# AN EXPLORATION OF ALTERNATIVE GULF OF ALASKA REX SOLE ASSESSMENT MODELS 

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## INTRODUCTION

The purpose of this document is to outline a proposed change from conducting assessments using the previously used rex sole assessment model framework to conducting assessments using Stock Synthesis version 3.24Y (SS3; Methot and Wetzel 2013).

Previous assessments were conducted using an ADMB-based age- and sex-structured population dynamics model with length-at-age, weight-at-length, maturity-at-age, and age-length transition matrices estimated outside of the model. The previous model estimated the log of mean recruitment, parameters for logistic age- and sex-specific selectivity curves for the fishery and survey, recruitment deviations, and yearly fishing mortality rates. The model included ages 3-20 (age 20 was a plus group) and excluded data for fish younger than age 3 and smaller than 9 cm in length.

SS3 is a flexible assessment model framework that extends the capabilities of the 2011 rex sole assessment model to address the concerns expressed by the GOA Plan Team, the SSC, and previous rex sole assessment authors. As an initial effort towards addressing these concerns, this document outlines a framework designed to begin to resolve these issues and transition the rex sole assessment to the SS3 framework. The new application allows for clear specification of alternative models that can easily deal with concerns and issues that have been raised. In particular:
(1) Length-at-age data or mean weight-at-age data can be included and used to help estimate growth (within the assessment model). This will facilitate incorporating recent data on GOA rex sole growth and allow comparisons of estimates done outside of the model.
(2) Alternative functional forms of selectivity curves are available and can be used to explore length-based and dome-shaped fishery selectivity for rex sole; the previous model used logistic age-based selectivity.
(3) Age-determination error can be included and evaluated; previously ages were assumed to be "perfect".
(4) Multiple survey and fishing fleets can easily be included in the model and hence allow for easy explorations of including ADF\&G bottom trawl survey data; the previous model accommodated only one fishery and one survey.
(5) Adjustments to account for mid-year weight-at-age can be allowed which would more accurately match biological processes that occur during the year relative to fishing and survey timing.
(6) Fishery age composition data can easily be applied and included.
(7) Options for modeling growth can account for different "morphs" and/or samples for different areas. Anecdotal evidence exists that there may be two GOA rex sole growth patterns (e.g., the eastern GOA may be different from other areas); this can be explored in the new application.

## SSC AND PLAN TEAM COMMENTS ON PREVIOUS ASSESSMENTS

SSC comment, 12/2011: The SSC appreciates the authors' responses to previous comments and looks forward to analysis of new growth data that may influence the assessment and may shed light on stock structure. Ultimately, if growth data point toward more than one GOA stock, then the approved stock separation template should be applied in the future for a more complete evaluation of stock structure. The SSC also looks forward to the incorporation of new fishery age composition data into the assessment model. The SSC supports the authors' expressed intentions to explore length-based approaches to survey and fishery selectivity, as well as alternative forms of the selectivity curve and exploration of potential environmental effects on recruitment. In this vein, environmental effects (e.g., temperature) on survey catchability might also be considered, as was done for several flatfish stocks in the Bering Sea.

Author Response: The SS3 framework will allow exploration of more than one growth pattern, internal estimation of growth, inclusion of fishery age composition data, length-based approaches to survey and fishery selectivity, alternative forms of the selectivity curve, and exploration of potential environmental effects on recruitment. An environmental effect on survey catchability could also easily be explored.

GOA Plan Team comment 11/2013: The survey averaging working group will continue to explore apportionment methods and the authors may consider incorporating their recommendations for apportionment contingent on the findings of this group.

Author Response: CRM will consider any requested alternative apportionment methods in the November SAFE document.

## ANALYTIC APPROACH: TRANSITION OF 2011 MODEL INTO AN EQUIVALENT SS3 MODEL

Matching population dynamics between models

## Mean recruitment

Several steps were taken to build an SS3 model with population dynamics that matched those of the 2011 model using deterministic models with no estimation of parameters and no recruitment deviations. First, the relationship between the log of mean recruitment estimated in the 2011 model $(\ln (\bar{R}))$ and the $\log$ of $R_{0}$ (unfished recruitment $\left(\ln \left(R_{0}\right)\right)$ that is estimated in SS3 was determined (Equation 1), where $M$ is natural mortality.

$$
\begin{equation*}
\ln \left(R_{0}\right)=\ln \left(\frac{2 \bar{R}}{1000}\right)+3 M \tag{1}
\end{equation*}
$$

The $\ln (\bar{R})$ estimated in the 2011 model refers to female mean recruitment of age 3 individuals, while $\ln \left(R_{0}\right)$ refers to total recruitment (males and females) of age 0 individuals in thousands; both models assume a $1: 1$ sex ratio (but any sex ratio can be specified in SS3; a different sex ratio would change Equation 1). Using Equation 1, equivalent deterministic runs were conducted
with fixed parameters at their maximum likelihood estimates (MLEs) from the 2011 model. This was to ensure that both models had the same behavior in the absence of estimation. Equation 1 was required to ensure that numbers at age 3 and above are the same in both models for an unfished population.

## Selectivity

The 2011 model assumed sex-specific age-based logistic selectivity functions for fishery and survey selectivity. Although SS3 has logistic, sex-specific selectivity, the specification of male logistic age-based selectivity in SS3 was difficult to cast into a logistic shape. Sex-specific length-based logistic selectivity can be specified such that selectivity can be estimated for both sexes while retaining the logistic shape, or age-based double normal selectivity curves could be specified with a large value for the standard deviation of the descending limb such that asymptotic, logistic-like, sex-specific selectivity could be estimated. In the interest of matching the 2011 model as closely as possible, the age-based, sex-specific double normal selectivity curves with no descending limbs with selectivity below age 3 set to 0 were used for fishery and survey selectivity curves. The fishery selectivity curves in SS3 were matched as closely as possible to the age-based logistic curves from the 2011 model for the purpose of comparing population dynamics between the models and are a near-exact match (but were logistic for the 2011 model and double-normal for the SS3 model; Figure 1). Deterministic runs conducted for rex sole using the fishery selectivity curves in Figure 1 led to nearly the same time series of SSB for both models (Figure 2), indicating that the population dynamics of the models are very similar. The difference in SSB between models that accumulates over time can be attributed to the small, irreconcilable differences between the fishery selectivity curves used in the 2011 model and those in SS3; the curves are different functional forms and hence will have different selectivity patterns.

## Stock-Recruitment

The 2011 model estimated recruits as median-unbiased recruitment deviations from their mean value. The SS3 model was configured similarly by specifying a Beverton-Holt stock-recruitment curve with a steepness of 1 . SS3 estimates mean-unbiased recruitment deviations by specifying $\sigma_{R}$ and applying a bias adjustment factor. For the deterministic runs, $\sigma_{R}$ was set to $1.0 \mathrm{E}-06$, and for runs when recruitment deviations were estimated, $\sigma_{R}$ was set to 0.60 . The 2011 model estimated recruits (age 3) freely (i.e. no $\sigma_{R}$ ) and this constitutes a difference between the models. Both the 2011 and SS3 models estimated early (1965-1981), main (1982-2008), and future period recruits (2009-2011) without any bias adjustment. The 2011 model assigned an arbitrary weight of 5 to the sum of squared log- recruitment deviations for the early period. Weighting schemes such as identified in Methot and Taylor (2011) are less arbitrary and preferred for estimating stock-recruitment related reference points used for management (e.g., $B_{0}$ ).

## Growth

The 2011 model used empirical estimates of maturity-at-age sex-specific somatic weight-at-age based on work done by Abookire (2006). SS3 also can use similar empirically specified values for the calculation of spawning stock biomass and biomass-at-age (Figure 4). A benefit of using the SS3 framework is the ability to specify and estimate growth parameters internally. When
growth parameters are specified (instead of age-specific schedules), small differences arise between models because SS3 uses the beginning of the year weight-at-age to calculate SSB (like in the 2011 model), but uses mid-year weight-at-age to calculate exploitable and survey biomass (the 2011 model uses beginning-of-the-year weight-at-age for all calculations).

In addition, age-length transition matrices were specified directly in the 2011 model whereas in SS3 they are computed internally from specified von-Bertalanffy growth curve parameters and CVs in length-at-age. To match population dynamics between models, the CVs of the youngest and oldest age classes were estimated externally and specified within SS3. The resulting agelength transition matrices output from SS3 runs were not an exact match to those used in 2011. A new age-length transition matrix, estimated within or outside of the model, should be estimated because length-at-age data collected after 1996 are not currently used in calculations of growth parameters. SS3 provides ample flexibility to explore growth relationships within the assessment model whereas this option was unavailable in the 2011 model.

## Biomass

Differences in total biomass will occur between the models because SS3 includes ages 0-2. However, SSB and survey biomass were shown to be very similar in determistic models when selectivity curves were as similar as possible and other parameters were fixed (Figure 2 and Figure 3).

## Timing

Both the SS3 and 2011 model calculated spawning stock biomass, survey biomass, and recruitment at the beginning of the year. SS3 calculates exploitable biomass in the middle of the year, but a vector for weight-at-age was manually provided to SS3 which forced the model to use beginning-of-the year weight-at-age in the exploitable biomass calculation to match the 2011 model as closely as possible (Figure 4).

## Data used in SS3 and the 2011 Model

The same data used in the 2011 rex sole assessment model (Stockhausen et al. 2011, page 634) were used in the SS3 model: survey biomass, survey age- and length-compositions (triennial for 1984-1999 and biennial for 2001-2011), fishery length-composition data (1985-2011), and catch history (1984-2011). An important difference between the 2011 model and SS3 is that the youngest age class in the 2011 model (age 3) represents only age 3 individuals, while SS3 population dynamics begin at age 0 and consider the lowest age and length bins of data to be the proportion of individuals ages 0-3 and lengths 0 -the upper limit of the lowest length bin, respectively. Therefore, age- and length-composition data must include ages 0-2 and any lengths no matter how small in SS3, while the 2011 model omitted data on ages 0-2 (and excluded data on fish smaller than 9 cm ). Ignoring this difference between models will result in extreme differences between expected and observed age- and length-compositions for the youngest age and length bins when selectivity at these ages and lengths is greater than 0 . Typically, the fact that SS3 included data on ages 0-2 likely would inform estimates of selectivity at the lowest ages and hence improve recruitment estimates (especially in the most recent years). However, very few fish were collected that were smaller than the lowest age and length bin sizes. This
difference between the models was taken into account by specifying survey and fishery selectivity to be 0 for ages $0-2$ (Figure 1 and Figure 7).

Parameter Estimation in SS3 and the 2011 Model

## Parameters Estimated Inside the Assessment Model

SS3 and 2011 model runs were conducted with estimation of the log of mean recruitment, recruitment deviations, fishing mortality rates (using the same empirical growth vectors in both models), and selectivity parameters. Selectivity parameters for the fishery and survey were estimated; the location of peak selectivity and the width of the ascending limb of the selectivity curves were estimated in SS3 and the age at $50 \%$ selection as well as the slope of the logistic selectivity curves were estimated in the 2011 model (Figure 1 and Figure 7).

## Likelihood component for survey biomass index

Table 2 lists the likelihood components used in SS3 and the 2011 model. The likelihood component for the survey biomass index and the data used to calculate the survey biomass likelihood component are the same for both models. The 2011 model and SS3 survey biomass values match almost exactly in a deterministic model with no estimation (Figure 3).

## Age- and length-composition likelihood components

The age- and length-composition likelihood components in SS3 are identical to those in the 2011 model. However, as noted above, the observations of survey proportions-at-age and proportions-at-length differ among models in that the data given to SS3 includes the data given to the 2011 model in addition to the proportions of age $0-2$ fish and lengths below 9 cm . Therefore, the values of these likelihood components will differ but should have similar influences on overall model fits.

## Recruitment likelihood components

Recruitment likelihood components differ slightly between models. The 2011 model estimates recruitment deviations freely whereas the SS 3 model is constrained by the $\sigma_{R}$ value specified. Both models allow for estimating early-period (1947-1983), main-period (1984-2008), and lateperiod (2008-2011) recruitment deviations with constraints based on prior distribution assumptions, but the 2011 model also includes the early period recruitment deviations in the likelihood component for the main-period (Table 2). Within SS3, specifications of recruitment "periods" within the time series having different constraints is limited. In the 2011 model, the recruitment deviations (in log-space) for the main and late periods are specified sum to 0 . As noted above, the 2011 model uses arbitrary penalty weights on early recruitment deviations whereas SS3 requires a more formal prior distribution specification to constrain recruitment estimates. However, 2011 model results were relatively insensitive to alternative early-period recruitment penalty weights. The inclusion of early-period recruitment deviations as a separate likelihood component as well as part of the main-period recruitment deviations likely contributes to slightly to differences in initial numbers of recruits and SSB. Differences between models are smallest when modeling early, main, and late-period recruitment deviations as simple deviations
(without a sum-to-0 constraint) in SS3; this seems to be the most accurate representation of the 2011 model in that the influence of the $\sigma_{R}$ value is minimized.

## ANALYTIC APPROACH: PROPOSED ALTERNATIVE SS3 MODELS FOR 2015

The SS3 model framework facilitates the potential for the following analyses to be conducted:

- Inclusion of fishery age composition data to improve estimation fishery selectivity curves.
- Updating estimates of growth within or outside of the model to include data collected after 1996.
- Conducting runs with alternative survey and fishery selectivity curves, such as length-based asymptotic curves and age- or length-based double normal selectivity curves.
- Re-calculating survey biomass estimates for 2001 using the random effects modeling approach to account for the missing Eastern GOA region survey biomass and CV estimates (as has been done for the 2013 Dover sole assessment model).
- Re-evaluating effective sample sizes for age- and length-composition data. There are abrupt year-to-year changes in age-compositions that occur in the observations that are likely due to observation error. Using such high effective sample sizes may exclude some process errors which should be considered.
- Including ageing error in the model: the previous assessment models ignored ageing error.


## RESULTS: TRANSITION OF 2011 MODEL INTO AN EQUIVALENT SS3 MODEL

The 2011 and SS3 models each estimated a similar time series of numbers at age 3 (considered recruits in the 2011 model; Figure 5). Numbers at age 3 in the last few years of the time series were the most different between the models. However, data available to estimate recruitment in these years were limited. SSB estimates in the most recent years were similar in the two models, but the SS3 model resulted in larger estimates for SSB than those estimated by the 2011 model before 2001 and very similar, but slightly smaller estimates after 2001 (Figure 6). Differences in fishery selectivity curves exist and may explain the differences in the trajectories of SSB (Figure 7). SS3 fishery selectivity is higher for ages 3-10 (Figure 7). The modeling of selectivity curves is perhaps the biggest difference between SS3 and the 2011 model and hence unsurprising that estimates would differ. In addition, the two models estimated age-based selectivity curves for the fishery based only on length composition data. The inexact match between the age-length transition matrices between the models may contribute to a larger mismatch in estimates of fishery selectivity. The estimated survey selectivity curves were very similar between models and likely informed by the survey age composition data and less influenced by the age-length transition matrix (Figure 7). Figure 8 shows that observed and predicted survey biomass for the 2011 and SS3 models were similar, but estimates of survey biomass in early years were slightly larger.

The fits to survey age composition data were almost identical among models (Figure 9 and Figure 10). The fits to survey length composition data were similar, but differed slightly because the age-length transition matrices differed between models (Figure 11 and Figure 12). Some differences exist in fits to fishery size composition data between models (Figure 13 and Figure
14). This can be attributed to inexact matches in both fishery selectivity curves and age-length transition matrices between models. The addition of age 0-2 and small length data included in the SS3 model likely contribute to differences in numbers at age 3 and selectivity parameter estimates. Testing the extent to which the additional data contribute to differences is difficult since they are omitted from the 2011 model.

## SUMMARY AND DISCUSSION OF DIFFERENCES BETWEEN THE SS3 MODEL AND 2011 MODEL

The differences between the configurations of the 2011 model and the SS3 model are:
(1) Both models used asymptotic selectivity curves, but the SS 3 selectivity curves were parameterized with a double-normal with no descending limb (the standard deviation for the descending limb was set to a very high value), while the selectivity curves for the 2011 model were logistic. In addition, the 2011 model re-normalized the selectivity curves such that the largest selectivity occurs at 1 . The asymptotic double-normal can approximate the logistic curve, but varied slightly. SS3 does not have an option for normalizing the selectivity curves such that the greatest selectivity is always equal to 1 , but the curve can be specified such that the peak value is at 1 . SS3 runs were conducted with a restriction that peak selectivity must equal 1 (and be asymptotic).
(2) The configuration of the likelihood components for early-period and main-period recruitment deviations differs between models and the 2011 model up-weighted the early period recruitment likelihood component by 5 x its value.
(3) SS3 population dynamics begin at age 0 and 2011 model dynamics begin at age 3. The SS3 model is given additional data, which consist of survey age-compositions for ages 0 2 and length-compositions for lengths $0-8 \mathrm{~cm}$.
(4) The estimated age-length transition matrices differ slightly.

## LITERATURE CITED

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Stockhausen, W.T., M.E. Wilkins and M.H. Martin. 2011. 6. Gulf of Alaska Rex Sole. In Stock Assessment and Fishery Evaluation Report for the Groundfish Resources of the Gulf of Alaska. pp. 547-628. North Pacific Fishery Management Council, P.O. Box 103136, Anchorage AK 99510.

## TABLES

Table 1. Symbols used in this document.

| Symbol | Meaning |
| :--- | :--- |
| $x$ | sex |
| $a$ | age |
| $f$ | fleet (fishery or survey) |
| $t$ | time |
| $S_{f, x, a}$ | Selectivity for fleet $f$, sex $x$, and age $a$ |
| $N_{t, x, a}$ | Numbers at age $a$, time $t$, and sex $s$ |
| $w_{a}$ | Weight at age $a$ |
| $Z_{t, x, a}$ | Total mortality at age $a$, sex $s$, and time $t$ |
| timing | The timing of the survey during the year |
| $I_{t, f}$ | Observed survey biomass at time $t$ for fleet $f$ |
| $S B_{t, f}$ | Predicted survey biomass at time $t$ for fleet $f$ |
| $C V_{t, f}$ | CV of observed survey biomass at time $t$ for fleet $f$ |
| $n_{t, x, f}$ | Number of age-composition observations at time $t$ for sex $x$ <br> and fleet $f$ |
| $p_{t, x, f, a}$ | Observed proportion at age $a$, time $t$, fleet $f$, and sex $x$ |
| $\hat{p}_{t, x, f, a}$ | Predicted proportion at age $a$, time $t$, fleet $f$, and sex $x$ |
| $n_{2, t, x, f}$ | Number of length-composition observations at time $t$ for <br> sex $x$ and fleet $f$ |
| $p_{t, x, f, l}$ | Observed proportion at length $l$, time $t$, fleet $f$, and sex $x$ |
| $\hat{p}_{t, x, f, l}$ | Predicted proportion at length $l$, time $t$, fleet $f$, and sex $x$ |
| $R_{t}$ | Estimated mean recruitment in year $t$ |
| $\sigma_{R}$ | Recruitment CV (specified in SS3 only) |
| $b_{t}$ | Bias adjustment factor at time $t$ (specified in SS3 only) |
| $\mathrm{C}_{t}^{\text {obs }}$ | Observed catch at time $t$ |
| $\hat{\mathrm{C}}_{t}$ | Predicted catch at time $t$ <br> only) |
| $\sigma_{t, f}$ |  |

Table 2. Likelihood components used in the 2011 and SS3 models. Numbers in the component column are likelihood component weightings for: (SS3, 2011 Model).

| Component | SS3 | 2011 Model |
| :---: | :---: | :---: |
| Survey biomass $\left(S B_{t, f}\right)$ equation | $\sum_{x} \sum_{a} S_{f, x, a} N_{t, x, a} w_{a} \exp \left(-\operatorname{timing}\left(Z_{t, r, a}\right)\right)$ | $\sum_{x} \sum_{a} S_{f, x, a} N_{t, x, a} w_{a}$ |
| Survey biomass likelihood $(1,1)$ | $\sum_{t \in \text { survey } f} \frac{\left(\ln \left(\mathrm{I}_{t, f}\right)-\ln \left(S B_{t, f}\right)\right)^{2}}{2 \ln \left(C V_{t, f}^{2}+1\right)}$ | As for SS3 |
| Age composition $(1,1)$ | $\sum_{t} \sum_{x} \sum_{a} n_{t, x, f} p_{t, x, f, a} \ln \left(\frac{p_{t, x, f, a}}{\hat{p}_{t, x, f, a}}\right)$ | As for SS3 |
| Length Composition $(0.5,0.5)$ | $\sum_{t} \sum_{x} \sum_{l} n_{2, t, x, f} p_{t, x, f, 1} \ln \left(\frac{p_{t, x, f, l}}{\hat{p}_{t, x, f, l}}\right)$ | As for SS3 |
| Main period recruits $(1,1)$ | $\frac{1}{2}\left(\sum_{t=1984}^{2008}\left(\frac{R_{t}^{2}}{\sigma_{R}^{2}}+b_{t} \ln \left(\sigma_{R}^{2}\right)\right)\right.$ (sum to 0 constraint) | $\left.\sum_{t=1947}^{2008} R_{t}^{2} \text { (sum to } 0 \text { constraint }\right)$ |
| Early period recruits $(1,2)$ | $\frac{1}{2}\left(\sum_{t=1947}^{1983}\left(\frac{R_{t}^{2}}{\sigma_{R}^{2}}+\ln \left(\sigma_{R}^{2}\right)\right)\right)$ | $\sum_{t=1947}^{1983} R_{t}^{2} \text { (sum to } 0 \text { constraint) }$ |
| Late period recruits $(1,3)$ | $\frac{1}{2}\left(\sum_{t=2009}^{2011}\left(\frac{R_{t}^{2}}{\sigma_{R}^{2}}+\ln \left(\sigma_{R}^{2}\right)\right)\right)$ | $\left.\sum_{t=2009}^{2011} R_{t}^{2} \text { (sum to } 0 \text { constraint }\right)$ |
| $\begin{aligned} & \text { Catch } \\ & (30,30) \end{aligned}$ | $\sum_{t} \frac{\left(\ln \left(\mathrm{C}_{t}^{\text {obs }}\right)-\ln \left(\hat{\mathrm{C}}_{t}\right)\right)^{2}}{2 \sigma_{t, f}^{2}}$ | $\sum_{t}\left(\ln \left(\mathrm{C}_{t}^{\text {obs }}\right)-\ln \left(\hat{\mathrm{C}}_{t}\right)\right)^{2}$ |

## FIGURES



Figure 1. Fishery and survey selectivity for rex sole used in deterministic runs to match population dynamics between the 2011 and SS3 models. Selectivity curves are fixed at MLEs for fishery selectivity from the 2011 model. The SS3 selectivity curves pictured were created using a double-normal selectivity functional form with no descending limb, attempting to match the selectivity from the 2011 model as closely as possible. The 2011 model selectivity curves are logistic with likelihood components forcing maximum selectivity to equal 1 by age 20 .


Figure 2. Spawning stock biomass for a deterministic run of the 2011 and SS3 models with parameters fixed at the MLEs for the 2011 rex sole model with rex sole catch history and no recruitment deviations. Fishery and survey selectivity curves for the SS3 model were forced to match the 2011 model as closely as possible, but the matches are not exact, resulting in a small difference in spawning stock biomass that accumulates over time (Figure 1).


Figure 3. Survey biomass for the 2011 model (black solid line) and the SS3 model (blue dashed line) for a deterministic run with no estimation, parameters fixed at the same values in both
models, and fishery and survey selectivity curves in both models fixed to the curves shown in Figure 1.


Figure 4. Maturity and weight-at-age for males and females (also used as mid-year weight at age) for the 2011 model and an equivalent SS3 model. The lines match perfectly because both models use empirical vectors for each of the three relationships.


Figure 5. Numbers at age 3 for the 2011 model (black line) and an equivalent SS3 run (blue line). Both models estimate the log of mean recruitment, recruitment deviations for 1984-2011, an early period of recruitment deviations starting in 1965, fishing mortality rates, and asymptotic selectivity parameters (logistic for the 2011 model and double-normal for SS3). Survey data for ages $0-2$ and lengths $0-9 \mathrm{~cm}$ are included in the SS3 model, but not the 2011 model.


Figure 6. Spawning stock biomass (solid lines) and asymptotic 95\% confidence intervals (dotted lines) for the 2011 model (black lines) and SS3 (blue lines) for an equivalent SS3 model. Both models estimate the log of mean recruitment, recruitment deviations for 1982-2011, an early period of recruitment deviations starting in 1965, fishing mortality rates, and asymptotic selectivity parameters (logistic for the 2011 model and double-normal for SS3). Survey data for ages $0-2$ and lengths $0-9 \mathrm{~cm}$ are included in the SS3 model, but not the 2011 model.


Figure 7. Fishery and survey selectivity for the 2011 model (solid lines) and an equivalent SS3 model run (dotted and dashed lines).


Figure 8. Observed survey biomass (black dots) with $95 \%$ asymptotic confidence intervals (vertical black lines) and predicted survey biomass from the 2011 model (black line) and an equivalent SS3 model (blue line).


Figure 9. Observed (2011 model; solid black lines) and predicted (dashed lines) survey proportions-at-age for the 2011 model (dashed black lines) and an equivalent SS3 model run (dashed blue lines) for females. The SS3 model included data for age $0-2$ individuals, while the 2011 model included data from ages 3-20.


Figure 10. As for Figure 9, but for males.


Figure 11. Observed (solid black lines) and predicted (dashed lines) fishery proportions-atlength for the 2011 model (dashed black lines) and an equivalent SS3 model run (dashed blue lines) for females.


Figure 12. As for Figure 11, but for males.


Figure 13. (1 of 3) Observed (solid black lines) and predicted (dashed lines) fishery proportions-at-length for the 2011 model (dashed black lines) and an equivalent SS3 model run (dashed blue lines) for females.


Length (cm)
Figure 13, continued (part 2 of 2).


Figure 14. As for Figure 13, but for males (part 1 of 2).


Figure 14, continue (part 2 of 2 ).

