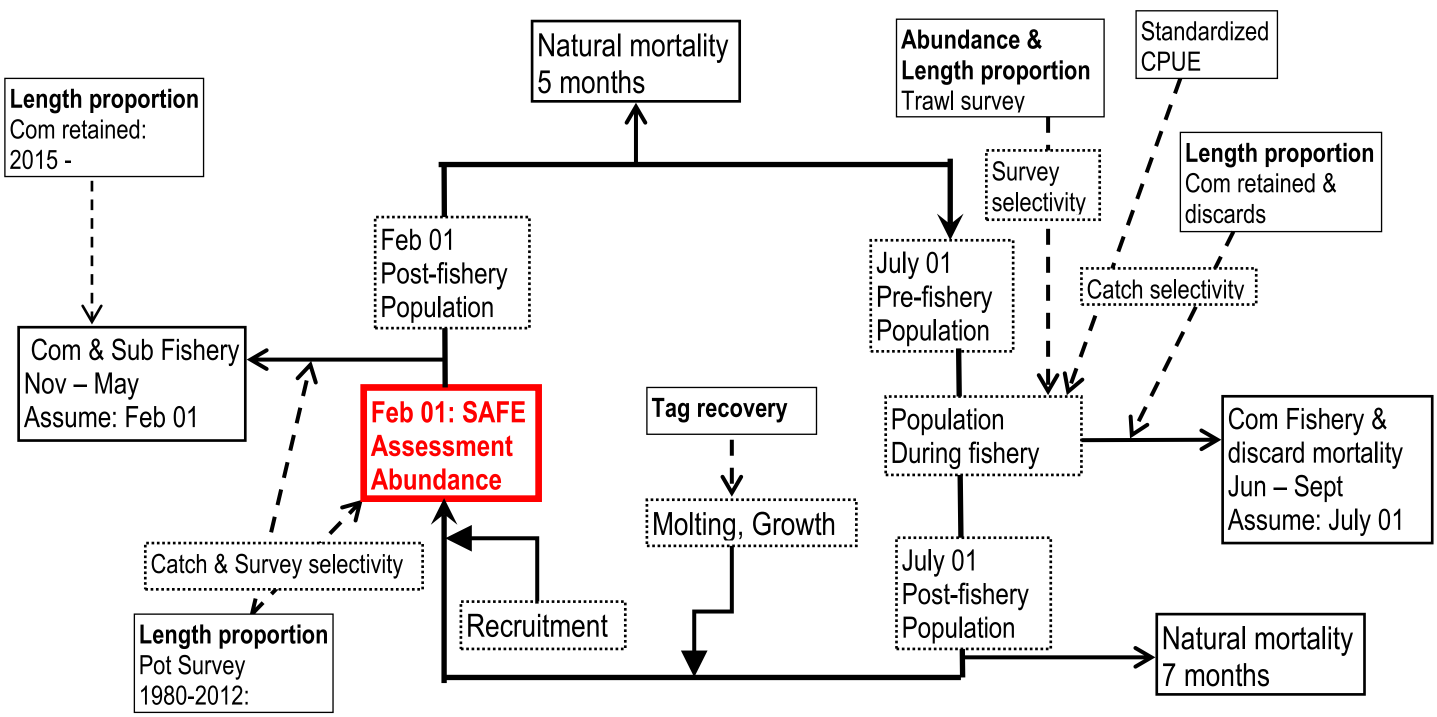
Appendix A. Description of the Norton Sound Red King Crab Model

a. Model description.

The model is an extension of the length-based model developed by Zheng et al. (1998) for Norton Sound red king crab. The model has 8 male length classes with model parameters estimated by the maximum likelihood method. The model estimates abundances of crab with CL ≥64 mm and with 10-mm length intervals (8 length classes, ≥134 mm) because few crab measuring less than 64 mm CL were caught during surveys or fisheries and there were relatively small sample sizes for trawl and winter pot surveys. The model treats newshell and oldshell male crab separately but assumes they have the same molting probability and natural mortality.

Norton Sound Red King Crab Modeling Scheme



Timeline of calendar events and crab modeling events:

- Model year starts February 1st to January 31st of the following year.
- Initial Population Date: February 1st 1976, consisting of only newshell crab.
- All winter fishery catch occurs on February 1st
- All summer fishery catch occurs on July 1st
- During 1976-2004, all legal crab caught in Commercial are retained.
- During 2004-2005, only commercially marketable legal crab caught in Commercial crabs are retained (i.e., high grading of crab ≥ 5 in CW).
- Winter Subsistence fishery retains all mature crab.

- Molting and recruitment occur on July 1st

Initial pre-fishery summer crab abundance on February 1st 1976:

Abundance of the initial pre-fishery population was assumed to consist of newshell crab to reduce the number of parameters, and estimated as

$$N_{w,l,1} = p_l e^{\log\_N_{76}} \qquad (1)$$

where length proportion of the first year (p_l) was calculated as

$$p_l = \frac{\exp(a_l)}{1+\sum_{l=1}^{n-1}\exp(a_l)} \text{ for } l = 1,..,n\text{-}1$$
$$p_n = 1 - \frac{\sum_{l=1}^{n-1}\exp(a_l)}{1+\sum_{l=1}^{n-1}\exp(a_l)} \qquad (2)$$

for model estimated parameters a_l.

Crab abundance on July 1st :

Summer (01 July) crab abundance of newshell and oldshell are of survivors of Winter (Feb 01) population from winter commercial and subsistence crab fisheries, and natural mortality from 01Feb to 01July.

$$N_{s,l,y} = (N_{w,l,y} - C_{w,y}P_{w,n,l,y} - C_{p,t}P_{p,n,l,y} - D_{w,n,l,y} - D_{p,n,l,y})e^{-0.42M_l}$$
$$O_{s,l,y} = (O_{w,l,y} - C_{w,y}P_{w,o,l,y} - C_{p,y}P_{p,o,l,y} - D_{w,o,l,y} - D_{p,o,l,y})e^{-0.42M_l} \qquad (3)$$

where

$N_{s,l,y}$, $O_{s,l,y}$: summer abundances of newshell and oldshell crab in length class l in year y ,
$N_{w,l,y}$, $O_{w,l,y}$: winter abundances of newshell and oldshell crab in length class l in year y,
$C_{w,t,y}$, $C_{p,t}$: total winter commercial and subsistence catches in year t,
$P_{w,n,l,y}$, $P_{w,o,l,y}$: Proportion of newshell and oldshell length class l crab in year y, harvested by winter commercial fishery,
$P_{p,n,l,y}$, $P_{p,o,l,y}$: Proportion of newshell and oldshell length class l crab in year y, harvested by winter subsistence fishery,
$D_{w,n,l,y}$, $D_{w,o,l,y}$: Discard mortality of newshell and oldshell length class l crab in winter commercial fishery in year y,

$D_{p,n,l,y}$, $D_{p,o,l,y}$: Discard mortality of newshell and oldshell length class l crab in winter subsistence fishery in year y,
M_l : instantaneous natural mortality in length class l,
0.42 : proportion of the year from Feb 1 to July 1 is 5 months.

Length proportion compositions of winter commercial retained catch ($P_{w,n,l,y}$, $P_{w,o,l,y}$) in year t were estimated as:

$$\begin{aligned}
&\text{1976-2007}\\
&P_{w,n,l,y} = N_{w,l,y}S_{w,l}P_{\lg,l} / \sum_{l=1}[(N_{w,l,y}+O_{w,l,y})S_{w,l}P_{\lg,l}]\\
&P_{w,o,l,y} = O_{w,l,y}S_{w,l}P_{\lg,l} / \sum_{l=1}[(N_{w,l,y}+O_{w,l,y})S_{w,l}P_{\lg,l}]\\
&\text{2008-present}\\
&P_{cw,n,l,y} = N_{w,l,t}S_{w,l}S_{wr,l} / \sum_{l}[(N_{w,l,y}+O_{w,l,y})S_{w,l}S_{wr,l}]\\
&P_{cw,o,l,y} = O_{w,l,t}S_{w,l}S_{wr,l} / \sum_{l}[(N_{w,l,y}+O_{w,l,y})S_{w,l}S_{wr,l}]
\end{aligned} \tag{4}$$

where
$P_{lg,l}$: the proportion of legal males in length class l ,
$S_{w,l}$: Selectivity of winter fishery pot.
$S_{wr,l}$: Retention probability of winter fishery

In the above, we assumed that all legal crabs were retained during 1976-2007 periods, and high grading has occurred since 2008 season.

The subsistence fisheries do not have a size limit; however, immature crab (< 94 mm) are generally not retained. Thus, we assumed proportion of length composition l = 1 and 2 as 0, and estimated length compositions ($l \geq 3$) as follows

$$\begin{aligned}
&P_{p,n,l,y} = N_{w,l,y}S_{w,l} / \sum_{l=3}[(N_{w,l,y}+O_{w,l,y})S_{w,l}]\\
&P_{p,o,l,y} = O_{w,l,y}S_{w,l} / \sum_{l=3}[(N_{w,l,y}+O_{w,l,y})S_{w,l}]
\end{aligned} \tag{5}$$

Crab abundance on Feb 1st :

The assessment model assumes that molting and growth occur immediately after summer fishery harvests, and that recruitment would occur between July 01 and Feb 01 of the next year. That is, the following events occur: (1) summery fishery, (2) summer fishery discards mortality, (3) molting and recruitment, and (4) natural mortality between July 01 and Feb 01. Those are formulated as follows:

Newshell Crab- Abundance of newshell crab of year t and length-class l ($N_{w,l,y}$) year-y consist of: (1) new and oldshell crab that survived the summer commercial fishery and molted, and (2) recruitment ($R_{l,y}$):

$$N_{w,l,y+1} = \sum_{l'=1}^{l'=l} G_{l',y}[(N_{s,l',y} + O_{s,l',y})e^{-y_c M_l} - C_{s,y-1}(P_{s,n,l',y} + P_{s,o,l',y}) - D_{l',y}]m_{l'} e^{-(0.58-y_c)M_l} + R_{l,y} \qquad (6)$$

Oldshell Crab- Abundance of oldshell crabs of year y and length-class l ($O_{w,l,y}$) consists of the non-molting portion of survivors from the summer fishery:

$$O_{w,l,y+1} = [(N_{s,l,y} + O_{s,l,y})e^{-y_c M_l} - C_{s,y}(P_{s,n,l,y} + P_{s,o,l,y}) - D_{l,y}](1 - m_l)e^{-(0.58-y_c)M_l} \qquad (7)$$

where

$G_{l',l}$: a growth matrix representing the expected proportion of crabs growing from length class l' to length class l
$C_{s,y}$: total summer catch in year y
$P_{s,n,l,y}$, $P_{s,o,l,y}$: proportion of summer catch for newshell and oldshell crab of length class l in year y,
$D_{l,y}$: summer discard mortality of length class l in year y,
m_l : molting probability of length class l,
y_c : the time in year from July 1 to the mid-point of the summer fishery,
0.58: Proportion of the year from July 1st to Feb 1st : 7 months = 0.58 year,
$R_{l,y}$: recruitment into length class l in year y.

Discards

Discards are crabs that were caught in summer and winter commercial and winter subsistence fisheries but were not retained.

Summer and winter commercial discards

In summer ($D_{l,t}$) and winter ($D_{w,n,l,t}$, $D_{w,o,l,t}$) commercial fisheries, sublegal males (<4.75 inch CW and <5.0 inch CW since 2008) are discarded. Those discarded crabs are subject to handling mortality. The number of discards was not directly observed, and thus was estimated from the model as: Observed Catch x (estimated abundance of crab that are not caught by commercial pot)/(estimated abundance of crab that are caught by commercial pot)

Model discard mortality in length-class l in year y from the summer and winter commercial pot fisheries is given by

$$D_{l,y} = C_{s,y} \frac{N_{s,l,y} S_{s,l}(1 - S_{r,n,l}) + O_{s,l,y} S_{s,l}(1 - S_{r,o,l})}{\sum_l (N_{s,l,y} S_{r,n,l} + O_{s,l,y} S_{r,o,l}) S_{s,l}} hm_s \qquad (8)$$

1977 – 2007   2008 – present   (9)

$$D_{w,n,l,y} = C_{w,y} \frac{N_{w,l,y} S_{w,l} (1-P_{lg,l})}{\sum_l (N_{w,l,y} + O_{w,l,y}) S_{w,l} P_{lg,l}} hm_w \quad D_{w,n,l,y} = C_{w,t} \frac{N_{w,l,y} S_{w,l} (1-S_{wr,l})}{\sum_l (N_{w,l,y} + O_{w,l,y}) S_{w,l} S_{wr,l}} hm_w$$

1977 – 2007 2008 – 2022

$$D_{w,o,l,y} = C_{w,y} \frac{O_{w,l,y} S_{w,l} (1-P_{lg,l})}{\sum_l (N_{w,l,y} + O_{w,l,y}) S_{w,l} P_{lg,l}} hm_w \quad D_{w,o,l,y} = C_{w,y} \frac{O_{w,l,y} S_{w,l} (1-S_{wr,l})}{\sum_l (N_{w,l,y} + O_{w,l,y}) S_{w,l} S_{wr,l}} hm_w \qquad (10)$$

where

hm_s: summer commercial handling mortality rate assumed to be 0.2,
hm_w: winter commercial handling mortality rate assumed to be 0.2,
$S_{s,l}$: Selectivity of the summer commercial fishery,
$S_{w,l}$: Selectivity of the winter commercial fishery,
$S_{r,l}$: Retention selectivity of the summer commercial fishery,
$S_{wr,l}$: Retention selectivity of the winter commercial fishery,

Winter subsistence discards

Discards (unretained) from the winter subsistence fishery are reported in a permit survey ($C_{d,y}$), though its size composition is unknown. We assumed that subsistence fishers discard all crabs of length classes 1 -2.

$$D_{p,n,l,y} = C_{d,y} \frac{N_{w,l,y} S_{w,l}}{\sum_{l=1}^{2} (N_{w,l,y} + O_{w,l,y}) S_{w,l}} hm_w \qquad (11)$$

$$D_{p,o,l,y} = C_{d,y} \frac{O_{w,l,y} S_{w,l}}{\sum_{l=1}^{2} (N_{w,l,y} + O_{w,l,y}) S_{w,l}} hm_w \qquad (12)$$

where

$C_{d,y}$: Winter subsistence discards

Recruitment

Recruitment of year y, R_y, is a stochastic process around the geometric mean, R_0:

$$R_y = R_0 e^{\tau_t}, \tau_y \sim N(0, \sigma_R^2) \qquad (13)$$

R_t of the last year was assumed to be an average of previous 5 years: $R_y = (R_{y-1} + R_{y-2} + R_{y-3} + R_{y-4} + R_{y-5})/5$.

R_t was assumed to be newshell crab of immature (< 94 mm) length classes 1 to r:

$$R_{r,y} = p_r R_y \tag{14}$$

where p_r takes multinomial distribution, same as equation (2)

Molting Probability

Molting probability for length class l, m_l, was estimated as an inverse logistic function of length-class mid carapace length (L) and parameters (α, β) where β corresponds to L_{50}.

$$m_l = \frac{1}{1+e^{\alpha(L-\beta)}} \tag{15}$$

Trawl net and summer commercial pot selectivity

Trawl and summer commercial pot selectivity was assumed to be a logistic function of mid-length-class, constrained to be 0.999 at the largest length-class (L_{max}):

$$S_l = \frac{1}{1+e^{(\alpha(L_{\max}-L)+\ln(1/0.999-1))}} \tag{16}$$

Winter pot selectivity,

Winter pot selectivity was assumed to be a dome-shaped with logistic function of length-class mid carapace length (L) and parameters (α, β) where β corresponds to L_{50}.

$$S_{w,l} = \frac{1}{1+e^{\alpha(L-\beta)}} \tag{17}$$

Selectivity of the first 3 length classes $S_{w,s}$ (S= l_1, l_2, l_3) were individually estimated.

Retention probability: Winter commercial, summer commercial

Winter and summer commercial retention probability was assumed to be a logistic function of length-class mid carapace length (L) and parameters (α, β) where β corresponds to L_{50}.

$$S_{r,l} = \frac{1}{1+e^{\alpha(L-\beta)}} \qquad (17)$$

Growth transition matrix

The growth matrix $G_{l',l}$ (the expected proportion of crab molting from length class l' to length class l) was assumed to be normally distributed:

$$G_{l',l} = \begin{cases} \dfrac{\int_{lm_l-h}^{lm_l+h} N(L \mid \mu_{l'}, \sigma^2) dL}{\sum_{l=1}^{n} \int_{lm_l-h}^{lm_l+h} N(L \mid \mu_{l'}, \sigma^2) dL} & \text{when } l \geq l' \\ 0 & \text{when } l < l' \end{cases} \qquad (18)$$

where

$$N(x \mid \mu_{l'}, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(L-\mu_{l'})^2}{\sigma^2} \right)$$
$$lm_l = L_1 + st \cdot l$$
$$\mu_l = L_1 + \beta_0 + \beta_1 \cdot l$$

Observation model

Summer trawl survey abundance

Modeled trawl survey abundance of year y ($B_{st,y}$) is July 1st abundance subtracted by summer commercial fishery harvest occurring from July 1st to the mid-point of summer trawl survey, multiplied by natural mortality occurring between the mid-point of commercial fishery date and trawl survey date, and multiplied by trawl survey selectivity. For the first year (1976) trawl survey, the commercial fishery did not occur.

$$\hat{B}_{st,y} = \sum_{l} [(N_{s,l,y} + O_{s,l,y}) e^{-y_c M_l} - C_{s,y} P_{c,y} (P_{s,n,l,y} + P_{s,o,l,y})] e^{-(y_{st}-y_c)M_l} S_{st,l} \qquad (19)$$

where
y_{st}: the time in year from July 1 to the mid-point of the summer trawl survey,
y_c: the time in year from July 1 to the mid-point for the catch before the survey, ($y_{st} > y_c$: Trawl survey starts after opening of commercial fisheries),
$P_{c,y}$: the proportion of summer commercial crab harvested before the mid-point of trawl survey date.
$S_{st,l}$: Selectivity of the trawl survey.

*Winter pot survey CPUE **(depleted)***

Winter pot survey cpue (f_{wy}) was calculated with catchability coefficient q and exploitable abundance:

$$\hat{f}_{wy} = q_w \sum_l [(N_{w,l,y} + O_{w,l,y}) S_{w,l}] \tag{20}$$

Summer commercial CPUE

Summer commercial fishing CPUE (f_y) was calculated as a product of catchability coefficient q and mean exploitable abundance, A_t minus one half of summer catch, C_t:

$$\hat{f}_y = q_i (A_y - 0.5C_y) \tag{21}$$

Because the fishing fleet and pot limit configuration changed in 1993, q_1 is for fishing efforts before 1993, q_2 is from 1994 to present.

Where A_y is exploitable legal abundance in year t, estimated as

$$A_y = \sum_l [(N_{s,l,y} + O_{s,l,y}) S_{s,l} S_{r,l}] \tag{22}$$

Summer pot survey abundance (**depleted**)

Abundance of y-th year pot survey was estimated as

$$\hat{B}_{p,y} = \sum_l [(N_{s,l,y} + O_{s,l,y}) e^{-y_p M_l}] S_{p,l} \tag{23}$$

Where

y_p: the time in year from July 1 to the mid-point of the summer pot survey.

Length composition

Summer commercial retained catch

Length compositions of the summer commercial catch for new and old shell crabs $P_{s,n,l,y}$ and $P_{s,o,l,y}$, were modeled based on the summer population, selectivity, and retention probability

$$\begin{aligned} \hat{P}_{s,n,l,y} &= N_{s,l,y} S_{s,l} S_{r,o,l} / A_t \\ \hat{P}_{s,o,l,y} &= O_{s,l,y} S_{s,l} S_{r,o,l} / A_t \end{aligned} \tag{24}$$

Retention probability is separated into two periods: 1977–2007 and 2008–2020 indicating before and after the start of high grading.

Summer commercial fishery discards (1977-1993)

Prior to 1993, Observer survey data contained length-shell composition of only discards.

Length/shell compositions of observer discards were modeled as

$$\begin{aligned}\hat{P}_{b,n,l,y} &= N_{s,l,y}S_{s,l}(1-S_{r,n,l}) / \sum_l [N_{s,l,y}(1-S_{r,n,l}) + O_{s,l,y}(1-S_{r,o,l})]S_{s,l} \\ \hat{P}_{b,o,l,y} &= O_{s,l,y}S_{s,l}(1-S_{r,o,l}) / \sum_l [N_{s,l,y}(1-S_{r,n,l}) + O_{s,l,y}(1-S_{r,o,l})]S_{s,l}\end{aligned} \quad (25)$$

Summer commercial fishery total catch (212-2019)

The 2012–2019 Observer survey had total as well as retained and discard length-shell composition, and total catch length-shell composition was fitted.

Length/shell compositions of observer total catch was modeled as

$$\begin{aligned}\hat{P}_{t,n,l,y} &= N_{s,l,y}S_{s,l} / \sum_l [(N_{s,l,y} + O_{s,l,y})S_{s,l}] \\ \hat{P}_{t,o,l,y} &= O_{s,l,y}S_{s,l} / \sum_l [(N_{s,l,y} + O_{s,l,y})S_{s,l}]\end{aligned} \quad (26)$$

Summer trawl survey

Proportions of newshell and oldshell crab, $P_{st,n,l,y}$ and $P_{st,o,l,y}$ were given by

$$\begin{aligned}\hat{P}_{st,n,l,y} &= \frac{[N_{s,l,y}e^{-y_c M_l} - C_{s,y}\ P_{c,y}\hat{P}_{s,n,l',y}]e^{-(y_{st}-y_c)M_l}S_{st,l}}{\sum_l [(N_{s,l,y} + O_{s,l,y})e^{-y_c M_l} - C_{s,y}P_{c,y}(\hat{P}_{s,n,l',y} + \hat{P}_{s,o,l',y})]e^{-(y_{st}-y_c)M_l}S_{st,l}} \\ \hat{P}_{st,o,l,y} &= \frac{[O_{s,l,y}e^{-y_c M_l} - C_{s,y}\ \hat{P}_{s,o,l',y}\ P_{c,y}]e^{-(y_{st}-y_c)M_l}S_{st,l}}{\sum_l [(N_{s,l,y} + O_{s,l,y})e^{-y_c M_l} - C_{s,y}P_{c,y}(\hat{P}_{s,n,l,y} + \hat{P}_{s,o,l,y})]e^{-(y_{st}-y_c)M_l}S_{st,l}}\end{aligned} \quad (27)$$

Winter pot survey

Winter pot survey length compositions for newshell and oldshell crab, $P_{sw,n,l,t}$ and $P_{sw,o,l,t}$ ($l \geq 1$) were calculated as

$$\hat{P}_{sw,n,l,y} = N_{w,l,y} S_{w,l} / \sum_l [(N_{w,l,y} + O_{w,l,y}) S_{w,l}]$$
$$\hat{P}_{sw,o,l,y} = O_{w,l,y} S_{w,l} / \sum_l [(N_{w,l,y} + O_{w,l,y}) S_{w,l}] \quad (28)$$

Winter commercial retained

Winter commercial retained length compositions for newshell and oldshell crab, $P_{cw,n,l,t}$ and $P_{cw,o,l,t}$ ($l \geq 1$) were calculated as

$$\hat{P}_{cw,n,l,y} = N_{w,l,y} S_{w,l} S_{wr,l} / \sum_l [(N_{w,l,y} + O_{w,l,y}) S_{w,l} S_{wr,l}]$$
$$\hat{P}_{cw,o,l,y} = O_{w,l,y} S_{w,l} S_{wr,l} / \sum_l [(N_{w,l,y} + O_{w,l,y}) S_{w,l} S_{wr,l}] \quad (29)$$

*Spring Pot survey 2012-2015 (**depleted**)*

Spring pot survey length compositions for newshell and oldshell crab, $P_{sw,n,l,t}$ and $P_{sw,o,l,t}$ ($l \geq 1$) were assumed to be similar to crab population caught by winter pot survey

$$\hat{P}_{sp,n,l,y} = N_{s,l,y} S_{w,l} / \sum_l [(N_{s,l,y} + O_{s,l,y}) S_{w,l}]$$
$$\hat{P}_{sp,o,l,y} = O_{s,l,y} S_{w,l} / \sum_l [(N_{s,l,y} + O_{s,l,y}) S_{w,l}] \quad (30)$$

Estimates of tag recovery

The proportion of released tagged length class *l'* crab recovered after *t-th* year with length class of *l* by a fishery of *s-th* selectivity (S_l) was assumed to be proportional to the growth matrix, catch selectivity, and molting probability (m_l) as

$$\hat{P}_{l',l,t,s} = \frac{S_l \cdot [X^t]_{l',l}}{\sum_{l=1}^{n} S_l \cdot [X^t]_{l',l}} \quad (31)$$

where X is a molting probability adjusted growth matrix with each component consisting of

$$X_{l',l} = \begin{cases} m_{l'} \cdot G_{l',l} & \text{when } l' \neq l \\ m_l \cdot G_{l',l} + (1\text{-}m_i) & \text{when } l' = l \end{cases} \quad (32)$$

c. Likelihood components.

Under assumptions that measurement errors of annual total survey abundances and summer commercial fishing efforts follow lognormal distributions, and each type of length composition has a multinomial error structure (Fournier and Archibald 1982; Methot 1989), the log-likelihood function is

$$\begin{aligned}
&\sum_{i=1}^{i=4}\sum_{y=1}^{y=n_i} K_{i,t}\left[\sum_{l=1}^{l=n} P_{i,l,y}\ln(\hat{P}_{i,l,y}+\kappa)-\sum_{l=1}^{l=n} P_{i,l,y}\ln(P_{i,l,y}+\kappa)\right] \\
&-\sum_{y=1}^{y=n_i}\frac{[\ln(q\cdot\hat{B}_{i,y})-\ln(B_{i,y})]^2}{2\cdot ln(CV_{i,y}^2+1)} \\
&-\sum_{y=1}^{y=n_i}\left[\frac{\ln\left[\ln(CV_y^2+1)+w_t\right]}{2}+\frac{[\ln(\hat{f}_y+\kappa)-\ln(f_y+\kappa)]^2}{2\cdot\left[\ln(CV_y^2+1)+w_t\right]}\right] \qquad (32) \\
&-\sum_{t=1}\frac{\tau_t^2}{2\cdot SDR^2} \\
&+W\sum_{s=1}^{s=2}\sum_{y=1}^{y=3}\sum_{l'=1}^{l'=n} K_{l',y,s}\left[\sum_{l=1}^{l=n} P_{l',l,y}\ln(\hat{P}_{l',l,y,s}+\kappa)-\sum_{l=1}^{l=n} P_{l',l,t}\ln(P_{l',l,y,s}+\kappa)\right]
\end{aligned}$$

where

i: length/shell compositions of:

1 triennial summer trawl survey,
2 annual winter pot survey,
3 summer commercial fishery retained,
4 summer commercial observer discards or total catch,
5 winter commercial fishery retained.

$K_{i,y}$: the effective sample size of length/shell compositions for data set i in year y,
$P_{i,l,y}$: observed and estimated length compositions for data set i, length class l, and year y.
κ : a constant equal to 0.0001,
CV: coefficient of variation for the survey abundance,
$B_{j,y}$: observed and estimated annual total abundances for data set i and year y,
F_y: observed and estimated summer fishery CPUE,
w^2_t: extra variance factor,
SDR: Standard deviation of recruitment = 0.5,
$K_{l',y}$: sample size of length class l' released and recovered after y-th in year,
$P_{l',l,y,s}$: observed and estimated proportion of tagged crab released at length l' and recaptured at length l, after y-th year by commercial fishery pot selectivity s,
W: weighting for the tagging survey likelihood = 0.5

b. Software used: AD Model Builder (Fournier et al. 2012).

d. Out of model parameter estimation framework:

i. Parameters Estimated Independently

M: Natural mortality

Natural mortality (M = 0.18) was based on an assumed maximum age, t_{max}, and the 1% rule (Zheng 2005):

$$M = -\ln(p)/t_{\max},$$

where p is the proportion of animals that reach the maximum age and is assumed to be 0.01 for the 1% rule (Shepherd and Breen 1992, Clarke et al. 2003). The maximum age of 25, which was used to estimate M for U.S. federal overfishing limits for red king crab stocks results in an estimated M of 0.18.

e. Definition of model outputs.

i. Mature male biomass (MMB) is on February 1st and is consisting of the biomass of male crab in length classes 4 to 8

$$MMB = \sum_{l=4} (N_{w,l,} + O_{w,l,}) wm_l$$

wm_l: mean weight of each length class.

ii. Recruitment: the number of males in length classes 1, 2, and 3.

f. OFL

The Norton Sound red king crab fishery consists of two distinct fisheries: winter and summer. The two fisheries are discontinuous with 5 months between the two fisheries during which natural mortalities occur. To incorporate this, the CPT in 2016 recommended the following formula:

OFL = winter harvest biomass (*Hw*) + summer harvest biomass (*Hs*) (1)

And

$$p = \frac{Hw}{OFL} \quad (2)$$

Where p is a specific proportion of winter crab harvest to total (winter + summer) harvest

At given fishery mortality (F_{OFL}), Winter harvest is a fishing mortality

$$Hw = (1 - e^{-x \cdot F})B_w \tag{3}$$

$$Hs = (1 - e^{-(1-x) \cdot F})B_s \tag{4}$$

where B_s is a summer crab biomass after winter fishery and x ($0 \leq x \leq 1$) is a fraction that satisfies the equation (2).

Since B_s is a summer crab biomass after winter fishery and 5 months of natural morality, ($e^{-0.42M}$)

$$\begin{aligned} B_s &= (B_w - Hw)e^{-0.42M} \\ &= (B_w - (1 - e^{-x \cdot F})B_w)e^{-0.42M} \\ &= B_w e^{-x \cdot F - 0.42M} \end{aligned} \tag{5}$$

Substituting *m* for *0.42M*, summer harvest is

$$\begin{aligned} Hs &= (1 - e^{-(1-x) \cdot F})B_s \\ &= (1 - e^{-(1-x) \cdot F})B_w e^{-x \cdot F - m} = (e^{-(x \cdot F + m)} - e^{-(F+m)})B_w \end{aligned} \tag{6}$$

Thus, OFL is

$$\begin{aligned} OFL &= Hw + Hs = (1 - e^{-xF})B_w + (e^{-(x \cdot F + m)} - e^{-(F+m)})B_w \\ &= (1 - e^{-xF} + e^{-(xF+m) \cdot} - e^{-(F+m) \cdot})B_w \\ &= [1 - e^{-(F+m) \cdot} - (1 - e^{-m \cdot})e^{-xF \cdot}]B_w \end{aligned} \tag{7}$$

Combining equations (2) and (7),

$$p = \frac{Hw}{OFL} = \frac{(1 - e^{-xF})B_w}{[1 - e^{-(F+m) \cdot} - (1 - e^{-m \cdot})e^{-xF \cdot}]B_w} \tag{8}$$

Solving equation (8) for *x*

$$\begin{aligned} (1 - e^{-xF}) &= p[1 - e^{-(F+m) \cdot} - (1 - e^{-m \cdot})e^{-xF \cdot}] \\ e^{-xF} - p(1 - e^{-m \cdot})e^{-xF \cdot} &= 1 - p[1 - e^{-(F+m) \cdot}] \\ [1 - p(1 - e^{-m \cdot})]e^{-xF \cdot} &= 1 - p[1 - e^{-(F+m) \cdot}] \\ e^{-xF \cdot} &= \frac{1 - p[1 - e^{-(F+m) \cdot}]}{1 - p(1 - e^{-m \cdot})} \end{aligned} \tag{9}$$

Combining equations (7) and (9), and substituting back m and

revised retained OFL is

$$OFL = B_w \left(1 - e^{-(F_{OFL} + 0.42M)} - (1 - e^{-0.42M}) \left(\frac{1 - p(1 - e^{-(F_{OFL} + 0.42M)})}{1 - p(1 - e^{-0.42M})} \right) \right) \tag{10}$$

Calculation of empirical F

From the equation (3) and (4) empirical F is derived as:

$$Hw = (1 - e^{-x \cdot F})B_w \qquad e^{-x \cdot F} = \left(1 - \frac{Hw}{B_w}\right) \tag{11}$$

$$Hs = (1 - e^{-(1-x) \cdot F})B_s \qquad e^{-F} = \left(1 - \frac{Hs}{B_s}\right)e^{-xF} \tag{12}$$

Combining (11) and (12)

$$e^{-F} = \left(1 - \frac{Hs}{B_s}\right)\left(1 - \frac{Hw}{B_w}\right) \qquad F = -ln\left(\left(1 - \frac{Hs}{B_s}\right)\left(1 - \frac{Hw}{B_w}\right)\right) \tag{13}$$

Where Bs and *Bw* were derived from the model. *Hs* and *Hw* are biomass of retained catch + the model derived discards mortality.

Appendix B

Norton Sound Red King Crab CPUE Standardization

Note: This is an update of model by G. Bishop (NPFMC 2013). Please see SAFE 2013 for more detailed descriptions.

Methods

Model

Let U_{ijk} denote the observed CPUE, U_0 the reference CPUE, P_{ij} a factor *i* at level *j*, and let X_{ij} take a value of 1 when the j^{th} level of the factor P_{ij} is present and 0 when it is not. The lognormal distribution of U_{ijk} (Quinn and Deriso 1999), can be denoted as:

$$U_{ijk} = U_0 \prod_i \prod_j P_{ij}^{X_{ij}} e^{\varepsilon_{ijk}} \qquad (1)$$

or

$$\ln(U_{ijk}) = \ln(U_0) + \sum_{i=1}^{p} \sum_{j=1}^{n_j - 1} X_{ij} \ln(P_{ij}) + \varepsilon_{ijk} \ .$$

where ε_{ijk}, ~ N(0, σ^2) observation error

Substituting $\ln(U_0)$ to β_0 and $\ln(P_{ij})$ to β_{ij}, we then obtain an additive GLM lognormal error distribution of U_{ijk}:

$$\ln(U_{ijk}) = \beta_0 + \sum_{i=1}^{p} \sum_{j=1}^{n_j - 1} X_{ij} \beta_{ij} + \varepsilon_{ijk} \ . \qquad (2)$$

Standardized CPUE was calculated as follows:

1. Divide the coefficients β_{ij} by their geometric mean $\bar{\beta}$ to obtain canonical coefficients:

$$\beta_i{}' = \frac{\beta_i}{\bar{\beta}} . \tag{3}$$

2. Exponentiate the result to obtain the arithmetic scale canonical coefficients:

$$b' = e^{\beta_i - \bar{\beta}}. \tag{4}$$

3. Subtract the year coefficient reference level to obtain standardized CPUE U_j for each year level *j* as:

$$U_{Yj} = e^{\beta'_{Yj} - \beta'_{Y0}} . \tag{5}$$

4. Base year CPUE index is calculated by eliminating all factors but *Year* in the GLM and following Equations (2) and (3), (4), and (5) above.

SE of the standardized CPUE is calculated as:

Standard errors of CPUE are standard errors of the Year coefficients, $\hat{\beta}_{yr}$. These are obtained from the square root of the diagonal elements of the estimated covariance matrix, cov $(\hat{\beta})$, i.e., $\sqrt{\boldsymbol{C'\emptyset C}}$.

where C = $X(X^T X)^{-1}$, C' is transpose of C; and $\emptyset = \boldsymbol{\sigma^2 I_n}$
where X is the matrix of predictor variables, I_n is the identity matrix, and σ is the standard error of the GLM fit.

Data Source & Cleaning

Commercial fishery harvest data were obtained from ADF&G fish ticket database, which included: Landing Date, Fish Ticket Number, Vessel Number, Permit Fishery ID, Statistical Area(s) fished, Effort, and Number and Pounds of Crab harvested (Table B2-1,2,3, Figure B2-1). The fish ticket database may have multiple entries of identical Fish Ticket Number, Vessel Number, Permit Fishery ID, and Statistical Area.

The following data cleaning and combining methods were conducted:

1. Sum crab number and efforts by Fish Ticket Number, Vessel Number, Permit Fishery ID, and Statistical Area.
2. Remove data with missing or zero values in Effort, Number of Crab, or Pounds of Crab; (these are considered true missing data).
3. Calculate CPUE as Number of Crab/Effort.

Data cleaning and censoring.

Norton Sound commercial red king crab fishery can be largely divided into three periods: large vessel operation (1977-1993), small vessel superexclusive (1994-2007), and small vessel superexclusive and high grading since 2008. The pre-superexclusive fishery consisted of a few large boats, fishing west of 167 longitude, and few deliveries, and the post-superexclusive fishery consists of many small boats operated by local fishermen, fishing east of 167 longitude and near shore, and delivering frequently (Figure B1). The post-superexclusive period can further be divided into pre and post high grading periods of 2008. The majority of commercially caught red king crab are sold to the Norton Sound Economic Development Corporation (NSEDC). Legal crab in Norton Sound is defined as male with carapace width (CW) greater than 4.75 inch. Since 20008 the NSEDC purchased only crab of CW 5 inch or greater.

Censoring data

During 1977-93 period, vessels of 1 year of operation and/or 1 delivery per year harvested 20-90% of crab (Table B2-5, Figure B2-2). For instance, all vessels made only 1 delivery in 1989, and in 1988 64% of crab were harvested by 1 vessel that made only 1 delivery. On the other hand, during the 1993-2022 period of post-superexclusive fishery status, the majority of commercial crab fishery and harvest was done by vessels with more than 5 years of operations and more than 5 deliveries per year. For 1977 – 1993, censoring was made for vessels of more than 2 years of operations. Increasing deliveries to more than one would result in no estimates for some years. For 1994 – 2022, the data were censored to vessels that fished more than 5 years and delivered crab more than 5 times per year.

Analyses

Norton Sound red king crab CPUE standardization

A GLM was constructed as

$$\ln(CPUE) = YR + VSL + MSA + WOY + PF$$

Where YR: Year, VSL: Vessel, MSA: Statistical Area, WOY: Week of Year, and PF: Permit vs open fishery (Table 1). All variables were treated as categorical. Inclusion of interaction terms was not considered because they were absent (SAFE 2013).

The fishery strata (PD) consisted of the 3 periods based on changes in fishery operations, and the model was run for each fishery period.

1977-1993: Large Vessel fishery
1994-2007: Small boat fishery
2008-2022: Small boat and high-grading fishery

For selection of the best model, forward and backward stepwise selection was conducted. (R step function)

```
fit <- glm(L.CPUE.NO ~ factor(YR) + factor(VSL) + factor(WOY) +
factor(MSA) + factor(PF),data=NSdata.C)
step <- step(fit, direction='both', trace = 10)
best.glm<-glm(formula(step), data=NSdata.C)
```

Table B-1. List of variables in the fish ticket database. Variables in bold face were used for generalized linear modeling.

Variable	Description
YR	Year of commercial fishery
VSL	Unique vessel identification number
Fish Ticket Number	Unique delivery to a processor by a vessel
PF	Unique Permit Fishery categories
PD	Fishery period: 1977-1992, 1993-2004,2005-2018
Statistical Area	Unique fishery area.
MOA	Modified statistical area, combining each statistical area into 4 larger areas: Inner, Mid, Outer, Outer North
Fishing Beginning Date	Date of pots set
Landing Date	Date of crab landed to processor
WOY	Week of Landing Date **(calculated)**
Effort	The number of pot lift
Crab Numbers	Total number of crabs harvested from pots
Crab Pounds	Total pounds of crab harvested from pots
ln(CPUE)	ln(Crab Numbers/Effort) **(calculated)**

Table B-2. Permit fisheries, descriptions, and years with deliveries for Norton Sound summer commercial red king crab harvest data.

Permit fishery	Type	Description	Years
K09Q	Open access	KING CRAB , POT GEAR VESSEL UNDER 60', BERING SEA	1994–2002
K09Z	Open access	KING CRAB , POT GEAR VESSEL UNDER 60', NORTON SOUND	1992–2022
K09ZE	CDQ	KING CRAB , POT GEAR VESSEL UNDER 60', NORTON SOUND CDQ, NSEDC	2000–2022
K09ZF	CDQ	KING CRAB , POT GEAR VESSEL UNDER 60', NORTON SOUND CDQ, YDFDA	2002–2004
K91Q	Open access	KING CRAB , POT GEAR VESSEL 60' OR OVER, BERING SEA	1978–1989
K91Z	Open access	KING CRAB , POT GEAR VESSEL 60' OR OVER, NORTON SOUND	1982–1994

Table B-3. Modified statistical area definitions used for analysis of Norton Sound summer commercial red king crab harvest data.

Modified statistical area	Statistical areas included
Inner	616331, 616401, 626331, 626401, 626402
Mid	636330, 636401, 636402, 646301, 646330, 646401, 646402
Outer	656300, 656330, 656401, 656402, 666230, 666300, 666330, 666401
Outer North	666402, 666431, 676300, 676330 ,676400, 676430, 676501, 686330

Table B-4. Final generalized linear model formulae and AIC selected for Norton Sound summer commercial red king crab fishery. The dependent variable is ln(CPUE) in numbers.

Periods: 1977-1993

Var	Df	Deviance	Resid DF	Resid Dev	AIC
YR	14	269.56	377	265.4	
MSA	3	11.91	374	253.5	
MOY	2	6.134	372	247.4	
					974.01

Periods: 1994-2007

Var	Df	Deviance	Resid DF	Resid Dev	AIC
VSL	43	451.6	2401	1465.6	
YR	14	232.8	2387	1232.8	
WOY	15	72.3	2372	1160.5	
MSA	3	24.1	2369	1130.4	
					8577.0

Periods: 2008-2023

Var	Df	Deviance	Resid DF	Resid Dev	AIC
YR	13	555.4	3489	2121.7	
VSL	43	329.3	3446	1792.3	
WOY	13	66.0	3433	1726.4	
MSA	3	27.0	3430	1699.3	
MOY	3	3.1	3427	1696.2	
					7554.7

Table B-5. Standardized (censored/full data), and scaled arithmetic observed CPUE indices.

Year	St. CPUE		Arithmetic
	CPUE	CV	CPUE
1977	2.03	0.32	2.06
1978	3.87	0.16	4.31
1979	1.30	0.23	1.78
1980	1.64	0.27	1.86
1981	0.57	0.19	0.72
1982	0.25	0.15	0.30
1983	0.50	0.18	0.65
1984	1.13	0.19	0.96
1985	0.69	0.17	0.66
1986	2.24	0.47	2.01
1987	0.88	0.33	0.68
1988	2.16	0.41	1.66
1989	0.99	0.29	0.79
1990	1.47	0.47	1.23
1991			
1992	0.17	0.22	0.18
1993	1.02	0.09	1.22
1994	0.43	0.17	0.79
1995	1.08	0.13	0.49
1996	1.01	0.09	0.64
1997	1.14	0.09	1.03
1998	1.3	0.13	0.74
1999	0.97	0.1	0.63
2000	2.08	0.11	1.56
2001	0.76	0.26	0.78
2002	0.76	0.1	1.23
2003	1.65	0.09	1.02
2004	1.36	0.07	1.59
2005	0.64	0.12	1.48
2006	0.93	0.1	1.62
2007	0.89	0.22	1.18
2008	1.27	0.05	1.14
2009	0.87	0.04	0.82
2010	1.27	0.05	1.06
2011	1.46	0.05	1.36
2012	1.29	0.04	1.25
2013	0.67	0.04	0.67
2014	1.01	0.04	0.98
2015	1.26	0.05	1.20
2016	1.1	0.05	1.20
2017	0.94	0.05	1.00
2018	0.54	0.05	0.67
2019	0.26	0.06	0.30
2020			
2021			
2022	1.41	0.07	1.61
2023	2.13	0.07	1.91

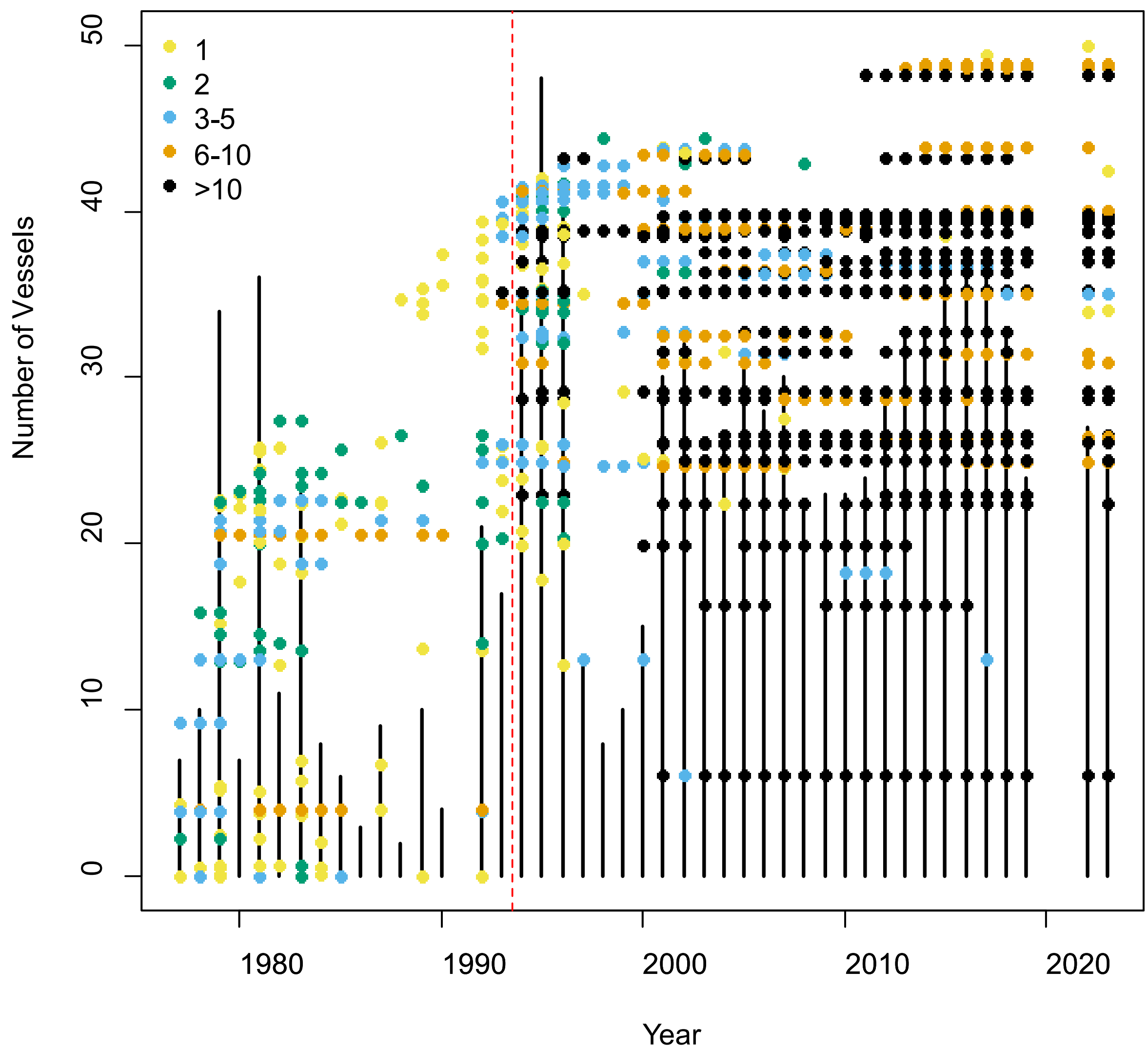


Figure B1. Number of fishing vessel (Vertical line) and distribution of unique vessel (dots) operated by year. Dot colors indicate the number of deliveries for each year by each vessel. Dashed red vertical line indicates a break between pre- (1977-1993) and post- (1994-2023) superexclusive fishery. No fishery occurred in 1993, and no fishery harvest occurred in 2020 and 2021.

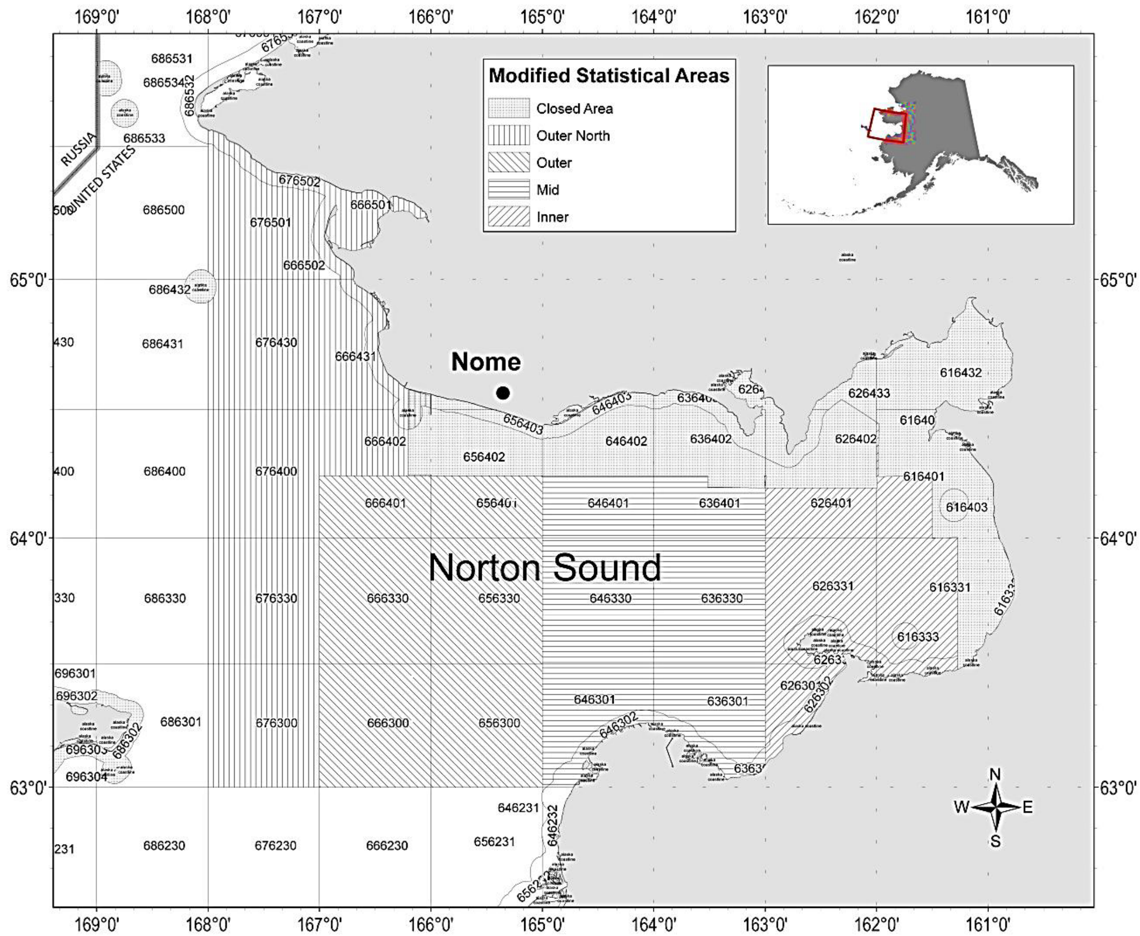


Figure A2-1. Closed area and statistical area boundaries used for reporting commercial harvest information for red king crab in Registration Area Q, Northern District, Norton Sound Section and boundaries of the new *Modified Statistical Areas* used in this analysis.

Appendix C

Norton Sound Red King Crab Summer Commercial Fishery Discard Estimation

Formal methodologies for estimating discards in the Norton Sound red king crab summer commercial fishery from observer data have not been established. Here, I describe a few methods and discuss pros and cons of each method.

Norton Sound Summer Commercial fishery observer coverage started in 2009 as a feasibility project, but formal data collection started in 2012 and terminated in 2019. **The main objective of the observer coverage was to gain information about the size composition of discarded crab, NOT to estimate total discards.** Because of this, carrying fishery observers was optional/voluntary and participation was **limited to vessels that are large enough to carry a fishery observer (a portion of the fleet are of a vessel length too small for an additional person). Thus, participating fishermen/vessels are NOT representative of the entire fleet.** The fishery observer worked as a crew member, but also recorded biological data including sex, carapace size, shell condition, etc. for all red king crab in selected pots. Fisherman sorted out discards and noted those individuals, and as such, **observed discarded crab are deemed accurate.** Because of the observer coverage is biased towards larger vessels, it is uncertain whether fishing behaviors of observed vessels are representative of unobserved vessels. Possible concerns include:

1. The participating fishermen have larger boats and are experienced. They may select better fishing grounds (e.g., higher number and proportions of legal-size crab relative to sub-legal size crabs). This leads to **higher CPUE and lower discards**.
2. The participating fisherman may allow observers when they expect higher discards. Additional free labor deckhand (i.e., observer) is always helpful. This leads to **higher discards**.
3. The participating fisherman may keep more (with catcher-seller permits) legal crab that are not accepted by NSEDC.
4. Unobserved small boat fisherman may keep more legal crab that are not accepted by NSEDC . (catcher-seller permits, personal-subsistence use).

Estimation Methods

Every discard estimation method is based on the following data (Table 1)

Observer survey data	Fish Ticket data
Sublegal crab discards (n_{sub}) and weight (w_{sub})	*NA*
Legal crab discards (n_{ld}) and weight (w_{ld})	*NA*
Legal crab retained (n_r) and weight (w_r)	Total Legal crab retained (N_R) and weight (W_R)

Female crab discards (n_f) and weight (w_f)	
Pot lifts (e)	Total Pot lifts (E)
Total discards ($n_d = n_{sub} + n_{ld}$) and weight ($w_d = w_{sub} + w_{ld}$)	*NA*
Total catch ($n_t = n_{sub} + n_{ld} + n_r$) and weight ($w_t = w_{sub} + w_{ld} + w_r$)	*NA*
Discards CPUE ($Cpue_d = n_d/e$) and by weight ($Cpue_d = w_d/e$)	*NA*
Total catch CPUE ($Cpue_t = n_t/e$) and by weight ($Cpue_t = w_t/e$)	*NA*
Discards/Retain ratio ($r_d = n_d/n_r$) and by weight ($r_d = w_d/n_r$)	*NA*
Discards size composition ($p_{dis,l}$)	*NA*

Note: female discards are not included because the NSRKC assessment model is male-only model.

LNR method

LNR method simply **expands observed discards CPUE (*$cpue_d$*)** to total pot lifts. This method assumes **that discarded crab are accurately accounted for** and that observed discards CPUE ($cpue_d$) is representative of all fishermen.

$$cpue_d = \frac{n_d}{e} \qquad D_{LNR} = cpue_d \cdot E \qquad (1)$$

LNR2 method

Observer bias corrected LNR method (LNR2) acknowledges that the observer discard CPUE may not be representative of all fishermen. Thus the CPUE is adjusted via taking retained CPUE by observed fishermen to all fishermen as follows:

Observed vessel retained catch $CPUE_{R,s} = \frac{N_{R.s}}{E_s}$ Entire fleet retained catch $CPUE_R = \frac{N_R}{E}$

Where $N_{R.s}$ and E_s are total number of retained crab and pot lifts of the observed fishermen from the fish ticket database, and N_R and E total number of retained crab and pot lifts by all fishermen. Then

$$D_{LNR2} = \left(\frac{CPUE_R}{CPUE_{R,s}}\right) \cdot D_{LNR} = \left(\frac{N_R}{E \cdot CPUE_{R,s}}\right) \cdot cpue_d \cdot E = \frac{cpue_d}{CPUE_{R,s}} N_R = r_{LNR2} \cdot N_R \quad (2)$$

Subtraction method

Subtraction method expands **total catch CPUE** and subtracts total retained catch. This method does **NOT** assume **accurate discarded crab** but assume **accurate total catch crab**

$$cpue_t = \frac{n_t}{e} \qquad D_{Sub} = cpue_t \cdot E - N_R$$

Subtraction2 method

Similar to LNR2, bias corrected Subtraction method is simply bias corrected total catch minus retained catch

$$D_{Sub2} = \left(\frac{CPUE_R}{CPUE_{R,s}}\right) \cdot cpue_t \cdot E - N_R = \left(\frac{cpue_t}{CPUE_{R,s}} - 1\right) \cdot N_R = r_{sub2} \cdot N_R \qquad (3)$$

Ratio method

The ratio method uses the identical method used in the assessment model, that multiplies the observed discards to retained catch ratio with total retained catch. This method assumes observed discards to retained ratio is accurate and representative.

$$D_{ratio} = \frac{n_d}{n_{lr}} N_R = r_d \cdot N_R \qquad (4)$$

Estimation of discard mortality biomass

One of the main objectives of estimating discard is calculating discard mortality biomass (Mb_{dis}) that is calculated as follows

$$Mb_{dis} = 0.2 \cdot D_n \cdot W_{dis} \qquad (5)$$

where, D_n is the number of discards, W_{dis} is average weight discarded crab, and 0.2 is assumed handling mortality rate.
W_{dis} is calculated as

$$W_{dis} = \sum_l p_{dis,l} \cdot wm_l \qquad (6)$$

where $p_{dis,l}$ is the proportion of discarded crab size class (l) and wm_l is average weight (lb) for each size class (Table 3).

Direct discard mortality biomass estimation method

Alternatively, the above methods can be converted directly to biomass using observed weights w_d and w_r or by using the equation (6), such that

$$w_d = n_d \sum_l p_{dis,l} \cdot wm_l \ , \ w_r = n_r \sum_l p_{r,l} \cdot wm_l \ , \ w_t = w_d + w_r,$$

$$CPUE_{R,s} = \frac{W_{R.s}}{E_s}, \qquad CPUE_R = \frac{W_R}{E}$$

Then all the above 5 methods can be converted to

LNR.lb method

$$cpue_d = \frac{w_d}{e} \qquad Mb_{LNR} = 0.2 \cdot cpue_d \cdot E$$

LNR2.lb method

$$Mb_{LNR2} = 0.2 \cdot \frac{cpue_d}{CPUE_{R,s}} W_R = 0.2 \cdot r_{LNR2} \cdot W_R$$

Sub.lb method

$$cpue_t = \frac{w_t}{e} \qquad Mb_{Sub} = 0.2 \cdot \left(cpue_t \cdot E - W_R \right)$$

Sub2.lb

$$Mb_{Sub2} = \left(\frac{cpue_t}{CPUE_{R,s}} - 1 \right) \cdot W_R = 0.2 \cdot r_{sub2} \cdot W_R$$

Ratio.lb

$$Mb_{ratio} = 0.2 \frac{w_d}{w_r} W_R = 0.2 \cdot r_{ratio} \cdot W_R$$

Results

Overall subtraction method appeared to give higher discard mortality than other methods. Between the number and lb methods, LNR and LNR.lb methods were identical, and discrepancies were under 5% for LNR2 and ratio methods. On the other hand, subtraction method (Sub, Sub2) had +/- 60% differences.

Discussion

As stated, the NSRKC observer survey was not designed or intended to estimate discards, and this estimation was conducted at the request of the CPT and SSC. Methods using CPUE (LNR, LNR2, Sub, Sub2) assumes that observed vessels are representative of the entire fleet. Difference between LNR and Subtraction method is that LNR method assumes that observed discards are accurate whereas subtraction method assumes that observed discards are biased but observed total catches are accurate. On the other hand, the ratio method assumes that observed discard proportions would represent total proportion or that every fisherman has a similar crab composition.

Estimates of discarded crab are more likely to be accurate on the observed vessels because retained and discarded crab are distinguished in cooperation with the fishermen. However, these estimates are likely biased low relative to the entire fleet because of the fact that observer coverage is voluntary and generally limited to larger boats which are generally more efficient in catching legal crab with fewer discards than those with small boats. In addition, fisherman may volunteer for observer coverage when catches are anticipated to be high. This is generally supported by fish ticket data where total season retained catch CPUE is generally higher by observed fishermen than unobserved fishermen (Table 2a,b). and retained catch CPUE is generally higher during periods when observers are on board. When observers were on board, fishermen went to different fishing areas from the rest of the fleet including those without observers (Table 4). Because of this nonuniformity in fishing behavior, total catch and discard estimation for the entire fishery is likely inaccurate and difficult to evaluate including the directionality of the bias. In the absence of TRUE observation, relative accuracies of the estimates among the 10 methods were highly uncertain. Furthermore, in the absence of objective criteria for selecting a method for estimation, it is difficult to choose the most appropriate method for the NSRKC fishery.

Table 1a. Observed pot lifts, catch, and total pot lifts and catch from 2012 to 2019

	Observer Survey						
Year	Pot lifts E	Sublegal n_{sub}	Legal retained n_r	Legal discards n_{ld}	Female n_f	*Discarded lb*	*Retained lb*
2012	82	1,025	1,112	177	155	1,404	3,210
2013	190	2,647	2,109	258	120	2,648	6,172
2014	141	1,472	1,752	315	103	2,684	5,252
2015	69	969	1,676	577	224	2,635	4,495
2016	67	264	1,700	169	877	710	4,840
2017	108	432	2,174	122	373	845	6,731
2018	77	547	1,095	10	573	678	3,583
2019	28	123	142	1	89	116	432

Table 1b Fish tickets

	All fishermen			Sampled fishermen		
Year	pot lifts E	Retained N_R	Retained lb	pot lifts E_s	Retained N_{Rs}	Retained lb
2012	10,041	161,113	475,990	3,595	52,185	154,444
2013	15,058	130,603	391,863	7,545	74,466	223,725
2014	10,124	129,656	389,004	3,729	53,741	161,573
2015	8,356	144,224	4,011,112	2,323	49,986	138,936
2016	8,009	138,997	420,159	1,882	45,225	135,581
2017	9,401	135,322	411,736	2,079	37,767	116,701
2018	8,797	89,613	298,396	2,494	26,031	88,095
2019	5,436	24,913	75,023	949	4,458	13,114

Table 2a. Estimated quantity: number method

Year	$cpue_d$	$cpue_t$	$CPUE_{R,s}$	$CPUE_R$	r_{LNR2}	r_{sub2}	r_d
2012	14.66	28.22	14.52	16.05	1.01	0.94	1.08
2013	15.29	26.39	9.87	8.67	1.55	1.67	1.38
2014	12.67	25.10	14.41	12.80	0.88	0.74	1.02
2015	22.41	46.70	21.52	17.26	1.04	1.17	0.92
2016	6.46	31.84	24.03	17.36	0.27	0.32	0.25
2017	5.13	25.26	18.17	14.33	0.28	0.39	0.25
2018	7.23	21.45	10.44	10.19	0.69	1.06	0.51
2019	4.43	9.50	4.70	4.58	0.94	1.02	0.87

Average	11.0	26.81	14.71	12.66	0.83	0.92	0.79

Table 2b. Estimated quantities: lb method

Year	$cpue_d$	$cpue_t$	$CPUE_{R,s}$	$CPUE_R$	r_{LNR2}	r_{sub2}	r_d
2012	17.13	56.28	42.96	47.40	0.40	0.31	0.44
2013	13.94	46.42	29.65	26.02	0.47	0.57	0.43
2014	19.04	56.29	43.33	38.41	0.44	0.30	0.51
2015	38.18	103.33	59.81	48.00	0.64	0.73	0.59
2016	10.59	82.83	72.04	52.46	0.15	0.15	0.15
2017	7.82	70.15	56.13	43.62	0.14	0.25	0.13
2018	8.81	55.34	35.32	33.92	0.25	0.57	0.19
2019	4.14	19.57	13.82	13.80	0.30	0.42	0.27
Average	14.96	61.27	44.13	37.96	0.35	0.41	0.34

Table 3 discarded crab size proportions ($p_{dis,l}$) and calculated W_{dis}.

Size class	34	44	54	64	74	84	94	104	114	124	134		W_{dis}
Average weight (lb) (wm_l)	0.09	0.18	0.32	0.52	0.82	1.20	1.70	2.32	2.99	3.69	4.37		
2012	0.00	0.01	0.12	0.20	0.12	0.16	0.28	0.10	0.01	0.00	0.00		1.17
2013	0.00	0.02	0.11	0.29	0.25	0.14	0.15	0.04	0.00	0.00	0.00		0.91
2014	0.00	0.00	0.01	0.04	0.10	0.27	0.43	0.13	0.01	0.00	0.00		1.50
2015	0.00	0.00	0.00	0.02	0.08	0.18	0.47	0.21	0.03	0.01	0.00		1.70
2016	0.00	0.00	0.01	0.04	0.05	0.17	0.53	0.18	0.02	0.00	0.00		1.64
2017	0.00	0.00	0.02	0.10	0.16	0.14	0.30	0.26	0.01	0.00	0.00		1.53
2018	0.00	0.00	0.04	0.09	0.18	0.36	0.30	0.02	0.00	0.00	0.00		1.22
2019	0.02	0.05	0.18	0.24	0.10	0.12	0.27	0.02	0.00	0.00	0.00		0.93
Average	0.00	0.01	0.06	0.13	0.13	0.19	0.34	0.12	0.01	0.00	0.00		1.33

Table 4. The number of discarded crab estimated by 5 methods via **number method**.

Year	D_{LNR}	D_{LNR2}	D_{Sub}	D_{Sub2}	D_{ratio}
2012	147,186	154,492	122,239	136,303	174,153
2013	230,229	202,324	266,770	230,229	179,896
2014	128,347	114,021	124,525	128,347	132,246
2015	187,223	150,175	245,965	187,223	133,037
2016	51,760	37,382	115,976	51,760	35,403
2017	48,424	38,212	103,125	48,424	34,484
2018	63,635	62,107	99,123	63,635	45,584
2019	24,074	23,486	26,729	24,074	21,755

Table 5a. Discard mortality (lb) by 5 methods via **number method**.

Year	*LNR*	*LNR2*	*Sub*	*Sub2*	*Ratio*
2012	34,395	36,102	28,565	31,851	40,696
2013	41,969	36,882	48,630	41,969	32,794
2014	38,560	34,256	37,411	38,560	39,731
2015	63,815	51,187	83,837	63,815	45,345
2016	16,968	12,255	38,020	16,968	11,606
2017	14,773	11,658	31,462	14,773	10,521
2018	15,492	15,120	24,131	15,492	11,097
2019	4,496	4,386	4,992	4,496	4,063

Table 5b. Discard mortality (lb) by 5 methods via **weight method.**

Year	*LNR.lb*	*LNR2.lb*	*Sub.lb*	*Sub2.lb*	*Ratio.lb*
2012	343,95	37,952	17,817	29,507	41,647
2013	41,969	36,833	61,419	44,313	33,624
2014	38,560	34,184	36,199	23,264	39,766
2015	63,815	51,218	92,456	58,370	47,025
2016	16,968	12,356	48,652	12,590	12,322
2017	14,773	11,479	50,099	20,564	10,338
2018	15,492	14,877	37,693	33,826	11,291
2019	4,496	4,490	6,267	6,239	4,021

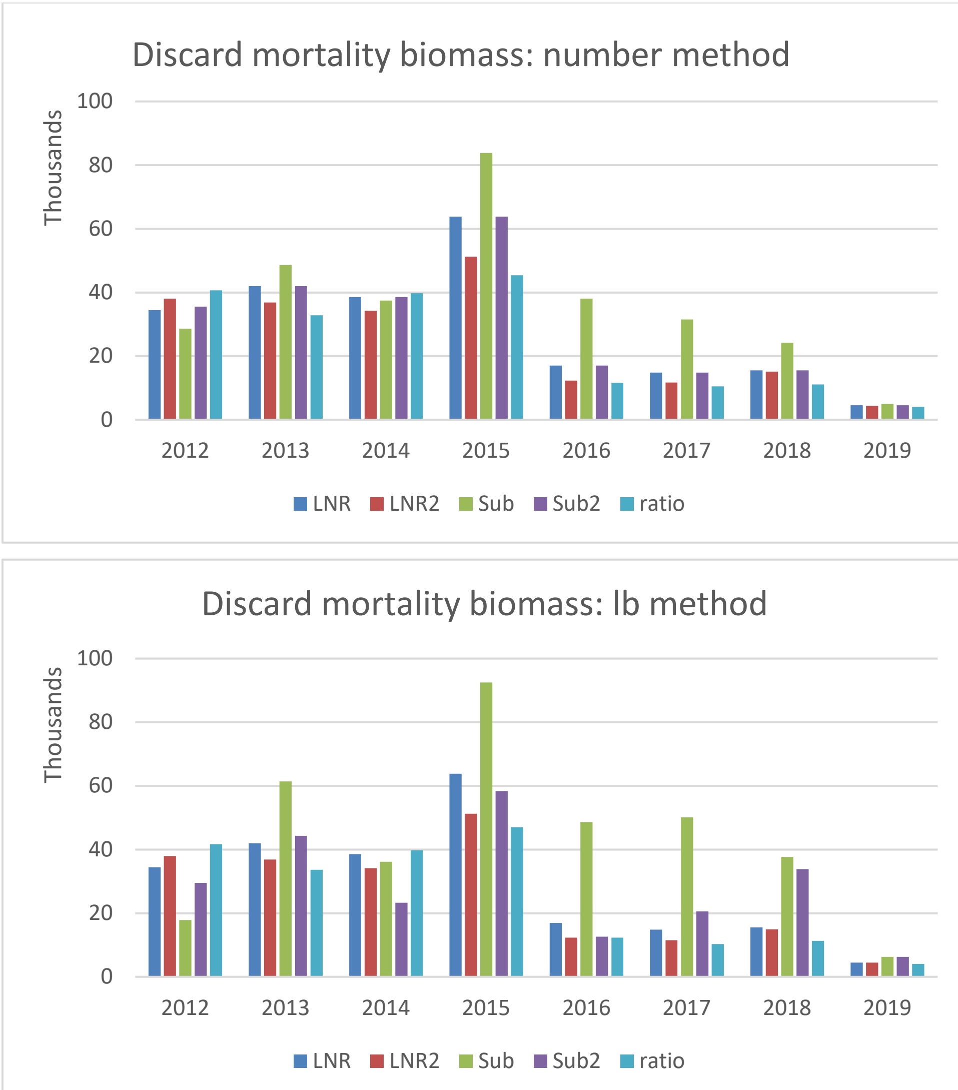


Figure 1. Discarded crab mortality biomass estimated by 5 proposed methods.

Discards Estimate without observer survey

Total catch OFL-ABC of NSRKC have been calculated since adoption of the NSRKC assessment model; however, it was not adopted because of the lack of discard estimate. Total catch OFL-ABC for NSRKC was set for the first time in 2020 based on the fact that discards could be estimated for 2012-2019, but in the same year the NSRKKC fishery observer program was terminated. This made it impossible to assess annual catch limit (ACL) overage for the NSRKC fishery. This prompted request by CPT-SSC to explore a method to estimate discards with NO DATA. Given that the NSRKC observer survey was not intended to estimate discards, developing a method is highly speculative.

There are 3 general approaches estimating discards for future fisheries in the absence of observer data:

1. Apply averages on observed retained catch and effort
2. Predict discards from observed retained catch and effort
3. Predict discards from observed crab size composition

Approaches 1 & 2

Approach 1

1. Apply averages of $cpue_d$, $cupe_t$, r_{LNR2}, r_{sub2} and r_d of the lb method (Table 2b)
2. Calculate average discards mortality/retained weight ratio of the 2012-2019 surveys.

Table 6: discard mortality weight/retained weight ratio of the 5 estimation methods.

Year	*LNR*	*LNR2*	*Sub*	*Sub2*	*Ratio*
2012	0.072	0.080	0.037	0.062	0.087
2013	0.107	0.094	0.157	0.113	0.086
2014	0.099	0.088	0.093	0.060	0.102
2015	0.159	0.128	0.230	0.146	0.117
2016	0.040	0.029	0.116	0.030	0.029
2017	0.036	0.028	0.122	0.050	0.025
2018	0.052	0.050	0.126	0.113	0.038
2019	0.060	0.060	0.084	0.083	0.054
Average	0.078	0.070	0.121	0.082	0.067

Approach 2: Construct a linear regression of predicting $cpue_d$, $cupe_t$, $CPUE_{Rs}$, and r_c from observed $CPUE_R$.

Table 7: linear regression equation

	Regression equation	R^2
$cpue_d$	$cpue_d = 0.4037+0.3834CPUE_R$	0.22
$cpue_t$	$cpue_t = -1.5427+1.655CPUE_R$	0.74
$CPUE_{Rs}$	$CPUE_{Rs} = -6.2385+1.3271CPUE_R$	0.87
r_d	*No correlation*	

In 2022, total potlift (E) was 5154, and total number of retained crab was 125042, total weight was 317173, and $CPUE_R$ was 61.54. Applying those, estimated quantities are as follows.

Table 8: average and predicted quantities for 2022 fishery

	Average	*Regression*
$cpue_d$	14.96	24.00
$cpue_t$	61.27	100.30
$CPUE_{Rs}$		75.43
r_{LNR2}	0.35	0.32
r_{sub2}	0.41	0.33
r_d	0.34	

Applying those to the equations, estimated discard mortality biomass (lb) of 2022 was

Table 9: The number of discards and regression method.

	LNR	*LNR2*	*Sub*	*Sub2*	*Ratio*
Regression	24,737	20,181	199,797	104,594	
Average	15,416	22,055	-272	26,041	21,355
Average lb	24,806	22,055	38,261	26,041	21,355

Approach 3: Predict discards from observed trawl survey crab size composition

Trawl survey selectivity method uses the same method for estimating discards (Appendix A, equations 8). **Trawl survey length proportion data as a proxy for true length proportions.** The model estimated trawl survey selectivity is 1.0 for all lengths. This assumes that trawl survey length composition equals NSRKC length proportion subject to fishery.

Discards length proportion $p_{dis,l}$ can be estimated by multiplying model estimated fishery selectivity (S_l) and 1- retention probability ($S_{ret,l}$)

$$p_{dis,l} = p_{twl,l} \cdot S_l \cdot \left(1 - S_{ret.l}\right)$$

Then calculate discards-retained ratio (r_{dis}) as

$$r_d = \frac{\sum_l p_{twl,l} \cdot S_l \cdot \left(1 - S_{ret,l}\right)}{\sum_l p_{twl,l} \cdot S_l \cdot S_{ret,l}}$$

The discard biomass unit (w_{dis}) is

$$W_{dis} = \frac{\sum_{l} p_{twl,l} \cdot S_l \cdot \left(1 - S_{ret,l}\right) \cdot wm_l}{\sum_{l} p_{twl,l} \cdot S_l \cdot \left(1 - S_{ret,l}\right)}$$

During the 2012-2019 periods, trawl survey occurred in 2014, 2017, 2018, and 2019. The table below shows trawl survey length proportion, and model estimated selectivity and retention probability from the 2021 assessment model

Table 10: Table: trawl survey size composition, fishery size selectivity (S_l), retention probability (S_{ret}), and estimated discard size composition.

Size	34	44	54	64	74	84	94	104	114	124	134
Trawl											
2014	0.01	0	0.01	0.01	0.07	0.14	0.25	0.27	0.14	0.06	0.02
2017	0.11	0.02	0.01	0.06	0.12	0.11	0.06	0.09	0.13	0.23	0.07
2018	0.02	0.33	0.42	0.08	0.05	0.02	0.02	0.01	0.01	0.01	0.02
2019	0	0	0.02	0.13	0.47	0.26	0.04	0.02	0.01	0.02	0.03
2022	0.12	0.03	0.04	0.14	0.15	0.15	0.14	0.12	0.07	0.03	0.01
S_l	0	0.01	0.04	0.12	0.33	0.64	0.86	0.96	0.99	1	1
S_{ret}	0	0	0	0	0	0	0.07	0.88	1	1	1
Discard											
2014	0	0	0.00	0.00	0.07	0.26	0.58	0.09	0	0	0
2017	0	0	0.00	0.04	0.22	0.40	0.27	0.00	0	0	0
2018	0	0.04	0.22	0.13	0.22	0.17	0.21	0.02	0	0	0
2019	0	0	0.00	0.04	0.42	0.45	0.09	0.01	0	0	0
2022	0	0.00	0.01	0.06	0.17	0.32	0.39	0.05	0	0	0

Comparing the estimated with observed, the estimated r_d tend to be higher than observed, especially 2018 and 2019.

Table 11 Comparisons of parameters between trawl survey method and ratio (number) method.

	r_d	W_{dis}	Ob.r_d	Ob.W_{dis}	*Pred* Mb_{dis}	Ob. Mb_{dis}
2014	0.75	1.57	1.00	1.50	30,300	38,967
2017	0.35	1.28	0.25	1.53	12,060	11,748
2018	1.54	0.92	0.51	1.22	25,238	10,421
2019	4.70	1.05	0.87	0.93	24,842	10,852
2022	1.40	1.34			47,024	

Comparison of methods

Putting the above methods together, 21 discard catch mortality were calculated. Total catch ranged from 0.35 to 0.39 million lb and below ABC of 0.4 million lb.

Table 12 estimates of 2022 total catch based on the 15 methods.

	2022 Total Catch (million lb)
Regression	
LNR	0.36
LNR2	0.36
Sub	0.54
Sub2	0.44
Average	
LNR	0.35
LNR2	0.36
Sub	0.34
Sub2	0.37
Ratio	0.36
Average lb	
LNR	0.36
LNR2	0.36
Sub	0.38
Sub2	0.37
Ratio	0.36
Trawl	0.39

Discussion

As presented the above, overage of ACL is highly depended on *ad hoc* estimation methods being selected. This suggests that a method has to be selected on the merit of scientific accuracy and precision before total catch is calculated. The 15 alternatives presented the above are examples and there could be alternative methods that would provide more accurate and precise estimates. Same as the discussion regarding selecting a method for estimating discards with data, objective criteria for selecting a method for estimating discards without data are not established, and thus author’s recommendation is not provided.

Regardless the method being ultimately selected, a question of jurisprudence should be answered first: “should ACL overage that has significant regulatory consequences be determined by an estimate based on NO data?”

The total ABC of NSRKC is calculated as

Norton Sound red king crab CPUE standardization

Total ABC = ABC_Buffer·(retained OFL + 0.2·discards OFL) = $Mb_{R,p} + Mb_{dis,p}$

Based on the preseason ABC, GHL is determined as

GHL < ABC_Buffer·(retained OFL) = $Mb_{R,p}$

Which assumes that discards morality (Mb_{dis}) would be

$$Mb_{dis} = \frac{Mb_{dis,p}}{Mb_{R,p}} \cdot Mb_R$$

And thus, the postseason total catch ($Mb_R + Mb_{dis}$) would be less than ABC unless Mb_R far exceeds GHL.

In reality; however, the projected discard mortality do not always match the observed one. During the 2012-2019 period, observed ratio of discard mortality/retained was up to 8.75 times greater than projected (Table).

Table: Projected and observed mort_lb and "observed" /predicted *mort_lb*$_b$ ratio during the 2012-2019 fisheries.

	2012	2013	2014	2015	2016	2017	2018	2019
Projected	0.010	0.019	0.028	0.045	0.047	0.042	0.037	0.059
Retrospective	0.062	0.091	0.110	0.069	0.035	0.029	0.039	0.083
Observed								
Obs. LNR	0.072	0.107	0.099	0.159	0.040	0.036	0.052	0.060
Obs. LNR2	0.080	0.094	0.088	0.128	0.029	0.028	0.050	0.060
Obs. Sub	0.037	0.157	0.093	0.230	0.116	0.122	0.126	0.084
Obs. Sub2	0.062	0.113	0.060	0.146	0.030	0.050	0.113	0.083
Obs. Ratio	0.087	0.086	0.102	0.117	0.029	0.025	0.038	0.054
Ob/Project ratio								
Retrospective	6.20	4.79	3.93	1.53	0.74	0.69	1.05	1.41
LNR	7.23	5.64	3.54	3.54	0.86	0.85	1.40	1.02
LNR2	7.97	4.95	3.14	2.84	0.63	0.66	1.35	1.01
Sub	3.74	8.25	3.32	5.12	2.46	2.90	3.41	1.42
Sub2	6.20	5.95	2.14	3.23	0.64	1.19	3.06	1.41
Ratio	8.75	4.52	3.65	2.61	0.62	0.60	1.02	0.91

For 2022, projected mort_lb was 0.058 and retrospective (model 21.0) mort_lb was 0.065, which can be translated into projected and retrospective total catch of 0.36 million lb.