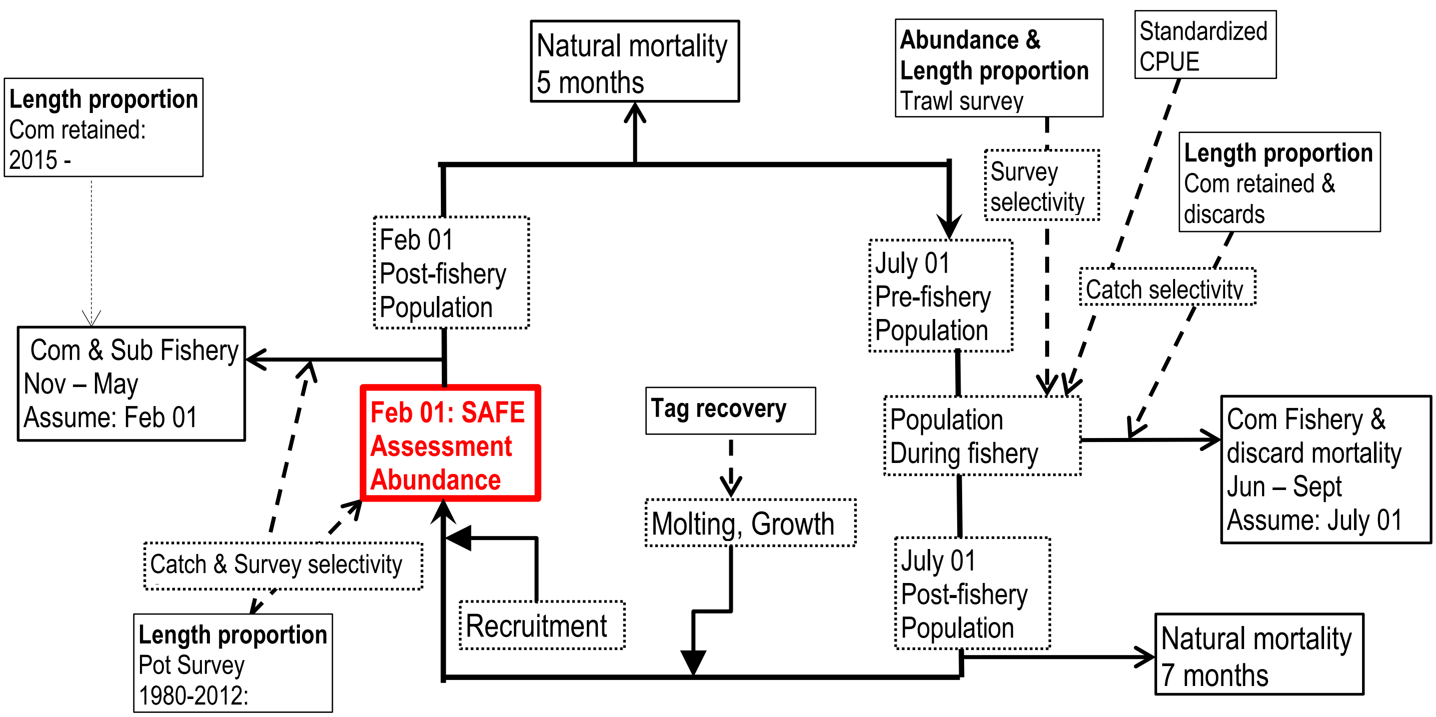
Appendix A. Description of the Norton Sound Red King Crab Model

a. Model description.

The model is an extension of the length-based model developed by Zheng et al. (1998) for Norton Sound red king crab. The model has 8 male length classes with model parameters estimated by the maximum likelihood method. The model estimates abundances of crab with CL ≥64 mm and with 10-mm length intervals (8 length classes, ≥134 mm) because few crab measuring less than 64 mm CL were caught during surveys or fisheries and there were relatively small sample sizes for trawl and winter pot surveys. The model treats newshell and oldshell male crab separately but assumes they have the same molting probability and natural mortality.

Norton Sound Red King Crab Modeling Scheme



Timeline of calendar events and crab modeling events:

- **Model year starts February 1st to January 31st of the following year.**
- **Initial Population Date: February 1st 1976, consisting of only newshell crab.**
- **All winter fishery catch occurs on February 1st**
- **All summer fishery catch occurs on July 1st**
- **During 1976-2004, all legal crab caught in Commercial are retained.**

- **During 2004-2005, only commercially marketable legal crab caught in Commercial crabs are retained (i.e., high grading of crab ≥ 5 in CW).**
- **Winter Subsistence fishery retains all mature crab.**
- **Molting and recruitment occur on July 1st**

Initial pre-fishery summer crab abundance on February 1st 1976:

Abundance of the initial pre-fishery population was assumed to consist of newshell crab to reduce the number of parameters, and estimated as

$$N_{w,1,1} = p_l e^{\log\_N_{76}} \tag{1}$$

where length proportion of the first year (p_l) was calculated as

$$\begin{aligned} p_l &= \frac{\exp(a_l)}{1+\sum_{l=1}^{n-1}\exp(a_l)} \text{ for } l = 1,..,n\text{-}1 \\ p_n &= 1 - \frac{\sum_{l=1}^{n-1}\exp(a_l)}{1+\sum_{l=1}^{n-1}\exp(a_l)} \end{aligned} \tag{2}$$

for model estimated parameters a_l.

Crab abundance on July 1st :

Summer (01 July) crab abundance of newshell and oldshell are of survivors of Winter (Feb 01) population from winter commercial and subsistence crab fisheries, and natural mortality from 01Feb to 01July.

$$\begin{aligned} N_{s,l,y} &= (N_{w,l,y} - C_{w,y}P_{w,n,l,y} - C_{p,t}P_{p,n,l,y} - D_{w,n,l,y} - D_{p,n,l,y})e^{-0.42M_l} \\ O_{s,l,y} &= (O_{w,l,y} - C_{w,y}P_{w,o,l,y} - C_{p,y}P_{p,o,l,y} - D_{w,o,l,y} - D_{p,o,l,y})e^{-0.42M_l} \end{aligned} \tag{3}$$

where

$N_{s,l,y}$, $O_{s,l,y}$: summer abundances of newshell and oldshell crab in length class l in year y ,
$N_{w,l,y}$, $O_{w,l,y}$: winter abundances of newshell and oldshell crab in length class l in year y,
$C_{w,t,y}$, $C_{p,t}$: total winter commercial and subsistence catches in year t,

$P_{w,n,l,y}$, $P_{w,o,l,y}$: Proportion of newshell and oldshell length class l crab in year y, harvested by winter commercial fishery,
$P_{p,n,l,y}$, $P_{p,o,l,y}$: Proportion of newshell and oldshell length class l crab in year y, harvested by winter subsistence fishery,
$D_{w,n,l,y}$, $D_{w,o,l,y}$: Discard mortality of newshell and oldshell length class l crab in winter commercial fishery in year y,
$D_{p,n,l,y}$, $D_{p,o,l,y}$: Discard mortality of newshell and oldshell length class l crab in winter subsistence fishery in year y,
M_l : instantaneous natural mortality in length class l,
0.42 : proportion of the year from Feb 1 to July 1 is 5 months.

Length proportion compositions of winter commercial retained catch ($P_{w,n,l,y}$, $P_{w,o,l,y}$) in year t were estimated as:

1976-2007

$$P_{w,n,l,y} = N_{w,l,y}S_{w,l}P_{\mathrm{lg},l} / \sum_{l=1}[(N_{w,l,y} + O_{w,l,y})S_{w,l}P_{\mathrm{lg},l}]$$

$$P_{w,o,l,y} = O_{w,l,y}S_{w,l}P_{\mathrm{lg},l} / \sum_{l=1}[(N_{w,l,y} + O_{w,l,y})S_{w,l}P_{\mathrm{lg},l}] \quad (4)$$

2008-present

$$P_{cw,n,l,y} = N_{w,l,t}S_{w,l}S_{wr,l} / \sum_{l}[(N_{w,l,y} + O_{w,l,y})S_{w,l}S_{wr,l}]$$

$$P_{cw,o,l,y} = O_{w,l,t}S_{w,l}S_{wr,l} / \sum_{l}[(N_{w,l,y} + O_{w,l,y})S_{w,l}S_{wr,l}]$$

where
$P_{lg,l}$: the proportion of legal males in length class l ,
$S_{w,l}$: Selectivity of winter fishery pot.
$S_{wr,l}$: Retention probability of winter fishery

In the above, we assumed that all legal crabs were retained during 1976-2007 periods, and high grading has occurred since 2008 season.

The subsistence fisheries do not have a size limit; however, immature crab (< 94 mm) are generally not retained. Thus, we assumed proportion of length composition l = 1 and 2 as 0, and estimated length compositions ($l \geq 3$) as follows

$$P_{p,n,l,y} = N_{w,l,y}S_{w,l} / \sum_{l=3}[(N_{w,l,y} + O_{w,l,y})S_{w,l}]$$

$$P_{p,o,l,y} = O_{w,l,y}S_{w,l} / \sum_{l=3}[(N_{w,l,y} + O_{w,l,y})S_{w,l}] \quad (5)$$

Crab abundance on Feb 1st :

The assessment model assumes that molting and growth occur immediately after summer fishery harvests, and that recruitment would occur between July 01 and Feb 01 of the next year. That is, the following events occur: (1) summery fishery, (2) summer fishery discards mortality, (3) molting and recruitment, and (4) natural mortality between July 01 and Feb 01. Those are formulated as follows:

Newshell Crab- Abundance of newshell crab of year t and length-class l ($N_{w,l,y}$) year-y consist of: (1) new and oldshell crab that survived the summer commercial fishery and molted, and (2) recruitment ($R_{l,y}$):

$$N_{w,l,y+1} = \sum_{l'=1}^{l'=l} G_{l',y}[(N_{s,l',y} + O_{s,l',y})e^{-y_c M_l} - C_{s,y-1}(P_{s,n,l',y} + P_{s,o,l',y}) - D_{l',y}]m_{l'} e^{-(0.58-y_c)M_l} + R_{l,y} \quad (6)$$

Oldshell Crab- Abundance of oldshell crabs of year y and length-class l ($O_{w,l,y}$) consists of the non-molting portion of survivors from the summer fishery:

$$O_{w,l,y+1} = [(N_{s,l,y} + O_{s,l,y})e^{-y_c M_l} - C_{s,y}(P_{s,n,l,y} + P_{s,o,l,y}) - D_{l,y}](1 - m_l)e^{-(0.58-y_c)M_l} \quad (7)$$

where

$G_{l',l}$: a growth matrix representing the expected proportion of crabs growing from length class l' to length class l
$C_{s,y}$: total summer catch in year y
$P_{s,n,l,y}$, $P_{s,o,l,y}$: proportion of summer catch for newshell and oldshell crab of length class l in year y,
$D_{l,y}$: summer discard mortality of length class l in year y,
m_l : molting probability of length class l,
y_c : the time in year from July 1 to the mid-point of the summer fishery,
0.58: Proportion of the year from July 1st to Feb 1st : 7 months = 0.58 year,
$R_{l,y}$: recruitment into length class l in year y.

Discards

Discards are crabs that were caught in summer and winter commercial and winter subsistence fisheries but were not retained.

Summer and winter commercial discards

In summer ($D_{l,t}$) and winter ($D_{w,n,l,t}$, $D_{w,o,l,t}$) commercial fisheries, sublegal males (<4.75 inch CW and <5.0 inch CW since 2008) are discarded. Those discarded crabs are subject to handling mortality. The number of discards was not directly observed, and thus was estimated from the model as: Observed Catch x (estimated abundance of crab that are not caught by commercial pot)/(estimated abundance of crab that are caught by commercial pot)

Model discard mortality in length-class l in year y from the summer and winter commercial pot fisheries is given by

$$D_{l,y} = C_{s,y} \frac{N_{s,l,y} S_{s,l} (1 - S_{r,n,l}) + O_{s,l,y} S_{s,l} (1 - S_{r,o,l})}{\sum_l (N_{s,l,y} S_{r,n,l} + O_{s,l,y} S_{r,o,l}) S_{s,l}} hm_s \tag{8}$$

1977 – 2007 2008 – 2022

$$D_{w,n,l,y} = C_{w,y} \frac{N_{w,l,y} S_{w,l} (1 - P_{lg,l})}{\sum_l (N_{w,l,y} + O_{w,l,y}) S_{w,l} P_{lg,l}} hm_w \qquad D_{w,n,l,y} = C_{w,t} \frac{N_{w,l,y} S_{w,l} (1 - S_{wr,l})}{\sum_l (N_{w,l,y} + O_{w,l,y}) S_{w,l} S_{wr,l}} hm_w \tag{9}$$

1977 – 2007 2008 – 2022

$$D_{w,o,l,y} = C_{w,y} \frac{O_{w,l,y} S_{w,l} (1 - P_{lg,l})}{\sum_l (N_{w,l,y} + O_{w,l,y}) S_{w,l} P_{lg,l}} hm_w \qquad D_{w,o,l,y} = C_{w,y} \frac{O_{w,l,y} S_{w,l} (1 - S_{wr,l})}{\sum_l (N_{w,l,y} + O_{w,l,y}) S_{w,l} S_{wr,l}} hm_w \tag{10}$$

where

hm_s: summer commercial handling mortality rate assumed to be 0.2,
hm_w: winter commercial handling mortality rate assumed to be 0.2,
$S_{s,l}$: Selectivity of the summer commercial fishery,
$S_{w,l}$: Selectivity of the winter commercial fishery,
$S_{r,l}$: Retention selectivity of the summer commercial fishery,
$S_{wr,l}$: Retention selectivity of the winter commercial fishery,

Winter subsistence discards

Discards (unretained) from the winter subsistence fishery are reported in a permit survey ($C_{d,y}$), though its size composition is unknown. We assumed that subsistence fishers discard all crabs of length classes 1 -2.

$$D_{p,n,l,y} = C_{d,y} \frac{N_{w,l,y} S_{w,l}}{\sum_{l=1}^{2} (N_{w,l,y} + O_{w,l,y}) S_{w,l}} hm_w \tag{11}$$

$$D_{p,o,l,y} = C_{d,y} \frac{O_{w,l,y} S_{w,l}}{\sum_{l=1}^{2} (N_{w,l,y} + O_{w,l,y}) S_{w,l}} hm_w \tag{12}$$

where

$C_{d,y}$: Winter subsistence discards

Recruitment

Recruitment of year *y*, R_t, is a stochastic process around the geometric mean, R_0:

$$R_y = R_0 e^{\tau_t}, \tau_y \sim N(0, \sigma_R^2) \tag{13}$$

R_t of the last year was assumed to be an average of previous 5 years: $R_y = (R_{y-1} + R_{y-2} + R_{y-3} + R_{y-4} + R_{y-5})/5$.

R_t was assumed to be newshell crab of immature (< 94 mm) length classes 1 to *r*:

$$R_{r,y} = p_r R_y \tag{14}$$

where p_r takes multinomial distribution, same as equation (2)

Molting Probability

Molting probability for length class *l*, m_l, was estimated as an inverse logistic function of length-class mid carapace length (*L*) and parameters (α, β) where β corresponds to L_{50}.

$$m_l = \frac{1}{1 + e^{\alpha(L-\beta)}} \tag{15}$$

Trawl net and summer commercial pot selectivity

Trawl and summer commercial pot selectivity was assumed to be a logistic function of mid-length-class, constrained to be 0.999 at the largest length-class (L_{max}):

$$S_l = \frac{1}{1 + e^{(\alpha(L_{\max} - L) + \ln(1/0.999 - 1))}} \tag{16}$$

Winter pot selectivity,

Winter pot selectivity was assumed to be a dome-shaped with logistic function of length-class mid carapace length (*L*) and parameters (α, β) where β corresponds to L_{50}.

$$S_{w,l} = \frac{1}{1+e^{\alpha(L-\beta)}} \tag{17}$$

Selectivity of the first 3 length classes $S_{w,s}$ (S= l_1, l_2, l_3) were individually estimated.

Retention probability: Winter commercial, summer commercial

Winter and summer commercial retention probability was assumed to be a logistic function of length-class mid carapace length (L) and parameters (α, β) where β corresponds to L_{50}.

$$S_{r,l} = \frac{1}{1+e^{\alpha(L-\beta)}} \tag{17}$$

Growth transition matrix

The growth matrix $G_{l',l}$ (the expected proportion of crab molting from length class l' to length class l) was assumed to be normally distributed:

$$G_{l',l} = \begin{cases} \dfrac{\int_{lm_l-h}^{lm_l+h} N(L \mid \mu_{l'}, \sigma^2)dL}{\sum_{l=1}^{n} \int_{lm_l-h}^{lm_l+h} N(L \mid \mu_{l'}, \sigma^2)dL} & \text{when } l \geq l' \\ 0 & \text{when } l < l' \end{cases} \tag{18}$$

where

$$N(x \mid \mu_{l'}, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(L-\mu_{l'})^2}{\sigma^2}\right)$$

$$lm_l = L_1 + st \cdot l$$

$$\mu_l = L_1 + \beta_0 + \beta_1 \cdot l$$

Observation model

Summer trawl survey abundance

Modeled trawl survey abundance of year y ($B_{st,y}$) is July 1st abundance subtracted by summer commercial fishery harvest occurring from July 1st to the mid-point of summer trawl survey, multiplied by natural mortality occurring between the mid-point of commercial fishery date and trawl survey date, and multiplied by trawl survey selectivity. For the first year (1976) trawl survey, the commercial fishery did not occur.

$$\hat{B}_{st,y} = \sum_{l}[(N_{s,l,y} + O_{s,l,y})e^{-y_c M_l} - C_{s,y}P_{c,y}(P_{s,n,l,y} + P_{s,o,l,y})]e^{-(y_{st}-y_c)M_l}S_{st,l} \quad (19)$$

where
y_{st}: the time in year from July 1 to the mid-point of the summer trawl survey,
y_c: the time in year from July 1 to the mid-point for the catch before the survey, ($y_{st} > y_c$: Trawl survey starts after opening of commercial fisheries),
$P_{c,y}$: the proportion of summer commercial crab harvested before the mid-point of trawl survey date.
$S_{st,l}$: Selectivity of the trawl survey.

Winter pot survey CPUE ***(depleted)***

Winter pot survey cpue (f_{wy}) was calculated with catchability coefficient q and exploitable abundance:

$$\hat{f}_{wy} = q_w \sum_{l}[(N_{w,l,y} + O_{w,l,y})S_{w,l}] \quad (20)$$

Summer commercial CPUE

Summer commercial fishing CPUE (f_y) was calculated as a product of catchability coefficient q and mean exploitable abundance, A_t minus one half of summer catch, C_t:

$$\hat{f}_y = q_i(A_y - 0.5C_y) \quad (21)$$

Because the fishing fleet and pot limit configuration changed in 1993, q_1 is for fishing efforts before 1993, q_2 is from 1994 to present.

Where A_t is exploitable legal abundance in year t, estimated as

$$A_y = \sum_{l}[(N_{s,l,y} + O_{s,l,y})S_{s,l}S_{r,l}] \quad (22)$$

Summer pot survey abundance (**depleted**)

Abundance of t-th year pot survey was estimated as

$$\hat{B}_{p,y} = \sum_{l}[(N_{s,l,y} + O_{s,l,y})e^{-y_p M_l}]S_{p,l} \quad (23)$$

Where

y_p: the time in year from July 1 to the mid-point of the summer pot survey.

Length composition

Summer commercial retained catch

Length compositions of the summer commercial catch for new and old shell crabs $P_{s,n,l,y}$ and $P_{s,o,l,y}$, were modeled based on the summer population, selectivity, and retention probability

$$\hat{P}_{s,n,l,y} = N_{s,l,y} S_{s,l} S_{r,o,l} / A_t \\ \hat{P}_{s,o,l,y} = O_{s,l,y} S_{s,l} S_{r,o,l} / A_t \quad (24)$$

Retention probability is separated into two periods: 1977–2007 and 2008–2020 indicating before and after the start of high grading.

Summer commercial fishery discards (1977-1993)

Prior to 1993, Observer survey data contained length-shell composition of only discards.

Length/shell compositions of observer discards were modeled as

$$\hat{P}_{b,n,l,y} = N_{s,l,y} S_{s,l} (1 - S_{r,n,l}) / \sum_l [N_{s,l,y}(1 - S_{r,n,l}) + O_{s,l,y}(1 - S_{r,o,l})] S_{s,l} \\ \hat{P}_{b,o,l,y} = O_{s,l,y} S_{s,l} (1 - S_{r,o,l}) / \sum_l [N_{s,l,y}(1 - S_{r,n,l}) + O_{s,l,y}(1 - S_{r,o,l})] S_{s,l} \quad (25)$$

Summer commercial fishery total catch (2008-present)

The 2012–2019 Observer survey had total as well as retained and discard length-shell composition, and total catch length-shell composition was fitted.

Length/shell compositions of observer total catch was modeled as

$$\hat{P}_{t,n,l,y} = N_{s,l,y} S_{s,l} / \sum_l [(N_{s,l,y} + O_{s,l,y}) S_{s,l}] \\ \hat{P}_{t,o,l,y} = O_{s,l,y} S_{s,l} / \sum_l [(N_{s,l,y} + O_{s,l,y}) S_{s,l}] \quad (26)$$

Summer trawl survey

Proportions of newshell and oldshell crab, $P_{st,n,l,y}$ and $P_{st,o,l,y}$ were given by

$$\hat{P}_{st,n,l,y} = \frac{[N_{s,l,y} e^{-y_c M_l} - C_{s,y} \ P_{c,y} \hat{P}_{s,n,l',y}] e^{-(y_{st}-y_c)M_l} S_{st,l}}{\sum_l [(N_{s,l,y} + O_{s,l,y}) e^{-y_c M_l} - C_{s,y} P_{c,y} (\hat{P}_{s,n,l',y} + \hat{P}_{s,o,l',y})] e^{-(y_{st}-y_c)M_l} S_{st,l}}$$
$$\hat{P}_{st,o,l,y} = \frac{[O_{s,l,y} e^{-y_c M_l} - C_{s,y} \ \hat{P}_{s,o,l',y} \ P_{c,y}] e^{-(y_{st}-y_c)M_l} S_{st,l}}{\sum_l [(N_{s,l,y} + O_{s,l,y}) e^{-y_c M_l} - C_{s,y} P_{c,y} (\hat{P}_{s,n,l,y} + \hat{P}_{s,o,l,y})] e^{-(y_{st}-y_c)M_l} S_{st,l}} \quad (27)$$

Winter pot survey

Winter pot survey length compositions for newshell and oldshell crab, $P_{sw,n,l,t}$ and $P_{sw,o,l,t}$ ($l \geq 1$) were calculated as

$$\hat{P}_{sw,n,l,y} = N_{w,l,y} S_{w,l} \Big/ \sum_l [(N_{w,l,y} + O_{w,l,y}) S_{w,l}]$$
$$\hat{P}_{sw,o,l,y} = O_{w,l,y} S_{w,l} \Big/ \sum_l [(N_{w,l,y} + O_{w,l,y}) S_{w,l}] \quad (28)$$

Winter commercial retained

Winter commercial retained length compositions for newshell and oldshell crab, $P_{cw,n,l,t}$ and $P_{cw,o,l,t}$ ($l \geq 1$) were calculated as

$$\hat{P}_{cw,n,l,y} = N_{w,l,y} S_{w,l} S_{wr,l} \Big/ \sum_l [(N_{w,l,y} + O_{w,l,y}) S_{w,l} S_{wr,l}]$$
$$\hat{P}_{cw,o,l,y} = O_{w,l,y} S_{w,l} S_{wr,l} \Big/ \sum_l [(N_{w,l,y} + O_{w,l,y}) S_{w,l} S_{wr,l}] \quad (29)$$

*Spring Pot survey 2012-2015 (**depleted**)*

Spring pot survey length compositions for newshell and oldshell crab, $P_{sw,n,l,t}$ and $P_{sw,o,l,t}$ ($l \geq 1$) were assumed to be similar to crab population caught by winter pot survey

$$\hat{P}_{sp,n,l,y} = N_{s,l,y} S_{w,l} \Big/ \sum_l [(N_{s,l,y} + O_{s,l,y}) S_{w,l}]$$
$$\hat{P}_{sp,o,l,y} = O_{s,l,y} S_{w,l} \Big/ \sum_l [(N_{s,l,y} + O_{s,l,y}) S_{w,l}] \quad (30)$$

Estimates of tag recovery

The proportion of released tagged length class *l'* crab recovered after *t-th* year with length class of *l* by a fishery of *s-th* selectivity (S_l) was assumed to be proportional to the growth matrix, catch selectivity, and molting probability (m_l) as

$$\hat{P}_{l',l,t,s} = \frac{S_l \cdot [X^t]_{l',l}}{\sum_{l=1}^{n} S_l \cdot [X^t]_{l',l}} \tag{31}$$

where X is a molting probability adjusted growth matrix with each component consisting of

$$X_{l',l} = \begin{cases} m_{l'} \cdot G_{l',l} \quad \text{when } l' \neq l \\ m_l \cdot G_{l',l} + (1\text{-}m_i) \;\; \text{when } l' = l \end{cases} \tag{32}$$

c. Likelihood components.

Under assumptions that measurement errors of annual total survey abundances and summer commercial fishing efforts follow lognormal distributions, and each type of length composition has a multinomial error structure (Fournier and Archibald 1982; Methot 1989), the log-likelihood function is

$$
\begin{aligned}
&\sum_{i=1}^{i=4}\sum_{y=1}^{y=n_i} K_{i,t}\left[\sum_{l=1}^{l=n} P_{i,l,y}\ln(\hat{P}_{i,l,y}+\kappa) - \sum_{l=1}^{l=n} P_{i,l,y}\ln(P_{i,l,y}+\kappa)\right] \\
&-\sum_{y=1}^{y=n_i}\frac{[\ln(q\cdot\hat{B}_{i,y})-\ln(B_{i,y})]^2}{2\cdot ln(CV_{i,y}^2+1)} \\
&-\sum_{y=1}^{y=n_i}\left[\frac{\ln\left[\ln(CV_y^2+1)+w_t\right]}{2}+\frac{[\ln(\hat{f}_y+\kappa)-\ln(f_y+\kappa)]^2}{2\cdot\left[\ln(CV_y^2+1)+w_t\right]}\right] \qquad (32) \\
&-\sum_{t=1}\frac{\tau_t^2}{2\cdot SDR^2} \\
&+W\sum_{s=1}^{s=2}\sum_{y=1}^{y=3}\sum_{l'=1}^{l'=n} K_{l',y,s}\left[\sum_{l=1}^{l=n} P_{l',l,y}\ln(\hat{P}_{l',l,y,s}+\kappa) - \sum_{l=1}^{l=n} P_{l',l,t}\ln(P_{l',l,y,s}+\kappa)\right]
\end{aligned}
$$

where

i: length/shell compositions of:

1 triennial summer trawl survey,
2 annual winter pot survey,
3 summer commercial fishery retained,
4 summer commercial observer discards or total catch,
5 winter commercial fishery retained.

$K_{i,y}$: the effective sample size of length/shell compositions for data set i in year y,
$P_{i,l,y}$: observed and estimated length compositions for data set i, length class l, and year y.
κ : a constant equal to 0.0001,
CV: coefficient of variation for the survey abundance,
$B_{j,y}$: observed and estimated annual total abundances for data set i and year y,
F_y: observed and estimated summer fishery CPUE,
w^2_t: extra variance factor,
SDR: Standard deviation of recruitment = 0.5,
$K_{l',y}$: sample size of length class l' released and recovered after y-th in year,
$P_{l',l,y,s}$: observed and estimated proportion of tagged crab released at length l' and recaptured at length l, after y-th year by commercial fishery pot selectivity s,
W: weighting for the tagging survey likelihood = 0.5

b. Software used: AD Model Builder (Fournier et al. 2012).

d. Out of model parameter estimation framework:

i. Parameters Estimated Independently

M: Natural mortality

Natural mortality (M = 0.18) was based on an assumed maximum age, t_{max}, and the 1% rule (Zheng 2005):

$$M = -\ln(p)/t_{\max},$$

where p is the proportion of animals that reach the maximum age and is assumed to be 0.01 for the 1% rule (Shepherd and Breen 1992, Clarke et al. 2003). The maximum age of 25, which was used to estimate M for U.S. federal overfishing limits for red king crab stocks results in an estimated M of 0.18. Among the 199 recovered crabs from the tagging returns during 1991-2007 in Norton Sound, the longest time at liberty was 6 years and 4 months from a crab tagged at 85 mm CL. The crab was below the mature size and was likely less than 6 years old when tagged. Therefore, the maximum age from tagging data is about 12, which does not support the maximum age of 25 chosen by the CPT.

Proportion of Legal-sized crab

Proportions of legal males (CW > 4.75 inches) by length group were estimated from the ADF&G trawl data 1996-2021.

e. Definition of model outputs.

i. Mature male biomass (MMB) is on **February 1st** and is consisting of the biomass of male crab in length classes 4 to 8

$$MMB = \sum_{l=4} (N_{w,l,} + O_{w,l,}) wm_l$$

wm_l: mean weight of each length class.

ii. Projected legal male biomass subject to winter and summer fishery OFL was calculated as winter biomass times summer commercial pot selectivity times proportion of legal crab. Though fishery size selectivity differs between winter and summer commercial, both fisheries were assumed to have the same selectivity because winter fishery is very small compared to summer fishery.

$$B_w = \sum_{l} (N_{w,l,} + O_{w,l,}) S_{s,l} S_{r,l} wm_l$$

iii. Recruitment: the number of males in length classes 1, 2, and 3.

f. OFL

The Norton Sound red king crab fishery consists of two distinct fisheries: winter and summer. The two fisheries are discontinuous with 5 months between the two fisheries during which natural mortalities occur. To incorporate this, the CPT in 2016 recommended the following formula:

$$OFL = \text{Winter harvest (Hw)} + \text{ Summer harvest (Hs)} \tag{1}$$

And

$$p = \frac{Hw}{OFL} \tag{2}$$

Where p is a specific proportion of winter crab harvest to total (winter + summer) harvest

At given fishery mortality (F_{OFL}), Winter harvest is a fishing mortality

$$Hw = (1 - e^{-x \cdot F})B_w \tag{3}$$

$$Hs = (1 - e^{-(1-x) \cdot F})B_s \tag{4}$$

where B_s is a summer crab biomass after winter fishery and x ($0 \le x \le 1$) is a fraction that satisfies equation (2).

Since B_s is a summer crab biomass after winter fishery and 5 months of natural morality, ($e^{-0.42M}$)

$$\begin{aligned} B_s &= (B_w - Hw)e^{-0.42M} \\ &= (B_w - (1 - e^{-x \cdot F})B_w)e^{-0.42M} \\ &= B_w e^{-x \cdot F - 0.42M} \end{aligned} \tag{5}$$

Substituting *0.42M* to *m*, summer harvest is

$$\begin{aligned} Hs &= (1 - e^{-(1-x) \cdot F})B_s \\ &= (1 - e^{-(1-x) \cdot F})B_w e^{-x \cdot F - m} = (e^{-(x \cdot F + m)} - e^{-(F+m)})B_w \end{aligned} \tag{6}$$

Thus, OFL is

$$\begin{aligned} OFL &= Hw + Hs = (1 - e^{-xF})B_w + (e^{-(x \cdot F + m)} - e^{-(F+m)})B_w \\ &= (1 - e^{-xF} + e^{-(xF+m) \cdot} - e^{-(F+m) \cdot})B_w \\ &= [1 - e^{-(F+m) \cdot} - (1 - e^{-m \cdot})e^{-xF \cdot}]B_w \end{aligned} \tag{7}$$

Combining equations (2) and (7),

$$p = \frac{Hw}{OFL_r} = \frac{(1 - e^{-xF})B_w}{[1 - e^{-(F+m) \cdot} - (1 - e^{-m \cdot})e^{-xF \cdot}]B_w} \tag{8}$$

Solving equation (8) for x

$$(1-e^{-xF}) = p[1-e^{-(F+m)\cdot}-(1-e^{-m\cdot})e^{-xF\cdot}] \tag{9}$$

$$e^{-xF} - p(1-e^{-m\cdot})e^{-xF\cdot} = 1- p[1-e^{-(F+m)\cdot}]$$

$$[1-p(1-e^{-m\cdot})]e^{-xF\cdot} = 1- p[1-e^{-(F+m)\cdot}]$$

$$e^{-xF\cdot} = \frac{1- p[1-e^{-(F+m)\cdot}]}{1-p(1-e^{-m\cdot})}$$

Combining equations (7) and (9), and substituting back,

revised retained OFL is

$$OFL = B_w\left(1-e^{-(F_{OFL}+0,42M)}-(1-e^{-0.42M})\left(\frac{1-p(1-e^{-(F_{OFL}+0.42M)})}{1-p(1-e^{-0.42M})}\right)\right)$$

Further combining equations (3) and (9), winter fishery harvest rate (Fw) is

$$Fw = (1-e^{-x\cdot F}) = 1-\frac{1- p[1-e^{-(F+m)\cdot}]}{1-p(1-e^{-m\cdot})} = \frac{1-p(1-e^{-m\cdot})-1+ p[1-e^{-(F+m)\cdot}]}{1-p(1-e^{-m\cdot})} \tag{10}$$

$$= \frac{p(e^{-m\cdot} - e^{-(F+m)\cdot})}{1-p(1-e^{-m\cdot})} = \frac{p(1- e^{-F\cdot})e^{-0.42M\cdot}}{1-p(1-e^{-0.42M\cdot})}$$

Summer fishery harvest rate (Fs) is

$$Fs = (e^{-(x\cdot F+m)}-e^{-(F+m)}) = (e^{-x\cdot F}-e^{-F})e^{-m} \tag{11}$$

$$= \left(\frac{1- p[1-e^{-(F+m)\cdot}]}{1-p(1-e^{-m\cdot})}-e^{-F}\right)e^{-m}$$

$$= \left(\frac{1- p[1-e^{-(F+m)\cdot}]-e^{-F}+p(e^{-F}-e^{-(F+m\cdot)})}{1-p(1-e^{-m\cdot})}\right)e^{-m}$$

$$= \left(\frac{1- p+ pe^{-(F+m)\cdot}-e^{-F}+pe^{-F}-pe^{-(F+m\cdot)}}{1-p(1-e^{-m\cdot})}\right)e^{-m}$$

$$= \frac{(1- p)(1-e^{-F})e^{-m}}{1-p(1-e^{-m\cdot})} = \frac{(1- p)(1-e^{-F})e^{-0.24M}}{1-p(1-e^{-0.24M\cdot})}$$