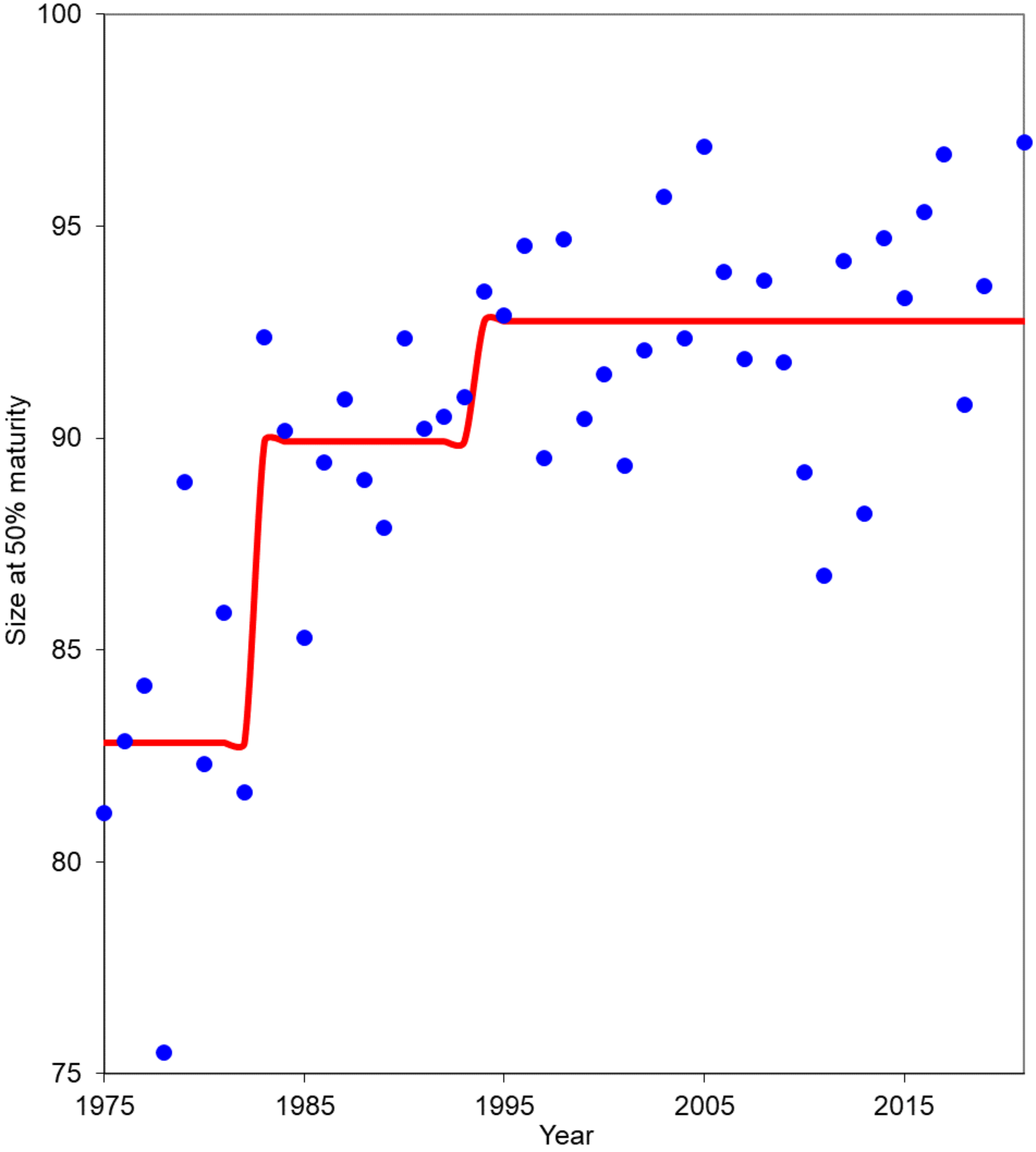

Figure A3. Estimated sizes at 50% maturity for Bristol Bay female red king crab from 1975 to 2021. Averages for three periods (1975-82, 1983-93, and 1994-2021) are plotted with a line.

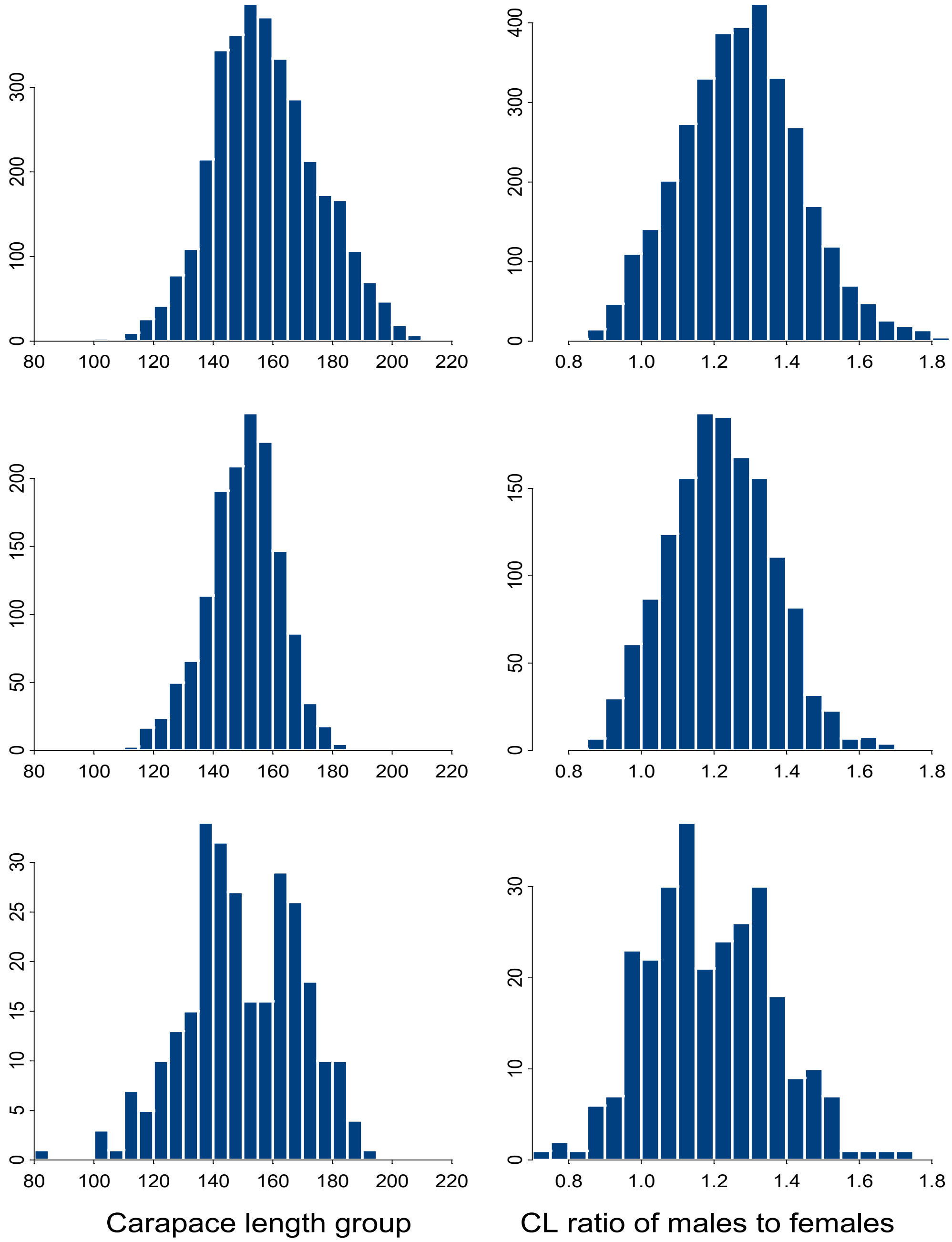

Figure A4. Histograms of carapace lengths (CL) and CL ratios of males to females for male shell ages ≤13 months of red king crab males in grasping pairs; Powell's Kodiak data. Upper plot: all locations and years pooled; middle plot: location 11; lower plot: locations 4 and 13. Sizes at maturity for Kodiak red king crab are about 15 mm larger than those for Bristol Bay red king crab. (Doug Pengilly, ADF&G, pers. comm.).

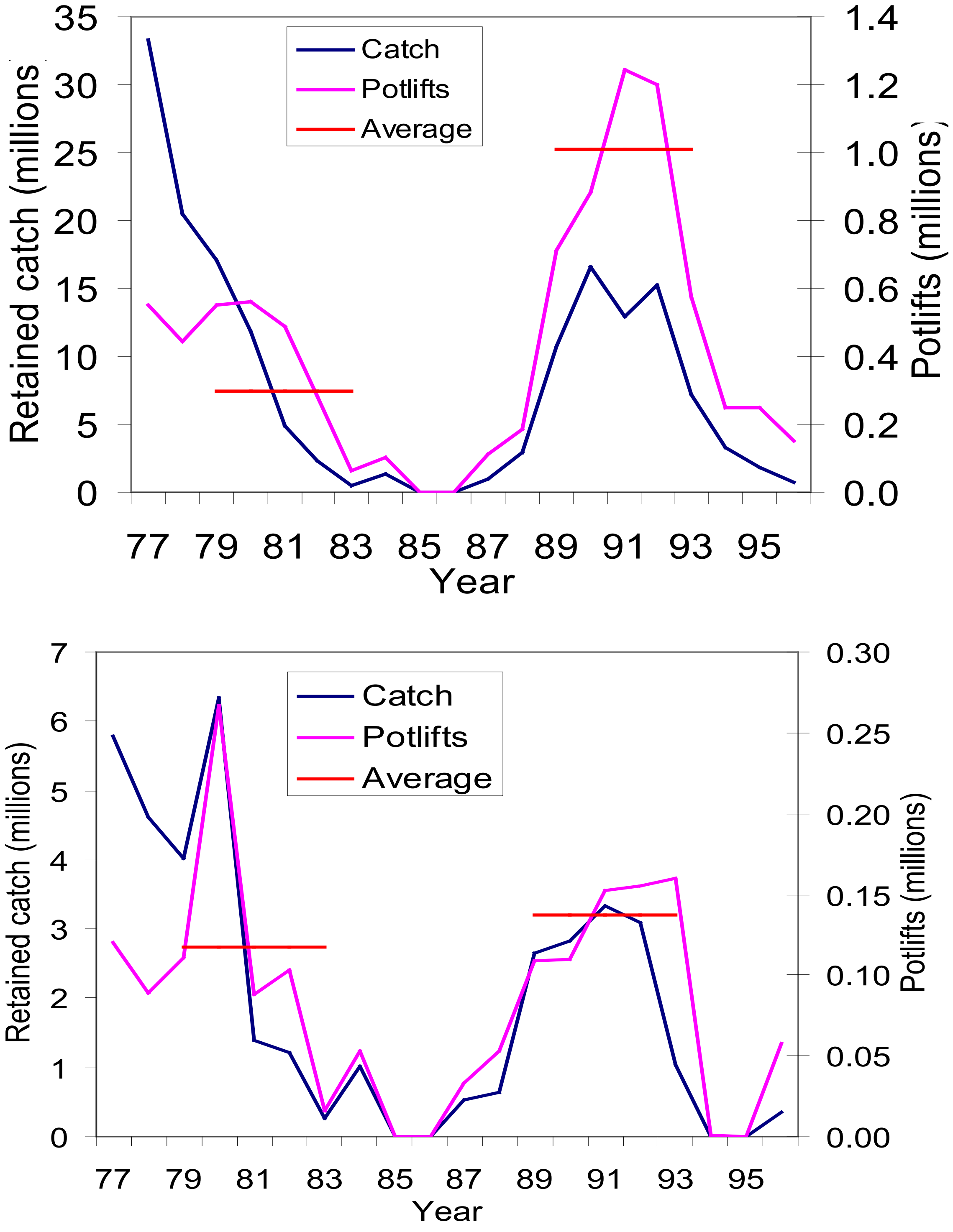

Figure A5. Tanner crab retained catch and potlifts for total eastern Bering Sea Tanner crab fishery (upper plot) and the Tanner crab fishery east of 163° W (bottom).

Appendix B. Data Files for Model 21.1b

See pdf posted.

Appendix C. Summary of the CIE Review of BBRKC in 2021

The virtual CIE review of the stock assessments for Bristol Bay red king crab and eastern Bering Sea snow crab was held online during March 22-26, 2021. The review was conducted by three independent experts: Drs. Yong Chen, Nick Caputi, and Billy Ernst. The review reports are at the end of this SAFE report. The followings are a brief summary of recommendations and plan to address these recommendations.

1. *Identifying the possible sources of the large retrospective patterns and/or develop alternative approaches to provide catch advice if retrospective patterns persistent and biased errors are too large for the assessments to be considered reliable. Conducted more studies to identify temporal trends and/or time blocks of parameters, such as natural mortality and survey catchability, to be incorporated in future stock assessments.*

   Reply: Temporal changes in parameters may play an important role for the large retrospective patterns, and some data conflict between NMFS surveys and 2007-2008 and 2013-2016 BSFRF surveys also contributes to them. We used model 21.2 to add another time block (2018-2019) of natural mortality. The Mohn's rho value for mature male biomass decreases from 0.347 for model 19.3d to 0.223 for model 21.2. We will further examine the retrospective patterns and develop alternative model scenarios to reduce the retrospective patterns for the CPT meeting in January/May 2022. Potential changes in natural mortality over time play a big role for the large retrospective patterns during recent years, and additional time blocks of parameters for recent years will be further evaluated.

2. *Survey performance/efficacy and selectivity curve evaluations in term of changes in distributions over time, and the stock area evaluation.*

   Reply: We would like to examine red king crab north of the management area of Bristol Bay sometime in the future to see whether they are part of the BBRKC stock. Hopefully, a tagging study can be conducted to examine the link between red king crab in these two areas. We have not seen the need for evaluating different kinds of survey selectivity curves now since large-size crab are generally inside the survey area. Some limited genetic and larval transport studies were conducted on the stock area in the past.

3. *Surveying the red king crab juvenile crab abundance in nearshore locations may provide an estimate of younger juvenile abundance where the year-class is better defined.*

Reply: We second this and have advocated this for a long time.

4. *Examining VAST results on effects on the stock assessment model.*

   Reply: We will continue to use VAST results as a model scenario to compare it to the other model scenarios.

5. *Evaluating commercial catch, effort, and CPUE for crab distributions, fishery performance, and population abundance relative to the trawl survey results and on impacts on survey timing and survey availability, and standardizing the CPUE for improvement, and conducting a depletion analysis.*

   Reply: Catch and bycatch are used in the model, the commercial CPUE is used to compare to the survey legal male abundance but not in the model, and fishing distributions and CPUE are often examined by ADF&G. The fishing season has been very short in the most years, so the depletion analysis may not be much useful. Trawl surveys generally cover all red king crab distribution areas except for nearshore areas. We just started CPUE standardization work and will try to incorporate the standardized CPUE in the assessment model in 2022.

6. *Extending estimates of sizes-at-50% maturity for females and examining the impacts of changes on mature female biomass estimates. Conducting a sensitivity study to examine impacts of changes at sizes-at-maturity for males on mature male biomass estimates.*

   Reply: We will update the estimates of sizes-at-50% maturity for females. Since the harvest strategy defines the sizes of mature females and males and the growth increments of males is not affected by changes in sizes-at-maturity, impacts of changes at sizes-at-maturity for males on mature male biomass estimates do not occur for the harvest strategy. The current defined size-at-maturity for males is for functional maturity and is much larger than the physiological mature sizes.

7. *A model run just using data from 1985 to avoid high natural mortality during the early 1980s.*

   Reply: We have planned to do this in 2022.

8. *Examining biological, environmental, and vessel performance data on the 2014 NMFS trawl survey to assess the survey abundance outlier and conducting a sensitivity study without the 2014 NMFS trawl survey data.*

Reply: During the CIE review, we conducted this sensitivity study. The NMFS and BSFRF have examined biological, environmental, and vessel performance data on the 2014 NMFS trawl survey extensively. It is unlikely that we would drop this data point in the stock assessments since there are several data points in the survey time series that are as unexpected as the 2014 data.

9. *Important to continue environmental SAFE reports.*

   Reply: We agree and hopefully it will be updated annually.

10. *Besides overfished and overfishing, using MMB, recruitment, trends in commercial catch and CPUE, legal-size abundance and total survey biomass, and the projections and near future outlook to summarize the stock status.*

    Reply: We will add these in our summary of the stock status.

11. *Modeling double bag experiment and BSFRF side-by-side survey data to improve the catchability prior.*

    Reply: This is a good suggestion. However, we do not use BSFRF side-by-side survey data to estimate the NMFS trawl catchability prior because we do not want to use these data twice since they are used in the model already.

12. *Conducting new tagging study to update the outdated tagging/return data used in the assessments.*

    Reply: We agree with this recommendation. Hopefully, tagging study will be conducted for BBRKC in the future.