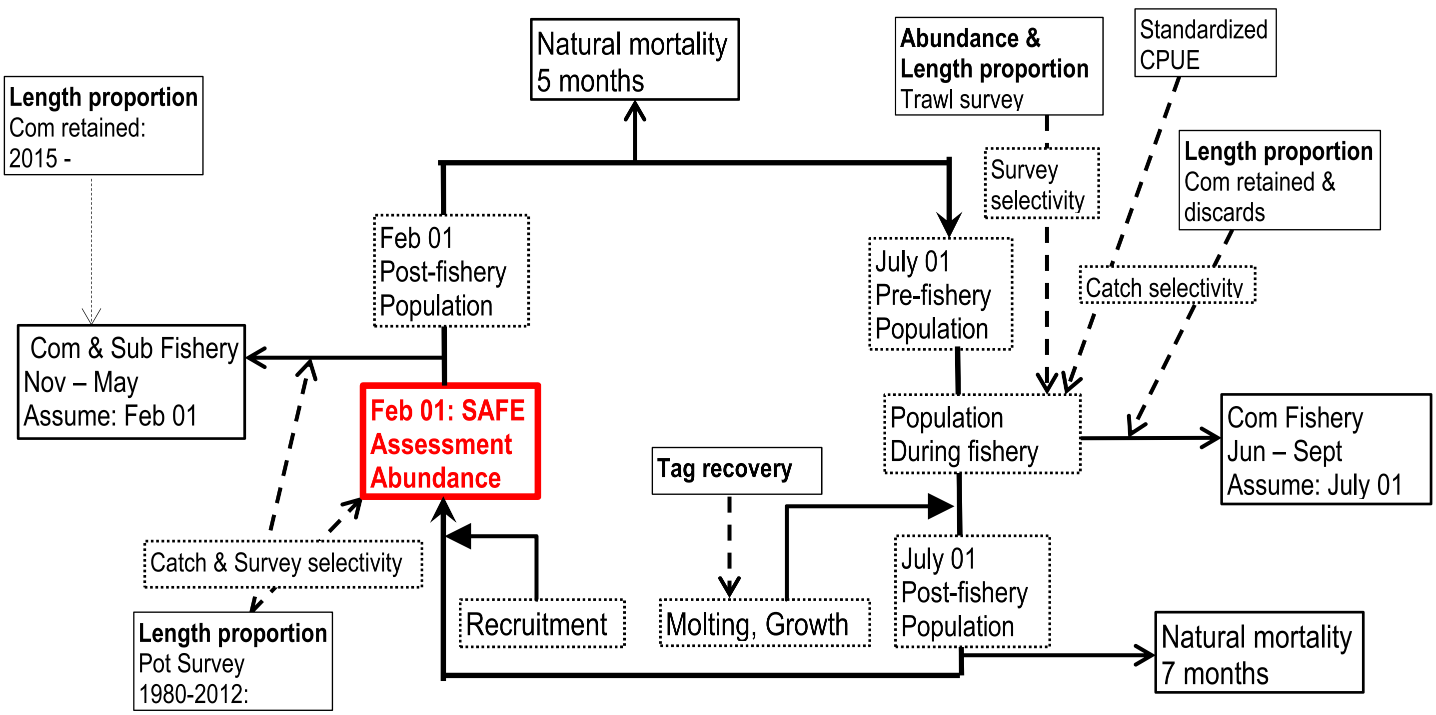
Appendix A. Description of the Norton Sound Red King Crab Model

a. Model description.

The model is an extension of the length-based model developed by Zheng et al. (1998) for Norton Sound red king crab. The model has 8 male length classes with model parameters estimated by the maximum likelihood method. The model estimates abundances of crab with CL ≥64 mm and with 10-mm length intervals (8 length classes, ≥134mm) because few crab measuring less than 64 mm CL were caught during surveys or fisheries and there were relatively small sample sizes for trawl and winter pot surveys. The model treats newshell and oldshell male crab separately but assumes they have the same molting probability and natural mortality.

Norton Sound Red King Crab Modeling Scheme



Timeline of calendar events and crab modeling events:

- **Model year starts February 1st to January 31st of the following year.**
- **All winter fishery harvest occurs on February 1st**
- **Molting and recruitment occur on July 1st**
- **Initial Population Date: February 1st 1976**

Initial pre-fishery summer crab abundance on February 1st 1976

Abundance of the initial pre-fishery population was assumed to consist of newshell crab to reduce the number of parameters, and estimated as

$$N_{l,1} = p_l e^{\log\_N_{76}} \quad (1)$$

where, length proportion of the first year (p_l) was calculated as

$$p_l = \frac{\exp(a_l)}{1 + \sum_{l=1}^{n-1} \exp(a_l)} \text{ for } l = 1,..,n\text{-}1$$
$$p_n = 1 - \frac{\sum_{l=1}^{n-1} \exp(a_l)}{1 + \sum_{l=1}^{n-1} \exp(a_l)} \quad (2)$$

for model estimated parameters a_l.

Crab abundance on July 1st

Summer (01 July) crab abundance of new and oldshells consists of survivors of winter commercial and subsistence crab fisheries and natural mortality from 01Feb to 01July:

$$N_{s,l,t} = (N_{w,l,t-1} - C_{w,t-1} P_{w,n,l,t-1} - C_{p,t} P_{p,n,l,t-1} - D_{w,n,l,t-1} - D_{p,n,l,t-1})e^{-0.42M_l}$$
$$O_{s,l,t} = (O_{w,l,t-1} - C_{w,t-1} P_{w,o,l,t-1} - C_{p,t} P_{p,o,l,t-1} - D_{w,o,l,t-1} - D_{p,o,l,t-1})e^{-0.42M_l} \quad (3)$$

where

$N_{s,l,t}$, $O_{s,l,t}$: summer abundances of newshell and oldshell crab in length class l in year t ,
$N_{w,l,t-1}$, $O_{w,l,t-1}$: winter abundances of newshell and oldshell crab in length class l in year $t-1$,
$C_{w,t-1}$, $C_{p,t-1}$: total winter commercial and subsistence catches in year $t-1$,
$P_{w,n,l,t-1}$, $P_{w,o,l,t-1}$: Proportion of newshell and oldshell length class l crab in year $t-1$, harvested by winter commercial fishery,
$P_{p,n,l,t-1}$, $P_{p,o,l,t-1}$: Proportion of newshell and oldshell length class l crab in year $t-1$, harvested by winter subsistence fishery,
$D_{w,n,l,t-1}$, $D_{w,o,l,t-1}$: Discard mortality of newshell and oldshell length class l crab in winter commercial fishery in year $t-1$,

$D_{p,n,l,t-1}$, $D_{p,o,l,t-1}$: Discard mortality of newshell and oldshell length class *l* crab in winter subsistence fishery in year *t-1*,
M_l : instantaneous natural mortality in length class *l*,
0.42 : proportion of the year from Feb 1 to July 1 is 5 months.

Length proportion compositions of winter commercial catch ($P_{w,n,l,t}$, $P_{w,o,l,t}$) in year *t* were estimated as:

$$
\begin{aligned}
P_{w,n,l,t} &= N_{w,l,t} S_{w,l} P_{lg,l} / \sum_{l=1} [(N_{w,l,t} + O_{w,l,t}) S_{w,l} P_{lg,l}] \\
P_{w,o,l,t} &= O_{w,l,t} S_{w,l} P_{lg,l} / \sum_{l=1} [(N_{w,l,t} + O_{w,l,t}) S_{w,l} P_{lg,l}]
\end{aligned} \tag{4}
$$

where
$P_{lg,l}$: the proportion of legal males in length class *l* ,
$S_{w,l}$: Selectivity of winter fishery pot.

Subsistence fishery does not have a size limit; however, crab of size smaller than length class 3 are generally not retained. Hence, we assumed proportion of length composition *l* = 1 and 2 as 0, and estimated length compositions ($l \geq 3$) as follows

$$
\begin{aligned}
P_{p,n,l,t} &= N_{w,l,t} S_{w,l} / \sum_{l=3} [(N_{w,l,t} + O_{w,l,t}) S_{w,l}] \\
P_{p,o,l,t} &= O_{w,l,t} S_{w,l} / \sum_{l=3} [(N_{w,l,t} + O_{w,l,t}) S_{w,l}]
\end{aligned} \tag{5}
$$

Crab abundance on Feb 1st

Newshell Crab: Abundance of newshell crab of year *t* and length-class *l* ($N_{w,l,t}$) year-t consist of: (1) new and oldshell crab that survived the summer commercial fishery and molted, and (2) recruitment ($R_{l,t}$) .

$$
N_{w,l,t} = \sum_{l'=1}^{l'=l} G_{l',l} [(N_{s,l',t-1} + O_{s,l',t-1}) e^{-y_c M_l} - C_{s,t} (P_{s,n,l',t-1} + P_{s,o,l',t-1}) - D_{l',t-1}] m_{l'} e^{-(0.58-y_c) M_l} + R_{l,t} \tag{6}
$$

Oldshell Crab: Abundance of oldshell crabs of year *t* and length-class *l* ($O_{w,l,t}$) consists of the non-molting portion of survivors from the summer fishery:

$$
O_{w,l,t} = [(N_{s,l,t-1} + O_{s,l,t-1}) e^{-y_c M_l} - C_{s,t} (P_{s,n,l,t-1} + P_{s,o,l,t-1}) - D_{l,t-1}] (1 - m_l) e^{-(0.58-y_c) M_l} \tag{7}
$$

where

$G_{l',l}$: a growth matrix representing the expected proportion of crabs growing from length class l' to length class l
$C_{s,t}$: total summer catch in year t
$P_{s,n,l,t}$, $P_{s,o,l,t}$: proportion of summer catch for newshell and oldshell crabs of length class l in year t,
$D_{l,t}$: summer discard mortality of length class l in year t,
m_l : molting probability of length class l,
y_c : the time in year from July 1 to the mid-point of the summer fishery,
0.58: Proportion of the year from July 1st to Feb 1st is 7 months is 0.58 year,
$R_{l,t}$: recruitment into length class l in year t.

Discards

Discards are crabs that were caught by fisheries but were not retained, which consists of summer commercial, winter commercial and winter subsistence.

Summer and winter commercial discards

In summer ($D_{l,t}$) and winter ($D_{w,n,l,t}$, $D_{w,o,l,t}$) commercial fisheries, sublegal males (<4.75 inch CW and <5.0 inch CW since 2005) are discarded. Those discarded crabs are subject to handling mortality. The number of discards was not directly observed, and thus was estimated from the model as: Observed Catch x (estimated abundance of crab that are not caught by commercial pot)/(estimated abundance of crab that are caught by commercial pot)

Model discard mortality in length-class l in year t from the summer and winter commercial pot fisheries is given by

$$D_{l,t} = C_{s,t}\frac{(N_{s,l,t}+O_{s,l,t})\,S_{s,l}\,(1-P_{lg,l})}{\sum_l (N_{s,l,t}+O_{s,l,t})\,S_{s,l}\,P_{lg,l}}hm_s \quad \text{(Baseline model)}$$

$$D_{l,t} = C_{s,t}\frac{(N_{s,l,t}+O_{s,l,t})S_{s,l}\,(1-S_{r,l})}{\sum_l (N_{s,l,t}+O_{s,l,t})\,S_{s,l}S_{r,l}}hm_s \quad \text{(Alternative model)} \qquad (8)$$

$$D_{w,n,l,t} = C_{w,t}\frac{N_{w,l,t}\,S_{w,l}\,(1-P_{lg,l})}{\sum_l (N_{w,l,t}+O_{w,l,t})\,S_{w,l}\,P_{lg,l}}hm_w \qquad (9)$$

$$D_{w,o,l,t} = C_{w,t}\frac{O_{w,l,t}\,S_{w,l}\,(1-P_{lg,l})}{\sum_l (N_{w,l,t}+O_{w,l,t})\,S_{w,l}\,P_{lg,l}}hm_w \qquad (10)$$

where

hm_s: summer commercial handling mortality rate assumed to be 0.2,
hm_w: winter commercial handling mortality rate assumed to be 0.2,
$S_{s,l}$: Selectivity of the summer commercial fishery,

$S_{w,l}$: Selectivity of the winter commercial fishery,
$S_{r,l}$: Retention selectivity of the summer commercial fishery,

Winter subsistence Discards

Discards (unretained) of winter subsistence fishery is reported in a permit survey ($C_{d,t}$), though its size composition is unknown. We assumed that subsistence fishers discarded all crabs of length classes 1 -2.

$$D_{p,n,l,t} = C_{d,t} \frac{N_{w,l,t} S_{w,l}}{\sum_{l=1}^{2}(N_{w,l,t} + O_{w,l,t}) S_{w,l}} hm_w \tag{11}$$

$$D_{p,o,l,t} = C_{d,t} \frac{O_{w,l,t} S_{w,l}}{\sum_{l=1}^{2}(N_{w,l,t} + O_{w,l,t}) S_{w,l}} hm_w \tag{12}$$

$C_{d,t}$: Winter subsistence discards catch,

Recruitment

Recruitment of year t, R_t, is a stochastic process around the geometric mean, R_0:

$$R_t = R_0 e^{\tau_t}, \tau_t \sim N(0, \sigma_R^2) \tag{13}$$

R_t of the last year was assumed to be an average of previous 5 years: $R_t = (R_{t-1} + R_{t-2} + R_{t-3} + R_{t-4} + R_{t-5})/5$.

R_t was assumed to be newshell crab of immature (< 94mm) length classes 1 to r:

$$R_{r,t} = p_r R_t \tag{14}$$

where r takes multinomial distribution, same as the equation (2)

Molting Probability

Molting probability for length class l, m_l, was estimated as an inverse logistic function of length-class mid carapace length (L) and parameters (α, β) where β corresponds to L_{50}.

$$m_l = \frac{1}{1+e^{\alpha(L-\beta)}} \tag{15}$$

Trawl net, summer commercial pot, retention selectivity

Trawl and summer commercial pot selectivity was assumed to be a logistic function of mid-length-class, constrained to be 0.999 at the largest length-class (L_{max}):

$$S_l = \frac{1}{1+e^{(\alpha(L_{\max}-L)+\ln(1/0.999-1))}} \tag{16}$$

Alternative Summer commercial pot, retention selectivity

Summer pot selectivity was assumed to be a logistic function of length-class mid carapace length (L) and parameters (α, β) where β corresponds to L_{50}.

$$S_{c,l} = \frac{1}{1+e^{-\alpha(L-\beta)}} \tag{16'}$$

Winter pot selectivity

Winter pot selectivity was assumed to be a dome-shaped with inverse logistic function of length-class mid carapace length (L) and parameters (α, β) where β corresponds to L_{50}.

$$S_{w,l} = \frac{1}{1+e^{\alpha(L-\beta)}} \tag{17}$$

Selectivity of the length classes $S_{w,s}$ (S= l_1, l_2) were individually estimated.

Growth transition matrix

The growth matrix $G_{l',l}$ (the expected proportion of crab molting from length class l' to length class l) was assumed to be normally distributed:

$$G_{l',l} = \begin{cases} \dfrac{\int_{lm_l-h}^{lm_l+h} N(L\,|\,\mu_{l'},\sigma^2)dL}{\sum_{l=1}^{n}\int_{lm_l-h}^{lm_l+h} N(L\,|\,\mu_{l'},\sigma^2)dL} & \text{when } l \geq l' \\ 0 & \text{when } l < l' \end{cases} \tag{18}$$

Where

$$N(x \mid \mu_{l'}, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(L-\mu_{l'})^2}{\sigma^2} \right)$$

$$lm_l = L_1 + st \cdot l$$

$$\mu_l = L_1 + \beta_0 + \beta_1 \cdot l$$

Observation model

Summer trawl survey abundance

Modeled trawl survey abundance of year t ($B_{st,t}$) is July 1st abundance subtracted by summer commercial fishery harvest occurring from July 1st to the mid-point of summer trawl survey, multiplied by natural mortality occurring between the mid-point of commercial fishery date and trawl survey date, and multiplied by trawl survey selectivity. For the first year (1976) trawl survey, the commercial fishery did not occur.

$$\hat{B}_{st,t} = \sum_l [(N_{s,l,t} + O_{s,l,t}) e^{-y_c M_l} - C_{s,t} P_{c,t} (P_{s,n,l,t} + P_{s,o,l,t})] e^{-(y_{st} - y_c) M_l} S_{st,l} \quad (19)$$

where
y_{st} : the time in year from July 1 to the mid-point of the summer trawl survey,
y_c: the time in year from July 1 to the mid-point for the catch before the survey, ($y_{st} > y_c$: Trawl survey starts after opening of commercial fisheries),
$P_{c,t}$: the proportion of summer commercial crab harvested before the mid-point of trawl survey date.
$S_{st,l}$: Selectivity of the trawl survey.

Winter pot survey CPUE

Winter pot survey cpue (f_{wt}) was calculated with catchability coefficient q and exploitable abundance:

$$\hat{f}_{wt} = q_w \sum_l [(N_{w,l,t} + O_{w,l,t}) S_{w,l}] \quad (20)$$

Summer commercial CPUE

Summer commercial fishing CPUE (f_t) was calculated as a product of catchability coefficient q and mean exploitable abundance minus one half of summer catch, A_t:

$$\hat{f}_t = q_i (A_t - 0.5 C_t) \quad (21)$$

Because the fishing fleet and pot limit configuration changed in 1993, q_1 is for fishing efforts before

1993, q_2 is from 1994 to present.

Baseline model

Where A_t is exploitable legal abundance in year *t*, estimated as

$$A_t = \sum_l [(N_{s,l,t} + O_{s,l,t}) S_{s,l} P_{lg,l}] \text{ (Baseline model)}$$
$$A_t = \sum_l [(N_{s,l,t} + O_{s,l,t}) S_{s,l} S_{r,l}] \text{ (Alternative model)} \quad (22)$$

Summer pot survey abundance (Removed from likelihood components)
Abundance of *t*-th year pot survey was estimated as

$$\hat{B}_{p,t} = \sum_l [(N_{s,l,t} + O_{s,l,t}) e^{-y_p M_l}] S_{p,l} \quad (23)$$

Where
y_p : the time in year from July 1 to the mid-point of the summer pot survey.
Length composition

Summer commercial catch

Length compositions of the summer commercial catch for new and old shell crabs $P_{s,n,l,t}$ and $P_{s,o,l,t}$, were modeled based on the summer population, selectivity, and legal abundance:

$$\hat{P}_{s,n,l,t} = N_{s,l,t} S_{s,l} P_{lg,l} / A_t$$
$$\hat{P}_{s,o,l,t} = O_{s,l,t} S_{s,l} P_{lg,l} / A_t \quad \text{(Baseline model)}$$
$$\hat{P}_{s,n,l,t} = N_{s,l,t} S_{s,l} S_{r,l} / A_t$$
$$\hat{P}_{s,o,l,t} = O_{s,l,t} S_{s,l} S_{r,l} / A_t \quad \text{(Alternative model)} \quad (24)$$

Summer commercial fishery discards (Base model)

Length/shell compositions of observer discards were modeled as

$$\hat{P}_{b,n,l,t} = N_{s,l,t} S_{s,l} (1 - P_{lg,l}) / \sum_l [(N_{s,l,t} + O_{s,l,t}) S_{s,l} (1 - P_{lg,l})]$$
$$\hat{P}_{b,o,l,t} = O_{s,l,t} S_{s,l} (1 - P_{lg,l}) / \sum_l [(N_{s,l,t} + O_{s,l,t}) S_{s,l} (1 - P_{lg,l})] \quad (25)$$

Summer commercial fishery total catch (Alternative model)

Length/shell compositions of observer discards were modeled as

$$\hat{P}_{t,n,l,t} = N_{s,l,t} S_{s,l} / \sum_l [(N_{s,l,t} + O_{s,l,t}) S_{s,l}]$$
$$\hat{P}_{t,o,l,t} = O_{s,l,t} S_{s,l} / \sum_l [(N_{s,l,t} + O_{s,l,t}) S_{s,l}] \quad (25')$$

Summer trawl survey

Proportions of newshell and oldshell crab, $P_{st,n,l,t}$ and $P_{st,o,l,t}$ were given by

$$\hat{P}_{st,n,l,t} = \frac{[N_{s,l,t}\, e^{-y_c M_l} - C_{s,t}\, P_{c,t}\, \hat{P}_{s,n,l',t}] e^{-(y_{st} - y_c) M_l} S_{st,l}}{\sum_l [(N_{s,l,t} + O_{s,l,t}) e^{-y_c M_l} - C_{s,t} P_{c,t} (\hat{P}_{s,n,l',t} + \hat{P}_{s,o,l',t})] e^{-(y_{st} - y_c) M_l} S_{st,l}}$$
$$\hat{P}_{st,o,l,t} = \frac{[O_{s,l,t}\, e^{-y_c M_l} - C_{s,t}\, \hat{P}_{s,o,l',t}\, P_{c,t}] e^{-(y_{st} - y_c) M_l} S_{st,l}}{\sum_l [(N_{s,l,t} + O_{s,l,t}) e^{-y_c M_l} - C_{s,t} P_{c,t} (\hat{P}_{s,n,l,t} + \hat{P}_{s,o,l,t})] e^{-(y_{st} - y_c) M_l} S_{st,l}} \quad (26)$$

Winter pot survey

Winter pot survey length compositions for newshell and oldshell crab, $P_{sw,n,l,t}$ and $P_{sw,o,l,t}$ ($l \geq 1$) were calculated as

$$\hat{P}_{sw,n,l,t} = N_{w,l,t} S_{w,l} / \sum_l [(N_{w,l,t} + O_{w,l,t}) S_{w,l}]$$
$$\hat{P}_{sw,o,l,t} = O_{w,l,t} S_{w,l} / \sum_l [(N_{w,l,t} + O_{w,l,t}) S_{w,l}] \quad (27)$$

Spring Pot survey 2012-2015

Winter pot survey length compositions for newshell and oldshell crab, $P_{sw,n,l,t}$ and $P_{sw,o,l,t}$ ($l \geq 1$) were assumed to be supper crab population caught by winter pot survey gears

$$\hat{P}_{sp,n,l,t} = N_{s,l,t} S_{w,l} / \sum_l [(N_{s,l,t} + O_{s,l,t}) S_{w,l}]$$
$$\hat{P}_{sp,o,l,t} = O_{s,l,t} S_{s,l} / \sum_l [(N_{s,l,t} + O_{s,l,t}) S_{w,l}] \quad (28)$$

Estimates of tag recovery

The proportion of released tagged length class *l'* crab recovered after *t-th* year with length class of *l* by a fishery of *s-th* selectivity (S_l) was assumed to be proportional to the growth matrix, catch selectivity, and molting probability (m_l) as

$$\hat{P}_{l',l,t,s} = \frac{S_l \cdot [X^t]_{l',l}}{\sum_{l=1}^{n} S_l \cdot [X^t]_{l',l}} \tag{29}$$

where X is a molting probability adjusted growth matrix with each component consisting of

$$X_{l',l} = \begin{cases} m_{l'} \cdot G_{l',l} & \text{when } l' \neq l \\ m_l \cdot G_{l',l} + (1\text{-}m_i) & \text{when } l' = l \end{cases} \tag{30}$$

b. Software used: AD Model Builder (Fournier et al. 2012).

c. Likelihood components.

Under assumptions that measurement errors of annual total survey abundances and summer commercial fishing efforts follow lognormal distributions and each type of length composition has a multinomial error structure (Fournier and Archibald 1982; Methot 1989), the log-likelihood function is

$$
\begin{aligned}
&\sum_{i=1}^{i=4}\sum_{t=1}^{t=n_i} K_{i,t}\left[\sum_{l=1}^{l=n} P_{i,l,t}\ln(\hat{P}_{i,l,t}+\kappa) - \sum_{l=1}^{l=n} P_{i,l,t}\ln(P_{i,l,t}+\kappa)\right] \\
&-\sum_{t=1}^{t=n_i}\frac{[\ln(q\cdot\hat{B}_{i,t}+\kappa)-\ln(B_{i,t}+\kappa)]^2}{2\cdot ln(CV_{i,t}^2+1)} \\
&-\sum_{t=1}^{t=n_i}\left[\frac{\ln\left[\ln(CV_t^2+1)+w_t\right]}{2}+\frac{[\ln(\hat{f}_t+\kappa)-\ln(f_t+\kappa)]^2}{2\cdot\left[\ln(CV_t^2+1)+w_t\right]}\right] \\
&-\sum_{t=1}\frac{\tau_t^2}{2\cdot SDR^2} \\
&+W\sum_{s=1}^{s=2}\sum_{t=1}^{t=3}\sum_{l'=1}^{l'=n} K_{l',t,s}\left[\sum_{l=1}^{l=n} P_{l',l,t}\ln(\hat{P}_{l',l,t,s}+\kappa) - \sum_{l=1}^{l=n} P_{l',l,t}\ln(P_{l',l,t,s}+\kappa)\right]
\end{aligned}
\qquad (32)
$$

where

i: length/shell compositions of :

1 triennial summer trawl survey,
2 annual winter pot survey,
3 summer commercial fishery retained catch,
4 observer discards or total catch during the summer fishery
5 spring pot survey.

$K_{i,t}$: the effective sample size of length/shell compositions for data set *i* in year *t*,
$P_{i,l,t}$: observed and estimated length compositions for data set *i*, length class *l*, and year *t.*
κ : a constant equal to 0.0001,
CV : coefficient of variation for the survey abundance,
$B_{i,k,t}$: observed and estimated annual total abundances for data set *i* and year *t*,
f_t : observed and estimated summer fishing CPUE,
w^2_t: extra variance factor,
SDR : Standard deviation of recruitment = 0.5,
$K_{l',t}$: sample size of length class *l'* released and recovered after *t-th* in year,
$P_{l',l,t,s}$: observed and estimated proportion of tagged crab released at length *l'* and recaptured at length *l*, after *t-th* year by commercial fishy pot selectivity *s*,
W: weighting for the tagging survey likelihood

It is generally believed that total annual commercial crab catches in Alaska are fairly accurately reported. Thus, total annual catch was assumed known.

d. Parameter estimation framework:

i. Parameters Estimated Independently

The following parameters were estimated independently: natural mortality (M =0.18), proportions of legal males by length group.

Natural mortality was based on an assumed maximum age, t_{max}, and the 1% rule (Zheng 2005):

$$M = -\ln(p)/t_{\max},$$

where p is the proportion of animals that reach the maximum age and is assumed to be 0.01 for the 1% rule (Shepherd and Breen 1992, Clarke et al. 2003). The maximum age of 25, which was used to estimate M for U.S. federal overfishing limits for red king crab stocks results in an estimated M of 0.18. Among the 199 recovered crabs from the tagging returns during 1991-2007 in Norton Sound, the longest time at liberty was 6 years and 4 months from a crab tagged at 85 mm CL. The crab was below the mature size and was likely less than 6 years old when tagged. Therefore, the maximum age from tagging data is about 12, which does not support the maximum age of 25 chosen by the CPT.

Proportions of legal males (CW > 4.75 inches) by length group were estimated from the ADF&G trawl data 1996-2011 (Table 11).

ii. Parameters Estimated Conditionally

Estimated parameters are listed in Table 10. Selectivity and molting probabilities based on these estimated parameters are summarized in Tables 11.

A likelihood approach was used to estimate parameters

e. Definition of model outputs.

i. Estimate of mature male biomass (MMB) is on **February 1st** and is consisting of the biomass of male crab in length classes 4 to 8

$$MMB = \sum_{l=4} (N_{w,l,} + O_{w,l,}) wm_l$$

wm_l: mean weight of each length class (Table 11).

ii. Projected legal male biomass for winter and summer fishery OFL was calculated as

$$Legal\_B = \sum_{l} (N_{w,l,} + O_{w,l,}) S_{s,l} P_{lg,l} wm_l \text{ Baseline model}$$

$$Legal\_B = \sum_l (N_{w,l,} + O_{w,l,}) S_{s,l} S_{r,l} wm_l \text{ Alternative model}$$

iii. Recruitment: the number of males in length classes 1, 2, and 3.

iv.

f. OFL

The Norton Sound red king crab fishery consists of two distinct fisheries: winter and summer. The two fisheries are discontinuous with 5 months between the two fisheries during which natural mortalities occur. To incorporate this fishery, the CPT in 2016 recommended the following formula:

$$OFL_r = \text{Winter harvest (Hw)} + \text{Summer harvest (Hs)} \quad (1)$$

And

$$p = \frac{Hw}{OFL_r} \quad (2)$$

Where *p* is a specific proportion of winter crab harvest to total (winter + summer) harvest

At given fishery mortality (F_{OFL}), Winter harvest is a fishing mortality

$$Hw = (1 - e^{-x \cdot F}) B_w \quad (3)$$

$$Hs = (1 - e^{-(1-x) \cdot F}) B_s \quad (4)$$

where B_s is a summer crab biomass after winter fishery and x ($0 \le x \le 1$) is a fraction that satisfies equation (2)

Since B_s is a summer crab biomass after winter fishery and 5 months of natural morality ($e^{-0.42M}$)

$$\begin{aligned} B_s &= (B_w - Hw) e^{-0.42M} \quad (5) \\ &= (B_w - (1 - e^{-x \cdot F}) B_w) e^{-0.42M} \\ &= B_w e^{-x \cdot F - 0.42M} \end{aligned}$$

Substituting *0.42M* to *m*, summer harvest is

$$\begin{aligned} Hs &= (1 - e^{-(1-x) \cdot F}) B_s \quad (6) \\ &= (1 - e^{-(1-x) \cdot F}) B_w e^{-x \cdot F - m} = (e^{-(x \cdot F + m)} - e^{-(F+m)}) B_w \end{aligned}$$

Thus, OFL is

$$OFL = Hw + Hs = (1 - e^{-xF})B_w + (e^{-(x \cdot F + m)} - e^{-(F+m)})B_w \quad (7)$$
$$= (1 - e^{-xF} + e^{-(xF+m)\cdot} - e^{-(F+m)\cdot})B_w$$
$$= [1 - e^{-(F+m)\cdot} - (1 - e^{-m\cdot})e^{-xF\cdot}]B_w$$

Combining (2) and (7),

$$p = \frac{Hw}{OFL_r} = \frac{(1 - e^{-xF})B_w}{[1 - e^{-(F+m)\cdot} - (1 - e^{-m\cdot})e^{-xF\cdot}]B_w} \quad (8)$$

Solving (8) for x

$$(1 - e^{-xF}) = p[1 - e^{-(F+m)\cdot} - (1 - e^{-m\cdot})e^{-xF\cdot}] \quad (9)$$
$$e^{-xF} - p(1 - e^{-m\cdot})e^{-xF\cdot} = 1 - p[1 - e^{-(F+m)\cdot}]$$
$$[1 - p(1 - e^{-m\cdot})]e^{-xF\cdot} = 1 - p[1 - e^{-(F+m)\cdot}]$$
$$e^{-xF\cdot} = \frac{1 - p[1 - e^{-(F+m)\cdot}]}{1 - p(1 - e^{-m\cdot})}$$

Combining (7) and (9), and substituting back,

revised retained OFL is

$$OFL = Legal\_B_w\left(1 - e^{-(F_{OFL} + 0,42M)} - (1 - e^{-0.42M})\left(\frac{1 - p(1 - e^{-(F_{OFL} + 0.42M)})}{1 - p(1 - e^{-0.42M})}\right)\right)$$

Further combining (3) and (9), Winter fishery harvest rate (Fw) i

$$Fw = (1 - e^{-x \cdot F}) = 1 - \frac{1 - p[1 - e^{-(F+m)\cdot}]}{1 - p(1 - e^{-m\cdot})} = \frac{1 - p(1 - e^{-m\cdot}) - 1 + p[1 - e^{-(F+m)\cdot}]}{1 - p(1 - e^{-m\cdot})} \quad (10)$$
$$= \frac{p(e^{-m\cdot} - e^{-(F+m)\cdot})}{1 - p(1 - e^{-m\cdot})} = \frac{p(1 - e^{-F\cdot})e^{-0.42M\cdot}}{1 - p(1 - e^{-0.42M\cdot})}$$

Summer fishery harvest rate (Fs) is

$$
\begin{aligned}
Fs &= (e^{-(x\cdot F+m)} - e^{-(F+m)}) = (e^{-x\cdot F} - e^{-F})e^{-m} \\
&= \left(\frac{1-p[1-e^{-(F+m)\cdot}]}{1-p(1-e^{-m\cdot})} - e^{-F} \right) e^{-m} \\
&= \left(\frac{1-p[1-e^{-(F+m)\cdot}] - e^{-F} + p(e^{-F} - e^{-(F+m\cdot)})}{1-p(1-e^{-m\cdot})} \right) e^{-m} \\
&= \left(\frac{1-p+pe^{-(F+m)\cdot} - e^{-F} + pe^{-F} - pe^{-(F+m\cdot)}}{1-p(1-e^{-m\cdot})} \right) e^{-m} \\
&= \frac{(1-p)(1-e^{-F})e^{-m}}{1-p(1-e^{-m\cdot})} = \frac{(1-p)(1-e^{-F})e^{-0.24M}}{1-p(1-e^{-0.24M\cdot})}
\end{aligned} \tag{11}
$$